AN INTRODUCTION TO HIGHLY INELASTIC LEPTON SCATTERING

AND RELATED PROCESSES

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INTRODUCTION

This talk contains little which is new and is not intended for experts. It is an introductory review; therefore the derivation of even some quite well-known results will be indicated but technicalities will be relegated to footnotes, or ignored.

Electron or neutrino scattering is said to be highly inelastic if the 'mass' and the lab. energy of the virtual photon or current (W meson?) which mediates the interaction are large compared to the mass of the target. In such extreme conditions the proton is violently broken up and (crudely speaking) we might hope to learn what is inside it. In space-time language, energetic heavy virtual photons (currents) probe the structure of the target at short distances.

The simplest possible highly inelastic lepton scattering experiments are those in which only the final lepton is observed, and this is all that has been done up to now in the case of incident electrons\(^1\). We shall only consider this simple situation. More sophisticated observations will require more sophisticated theories.

Section 1 is devoted to highly inelastic electron scattering; after a review of the kinematics, various models are discussed. In Section 2 highly inelastic neutrino and antineutrino scattering are discussed following the same plan. In Section 3 we consider attempts to extend the models and ideas to related processes involving heavy time-like photons, such as \( pp \rightarrow \mu^+\mu^- \ldots \) and \( e^+e^- \rightarrow p \ldots \). Finally, in Section 4 some features of the different models considered are compared. Occasional reference to Table 2 (where the comparison of models is summarized) may make it easier to read this paper.

1. INELASTIC ELECTRON SCATTERING

1.1 Kinematics

We shall consider the process in which an electron scatters from a nucleon and only the final electron is detected. Treating the electromagnetic interaction to lowest order the process is represented by the Feynman diagram:
where $E(E')$ is the lab. energy of the incident (final) electron, $\theta$ the lab. scattering angle and $k(k')$, $P$, $q$ and $P_F$ the four momenta of the initial (final) electron, the target nucleon, the virtual photon and the undetected hadrons respectively.

Applying Feynman's rules the total cross-section may be written$^2$:

$$
\sigma \sim \sum_F \sum_{\nu} \left| J_{\mu}^L(0) \frac{i}{q^2} < F | J_{\nu}^L(0) | P > \right|^2 \delta^4(k' + P_F - k - p) \\
\sim \frac{1}{q^4} m_{\mu\nu} - W_{\mu\nu}
$$

where $J_{\mu}^L(0) = \bar{u}(k') \gamma_{\mu} u(k)$ is the electron current, $J_{\mu}$ the hadron current, the sum over $F$ runs over all final states, $\sum_{\nu}$ indicates an average over the initial spins and$^3$:

$$
m_{\mu\nu} = \frac{1}{2} \sum_{\text{spins}} j_{\mu} j_{\nu}^* = \frac{1}{2} \text{Tr}(\mathbb{I} + m_e) \gamma_{\mu}(k' + m_e) \gamma_{\nu}$$

$$
= 2(k_{\mu\nu} k_{\nu\mu} - g_{\mu\nu} k \cdot k').
$$

$$
W_{\mu\nu} = \frac{1}{2} \sum_F < P | J_{\mu}^+(0) | F > < F | J_{\nu}^+(0) | P > (2\pi)^3 \delta^4(p + P - P_F)
$$

$$
= \frac{1}{2} \sum \frac{d^4 x}{4\pi} e^{i q \cdot x} < P | J_{\mu}^+(x) J_{\nu}^+(0) | P >
$$

$$
q = k - k' = P_F - P.
$$
In calculating $m_{\mu\nu}$ we have put $m_e = 0$; an approximation which will be used henceforth. Note that $W_{\mu\nu}$ can be written

$$W_{\mu\nu} = \int \frac{d^4x}{4\pi} e^{iq^\ast x} \langle P | [J^+_\mu(x), J^-_\nu(0)] | P >$$

$$= \frac{1}{2\pi} \text{Im} T_{\mu\nu}$$

$$(3)$$

$$T_{\mu\nu} = \int d^4x \ e^{iq^\ast x} \theta(x_0) \langle P | [J^+_\mu(x), J^-_\nu(0)] | P >$$

(the average over spins being understood from now on). The first step is permissible because $q_0 > 0$; the extra term added in the commutator only gets contributions from intermediate states with lab. energy $[M_P - (E - E')] < M_P$ and therefore vanishes. $T_{\mu\nu}$ is the amplitude involved in the forward Compton scattering of a virtual photon $[\gamma(q) + P + \gamma(q') + P]$; Eq. (3) is just the optical theorem - the imaginary part of the forward Compton amplitude gives the absorption cross-section for the virtual photon.

Since the electromagnetic current is conserved:

$$q_\mu W_{\mu\nu} = W_{\mu\nu} q_\nu = 0 .$$

The vectors $q_\mu$ and $P_\mu$ and the tensors $g_{\mu\nu}$ and $\epsilon_{\mu\nu\alpha\beta} q_\alpha P_\beta$ are at our disposal. Since $W_{\mu\nu}$ has positive parity the most general form we can construct which satisfies the conservation conditions is:

$$W_{\mu\nu} = - (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) W_1 + \frac{1}{M^2} (P_\mu - q_\mu \frac{q^\ast P}{q^\ast q} P_\nu - q_\nu \frac{q^\ast P}{q^\ast q}) W_2$$

$$(4)$$

where $W_{1,2}$ are functions of the only independent Lorentz scalars available: $\nu = q^\ast P = M(E - E')$ and $q^2 = -4E' \sin^2 \theta/2$ (we shall also use $Q^2 = -q^2$).

$W_{\mu\nu}$ must be finite at $q^2 = 0$ (cf. Eq. (2)); Eq. (4) therefore implies that:

$$W_2 \sim 0(q^2) \quad \text{as} \quad q^2 \to 0$$

$$(5)$$

$$\left( W_1 + \frac{(q^\ast P)^2}{M^2 q^2} W_2 \right) \sim 0(q^2) .$$
Combining Eqs. (1) and (4) and putting in all the correct factors gives

\[ \frac{\pi}{MEET} \frac{d^2 \sigma}{d\Omega dE'} = \frac{d^2 \sigma}{dq^2 |d\nu} = \frac{4\pi q^2}{q^2 M^2} \frac{E'}{E} \cos^2 \frac{\theta}{2} W_2 + 2 \sin^2 \frac{\theta}{2} W_1. \] (6)

(This expression is not quite as singular as it looks at \( q^2 = 0 \) since \( \sin^2 \theta/2 \propto q^2 \) and \( W_2 \propto q^2 \) as \( q^2 \to 0 \)). If follows from Eq. (6) that the \( W_i \) must be real (this can also be seen from the definition of \( W_{\mu \nu} \)). By considering \( W_{\mu \nu}^* \), and using the reality of the \( W_i \) and the fact that \( J_\mu \), which represents a classical observable, is hermitian, we obtain the crossing relation

\[ W_i(\nu, q^2) = -W_i(-\nu, q^2). \] (7)

It is useful to define the total absorption cross-sections for longitudinal and transverse virtual photons, \( \sigma_L \) and \( \sigma_T \). First we define normalized polarization vectors \( \epsilon^L_\mu \) for the virtual photons, which satisfy the gauge condition \( q_\mu \epsilon^L_\mu = 0 \):

\[ \epsilon^L_\mu = \frac{1}{\sqrt{-q^2}} \left( q_3, 0, 0, q_0 \right) \]

\[ \epsilon^{T+}_\mu = \frac{1}{\sqrt{2}} \left( 0, 1, \pm i, 0 \right) \left[ q_\mu = (q_0, 0, 0, q_3) \right]. \]

Contracting \( \epsilon^{\mu}_\mu \) with \( W_{\mu \nu} \) and putting in the correct factors gives

\[ \sigma_T = \frac{4\pi^2 \alpha W_1}{\nu + q^2/2} \]

\[ \sigma_L = \frac{4\pi^2 \alpha}{\nu + q^2/2} \left[ W_2 \left( 1 - \nu^2/M^2 q^2 \right) - W_1 \right] \] (8)

where (by convention) we have used the flux factor for a real photon which gives a final state with the same invariant mass \( (q + P)^2 \).

Note that it follows from Eq. (5) that \( \sigma_L \propto q^2 \) as \( q^2 \to 0 \), which reflects the fact that real longitudinal photons do not exist; \( \sigma_T(q^2 = 0) \) is the total photoabsorption cross-section for real photons.
Since $\sigma_T$ and $\sigma_L$ are $> 0$, we see that:

$$W_2(1 - \nu^2/q^2M^2) \geq W_1 \geq 0 .$$

(9)

Before passing from kinematics to physics it is convenient to consider the kinematic diagram in Fig. 1. Note that the "missing mass" $M^*$ is given by

$$M^* = (q + P)^2 = -q^2 + 2\nu + M^2.$$  

For elastic scattering $M^* = M$ (line I). Single $\pi$ production gives the lightest inelastic state, with a threshold at $M^* = M + M_\pi$ (line II); lines of fixed missing mass (e.g. resonances) are just displaced parallel to this (line III). The experiments are usually carried out at fixed incident energy and scattering angle (line IV); note that to reach large $\nu$ and $q^2$ experiments are needed at large $E$ and large angles.

1.2 Scale invariance and the data

Bjorken argued$^5$) that the limits of $W_1$ and $\nu W_2$ exist as $\nu$ and $|q^2|$ tend to infinity with their ratio fixed; i.e.

$$W_1(\nu, q^2) \xrightarrow{\nu \to \infty} F_1(\omega)$$

$$\frac{\nu W_2}{\omega} (\nu, q^2) \xrightarrow{\nu \to \infty} F_2(\omega)$$

(10)

$$\omega = \frac{2\nu}{-q^2} = 2\nu/q^2 .$$

This hypothesis is known as scale invariance. To understand why, consider a theory in which all masses and coupling constants with dimensions are zero, so that no natural scale is defined (and the theory is invariant under a scale transformation of all coordinates and momenta $x \to \lambda x$, $P \to \frac{1}{\lambda} P$). Since $W_1$ and $W_2$ are dimensionless $[\text{cf. Eq. (6)}]$, Eq. (4) shows that in such theories $W_1$ can only be a function of $\nu/q^2$ while $(1/M^2)W_2$ must be replaced by $1/\nu$ times a function of $\nu/q^2$; this is just the limiting behaviour in Eq. (10).

In the real world masses are not zero but one might imagine that they become irrelevant as $\nu$ and $Q^2 \to \infty$, which would give Bjorken's result \[\text{[Eq. (10)]}\]. We would then expect the limit in Eq. (10) to be reached
when $\nu >>$ the largest relevant mass, which could be very large as heavy resonances exist and the hadronic spectrum may be unbounded. However, the form factors for producing resonances fall rapidly with $Q^2$ so that individual resonances could be irrelevant at large $Q^2$ if the non-resonant background falls less rapidly.

The data will be discussed by Brasse at this meeting, so that we will only briefly review a few points here:

1) $\sigma/\sigma_{\text{Mott}} = W_2 + 2W_1 \tan^2 \theta/2$ is shown in Fig. 2 plotted against $Q^2$ for fixed missing mass $W$ \textsuperscript{6).} Note the enormous difference between the inelastic and elastic form factor, which is also shown.

2) It is customary and convenient to discuss the data in terms of the two independent functions $\nu W_2(\nu, Q^2)$ and $R(\nu, Q^2) = \sigma_T/\sigma_L$ (rather than $W_1$ and $W_2$). In the region investigated up to now, $R$ seems to be rather independent of $\nu$ and $Q^2$ and $= 0.2 \pm 0.2$ \textsuperscript{7,8).}

3) $\nu W_2$ (measured in the experiment at 10°) is plotted against $\omega$ in Fig. 3 \textsuperscript{6).} Since it is now known that $R$ is small, the lower curve is the relevant one. Excluding a few points with low $Q^2$ $[Q^2 = 4E^2 \sin^2 \theta/2/(1 + 8E\omega \sin^2 \theta/2)]$, the data do seem to lie on a single curve, in agreement with the hypothesis of scale invariance.

4) There exists data up to $\omega \sim 40$ and there is an indication that $\nu W_2$ may be falling with $\omega$ \textsuperscript{6).} However, the points at large $\omega$ correspond to small $Q^2$ so that the limiting curve could be higher, and $F_2(\omega) \propto \omega$ const. is not excluded.

(The latest preliminary SLAC-MIT data are given by R.E. Taylor in Proc. Conf. on Expectations for Particle Reactions at the New Accelerators, Wisconsin report 1970).

We shall assume from now on that $W_1$ and $\nu W_2$ do tend to scale invariant limits with $R \approx 0$. It should be emphasized that this is not proved by the data \textsuperscript{[a point stressed by Nauenberg\textsuperscript{9)]}. However, the data certainly encourage speculation on the very interesting hypothesis of scale invariance.
1.3 Models

We shall now discuss some of the models proposed for highly inelastic electron scattering\(^{10}\). We begin with the most concrete models and later mention more abstract considerations whose content is, perhaps, more easily assimilated if simple models are kept in mind.

**The parton model\(^{11,12,13}\)**

The parton model will be discussed by Jengo at this meeting. However, since the author is enamoured of its heuristic value, a lengthy discussion of the model is included in the written version of this talk.

The parton model is based on the following arguments and assumptions:

1) As \(E \to \infty\), in the electron-proton centre of mass system \(|\vec{P}| \to \infty\).

Therefore, because of relativistic time dilation, the lifetime of the virtual states of the proton \(\tau_L\) also \(\to \infty\). However, the time of interaction \(t \propto 1/\sqrt{Q^2}\) so that in the limit of large \(Q^2\), \(t \propto \tau_L\); the photon 'sees' the proton 'frozen' in one virtual state during the interaction. Since the virtual state is long lived the corresponding energy denominators are small; the virtual particles (or 'partons') are nearly real and it is assumed that they can be treated as free particles during the interaction.

2) The momentum of the partons perpendicular to \(\vec{P}\) is supposed to be small (\(|\vec{P}_T| \ll |\vec{P}|\)). This is reasonable since we know that if the proton is broken up by collision with another hadron the transverse momenta of the secondaries are small. (Furthermore, if the \(|\vec{P}_T|\) are cut off, the probability of 'putting the proton together again' will be small when a photon with large \(|\vec{q}_T|\) is absorbed by a constituent. Since \(|\vec{q}_T| \sim \sqrt{Q^2}\) this implies a rapidly decreasing elastic form factor).

It follows from assumption 2 that the scattering from the constituents is incoherent as \(Q^2 \to \infty\); the amplitude for the process in which the virtual photon is absorbed by parton 'a' leaving 'b', 'c'... with small transverse momenta cannot interfere with the amplitude for the process in which the photon is absorbed by parton 'b' leaving 'a', 'c'... with small transverse momenta.
Hence we have the fundamental prescription of the parton model: in the deep inelastic region the proton behaves like a free gas of 'bare' constituents or partons - the inelastic cross-section is obtained by adding incoherently the cross-section for scattering from each of the partons (which, being the constituents of virtual states, are just the bare particles of a field theory).

The situation is easily visualized:

\[
\text{proton} \quad \text{'}bare' \text{ partons} \quad \text{'}dressed' \text{ hadrons}
\]

We can now begin to calculate \( W_2 \) for example, by adding up the \( W_2 \)'s from each parton configuration:

\[
W_2 = \sum_N P_N \sum_{i=1}^N \int f_1^N(x) W_2^i(x,\nu,q^2) dx
\]

where:

\[
P_N \quad - \text{probability that there are } N \text{ partons}
\]

\[
i \quad - \text{labels partons}
\]

\[
f_1^N(x) \quad - \text{probability of parton } i \text{ having a longitudinal momentum } x|\vec{P}|.
\]

\[
\left( \sum_N P_N = 1, \int f_1^N(x)dx = 1 \right).
\]

Note that the final state interaction which 'dresses' the partons has been ignored. It turns one complete set of states into another and is therefore unity provided the states have the same energy; this is approximately true as the partons are assumed to be 'nearly free' in the \( |\vec{P}| \to \infty \) frame.
The \(i\)th parton has momentum \(\not{p}_i = x_i \not{P} + \not{p}_{iT}\). Since \(\not{p}_{iT} \ll \not{P}\) by assumption, \(P_{i0} = x_i P_0\) and therefore \(P_{i,\mu} = x_i P_{\mu}\) as \(\not{P} \to \infty\). The partons are almost free (assumption 1) and therefore \(W_2\) is proportional to \(\delta(q + P_i)^2 - m_i^2 \approx \delta(q^2 + 2x_i\nu)\). \(W_2\) is just the charge form factor of the parton so that, whatever the spin\(^{15}\):

\[
W_2^i = 2x_i Q_i^2 \delta(q^2 + 2x_i\nu) \quad (12)
\]

Substituting this in Eq. (11) gives:

\[
\nu W_2 = \sum_N P_N \sum_i Q_i^2 f_i^N (Q^2/2\nu) (Q^2/2\nu)
\]

\[
= \frac{1}{\omega} \sum_N P_N \sum_i Q_i^2 f_i^N (1/\omega)
\]

\[
(\omega = 2\nu/Q^2) \quad (13)
\]

i.e. parton models automatically have scale invariance because of the mass-shell \(\delta\) function and the assumption that partons are point-like.

The other form factor \(R(\nu, q^2) = \sigma_L/\sigma_T\) depends on the spin of the parton. Consider the parton - photon interaction in the Breit frame:

\[
\begin{align*}
\vec{q} & \rightarrow \vec{p} = -\vec{q}/2 \\
\vec{q} & \rightarrow \vec{p}' = \vec{q}/2
\end{align*}
\]

If the parton has spin zero its helicity is zero before and after the interaction and therefore \(\sigma_T = 0, R = \infty\) (this result is rather obvious as spinless partons have only a charge form factor). If the parton has spin \(\frac{1}{2}\) its helicity is unchanged by the electromagnetic current when it is highly relativistic but, since its direction is reversed, the photon must impart one unit of helicity; therefore \(\sigma_L = 0, R = 0\) in this case\(^{16}\). For partons with higher spin intermediate results obtain for \(R\).

The experimental result \(R = 0.2 \pm 0.2\) excludes the possibility that a large fraction of the charged partons have spin zero. We shall assume that the partons have spin \(\frac{1}{2}\); it is then tempting to assume that they are quarks, since this is a natural way to obtain the results of current algebra.
The functions $P_N$ and $f(x)$ must be specified before detailed calculations can be made. We only remark that:

1) Since $f(x) \leq 1$ (being a probability) it follows from Eq. (13) that
   \[ \nu W_2 = 0 \text{ as } \omega \to \infty \text{ unless } \sum_{N} \nu W_2 \to \text{ const.} \]
   This can be achieved in the parton model but it requires non-zero contributions from configurations with many partons $[P_N \sim 1/N^2 \text{ is needed in a simple model}^{13}]$.

2) It is rather natural to assume that the proton's momentum is equally distributed among the partons on average, i.e.:
   \[ \int x f_N(x) dx = \frac{1}{N} \tag{14} \]
   It then follows from Eq. (13) that:
   \[ \int \nu W_2 d(1/\omega) = \sum_{N} P_N \frac{\sum_{i=1}^{N} Q_i^2}{N} \tag{15} \]
   \[ = \text{Mean square charge/parton} \]

   Experimentally, the left-hand side is $\approx 0.18$ (assuming $\nu W_2 = \text{const.}$ above the measured region). This is rather small; consider a model in which the proton is made of three valence quarks plus an SU(3) symmetric sea of quarks and antiquarks so that:
   \[ \sum_{i} Q_i^2 = 1 + (N - 3) \frac{2}{9} \tag{16} \]
   \[ \sum_{N} P_N \frac{\sum_{i=1}^{N} Q_i^2}{N} = \frac{2}{9} + \frac{1}{3} \left( \frac{1}{N} \right) \geq \frac{2}{9} \]

   which is larger than the experimental value. However, in a more realistic model neutral particles ('gluons') might be present to bind the quarks. Assuming, for example, that three of the $N$ partons are 'valence' quarks and, on average, $(1 - \varepsilon)(N - 3)$ are gluons, the rest being the $Q - \bar{Q}$ sea, the data give:
\( \langle \frac{1}{N} \rangle < 0.18 \)
\( \varepsilon < 0.72 \)

so that many partons are required.

In this simple model

\[
\int \nu \ W_2^P \ d\left(\frac{1}{\omega}\right) = \frac{2\varepsilon}{9} + \left(1 - \frac{2\varepsilon}{3}\right) \langle \frac{1}{N} \rangle
\]

(17)

\[
\int \nu \ W_2^N \ d\left(\frac{1}{\omega}\right) = \frac{2\varepsilon}{9} + \left(\frac{2}{3} - \frac{2\varepsilon}{3}\right) \langle \frac{1}{N} \rangle
\]

so that \( \varepsilon \) and \( \langle \frac{1}{N} \rangle \) can be calculated (and used to make predictions for \( \nu(\bar{\nu}) \) scattering — cf. Section B) once \( W_2^N \) is known from the experiments now in progress at SLAC with a deuterium target.

In a series of papers\(^{17}\) Drell and collaborators have derived a parton model from a canonical field theory of pions and nucleons endowed \textit{ab initio} with a cut-off in transverse momenta (the partons being bare pions and nucleons). The existence of this model shows the consistency of the intuitive approach\(^{11,12,13}\) followed here, but we will not go into the details since they are rather technical. The great advantage of the model is that it allows an investigation of related processes such as \( e^+ e^- \rightarrow p + \) (cf. Section C).

The Vector Meson Dominance (VMD) model

VMD rests on the assumption that the electromagnetic current is related to the source currents of the \( \rho, \omega \) and \( \phi \) mesons by:

\[
\langle F|J_\mu|P \rangle = \sum_V \frac{M_V^2}{f_V} \frac{1}{q^2 - M_V^2} \langle F|J^V_\mu|P \rangle .
\]

Diagrammatically:
The model is given content by simple assumptions about how \( \langle F | J^V_\mu | P \rangle \) varies with \( q^2 \) away from the vector meson poles. One cannot simply assume that \( \langle F | J^V_\mu | P \rangle \) is constant as this would contradict the current conservation condition \( q_\mu \langle F | J^V_\mu | P \rangle = 0 \). In fact there is a fundamental ambiguity in choosing what function might vary little away from the poles. Sakurai proposed\(^{16}\) that the tensor

\[
\sum_F \langle F | J^P_i | F \rangle \langle F | J^P_j | F \rangle \delta^4(P_F - P - q)
\]  

(18)

(where \( i \) and \( j \) are space indices) evaluated in the proton's rest frame varies little with \( q^2 \) at fixed \( s = (q + P)^2 \)\(^{19}\) (henceforth, we will only write the isovector (\( \rho \) part, for simplicity). This gives:

\[
\sigma_T(q^2, s) = \left( \frac{e}{F_\rho} \right)^2 \left( \frac{M_\rho^2}{q^2 - M_\rho^2} \right)^2 \sigma^T_{\rho\rho}(s)
\]

(19)

\[
\sigma_L(q^2, s) = \left( \frac{e}{F_\rho} \right)^2 \left( \frac{M_\rho^2}{q^2 - M_\rho^2} \right)^2 \frac{Q^2}{M_\rho^2} \left( \frac{s - M^2}{2M_\rho} \right)^2 \xi(s) \sigma^T_{\rho\rho}(s)
\]

where

\[
\xi(s) = \frac{\sigma^L_{\rho\rho}(s)}{\sigma^T_{\rho\rho}(s)}
\]

(20)

is assumed constant and treated as a parameter. Equation (19) implies scale invariance for \( \nu W_2 \) in the large \( \nu \), fixed \( \omega \), limit and that \( \nu W_2 \to \text{const. as } \omega \to \infty \) if \( \sigma^T_{\rho\rho}(s) \to \text{const.} \), in agreement with Regge models (see below). Unfortunately, it is in gross disagreement with experiment in the region when \( \sigma_T \) and \( \sigma_L \) have been separated\(^7\) because \( \sigma_L/\sigma_T \sim Q^2 \) (unless \( \xi = 0 \), which would be surprising) and because \( \sigma_T \) is wrong.

Sakurai and Chou\(^{20}\) have stressed that the model should, perhaps, only be expected to work when \( s >> Q^2 \) so that other dynamical variables (such as the Mandelstam \( u \) variable = \( 2M^2 + 2Q^2 - s \)) do not vary much during the continuation to the \( \rho \) pole. Since \( \sigma_T \) and \( \sigma_L \) have not been
separated in the region $s \gg Q^2$ they conclude that the model is untested. Nauenberg has pointed out\textsuperscript{21} that the model is nevertheless in great difficulty. Equation (19) is commonly believed to hold quite well at $Q^2 = 0$\textsuperscript{22}. However, the ratio of the left and right-hand sides is very different from one for $Q^2 \gg 1 \text{ GeV}^2$; Nauenberg\textsuperscript{21} got a good fit with this ratio parametrized as $\Gamma = 1 + b_\rho(s) (q^2/M^2_\rho - 1)$ with $b_\rho \approx 1$. If Eq. (19) ($\Gamma = 1$) is accepted in $0 \ll -Q^2 \ll M^2_\rho$, a large change of behaviour must occur in the unexplored region $0 \ll Q^2 \ll 1 \text{ GeV}^2$. This casts doubt on the idea that the model will work in this region and on the significance of its apparent success at $Q^2 = 0$.

VMD may be saved, of course, by replacing Sakurai's hypothesis by another. An obvious choice might be to assume that the $\rho$ form factors $V_1$ and $V_2$ (analogous to $W_1$ and $W_2$) in the tensor decomposition of $\sum_F < F|J^\rho_\mu(F) > < F|J^\rho_\nu(F) > \delta^\rho(\not{p} - \not{F} - q)$ are independent of $q^2$; however, this would lead to results for $W_1$ and $W_2$ which do not have the required kinematical zeros at $q^2 = 0$ [Eq. (5)]. $\rho$ form factors $\bar{V}_1$ and $\bar{V}_2$ which are free from kinematical zeros and singularities can be introduced but (with the simplest choice) the conditions $\sigma_T > 0$ and $\sigma_L > 0$ are not satisfied if $\bar{V}_1$ and $\bar{V}_2$ are constant in $q^2$. This led Tung\textsuperscript{23} to introduce the most general $\bar{V}_1$ and $\bar{V}_2$ such that $\sigma_T > 0$ and $\sigma_L > 0$ if $\bar{V}_1$ and $\bar{V}_2$ are constant in $q^2$; even this 'generalized VMD' is highly non-unique\textsuperscript{24,25}.

[Note that the supposedly successful relation for $\sigma_T$ {Eq. (19)} is not obtained in general.]

It would be interesting if form factors could be found which are constant in $q^2$ and if these form factors could be shown to have special significance.

**Diffraction models**

If we think of the virtual photon as an incident particle of mass $q^2$ we can consider the Regge limit $\gamma + \infty$, with $q^2$ fixed, just as in a hadronic reaction. We know that in this limit total cross-sections behave as $\sigma \propto (\alpha^{(0)})^{-1}$, where $\alpha(t)$ is the leading trajectory function when the incident particles scatter elastically. This behaviour for $\sigma_T$ and $\sigma_L$ [Eq. (8)] gives\textsuperscript{25}:
\[ W_1(\nu, q^2) \xrightarrow{\nu \to \infty} \beta_1(q^2)\nu^{\alpha(0)} \]
\[ W_2(\nu, q^2) \xrightarrow{\nu \to \infty} \beta_2(q^2)\nu^{\alpha(0) - 2}. \]  

(21)

Since no quantum numbers need be exchanged in the t channel in elastic \( \gamma p \) scattering, we expect that the leading trajectory is the Pomeron with \( \alpha(0) = 1 \). Experimentally it does seem that the photoabsorption cross-section \( c_T^{\gamma p}(\nu, q^2 = 0) \xrightarrow{\nu \to \infty} \) const. and \( c_T^{\gamma n}(\nu, q^2 = 0) \) is consistent with the hypothesis that it tends to the same value\(^{27} \). \([\text{Actually, the point } \alpha(0) = 1 \text{ is a nonsense point for } W_2 \text{ and the Pomeron decouples unless a fixed pole is present which restores its contribution} - \text{see footnote (28). Fixed poles may be present theoretically}^{29} \).\]

Abarbanel, Goldberger and Treiman\(^{30} \) and also Harari\(^ {31} \) have conjectured that the leading Regge pole at finite \( Q^2 \) continues to dominate as \( Q^2 \to \infty \) with \( \nu/Q^2 \) fixed \( \gg 1 \). Scale invariance then requires:

\[ \beta_1(q^2) \sim (q^2)^{-\alpha(0)}, \beta_2(q^2) \sim (q^2)^{1-\alpha(0)} \]  

in this limit, which implies \( F_1(\omega) \sim \omega \) and \( F_2(\omega) \sim \text{const.}, \) since \( \alpha(0) = 1 \) for the leading trajectory. The sum of ladder graphs in a simple theory with a scalar nucleon and a scalar photon gives \( \beta(q^2) \sim (q^2)^{\alpha(0) - 1} \) \(^{30} \), which is sufficiently close to the required result to encourage the hope that it would be obtained if spin were put into the model\(^ {32} \).

Harari's model\(^ {31} \)

Harari has pursued the analogy with hadron reactions and considered the finite energy sum rule:

\[ \int_0^N \nu W_2(q^2, \nu) d\nu = \sum_i \beta_i(q^2) N \alpha_i^{(0)} \]  

(22)

where the sum goes over all Regge trajectories and \( N \) is the energy above which the amplitude is assumed to be Regge behaved. If the left-hand side is dominated by resonances then, according to our knowledge of resonant form factors, it falls rapidly with \( Q^2 \). Equation (22) then implies that the \( \beta_i(q^2) \) also falls rapidly with \( |q^2| \), which would not lead to scale invariance and would contradict the data. To avoid this dilemma
Harari suggested\textsuperscript{31}) that the non-resonant background term on the left of Eq. (22) should be kept and associated with the Pomeron on the right, while the resonance contributions should be associated with the other ('ordinary') Regge terms. He assumes that the background contribution does not vary with $Q^2$ (at least at large $Q^2$); it follows that the Pomeron contribution is scale invariant while the rest is negligible at large $Q^2$ as it falls off like the square of resonance form factors.

Two criticisms can be made of this conjecture:

1) The Harari-Freund\textsuperscript{31,33}) association of Pomeron ↔ background and 'ordinary' trajectories ↔ resonances was successfully tested by Harari and Zarmi\textsuperscript{34}) in hadronic processes. However, a small non-resonant background contribution to the 'ordinary' trajectories could be present; it might be negligible at $Q^2 = 0$ compared to resonant contributions but dominant at large $Q^2$.

2) A more serious criticism is that Regge behaviour is usually assumed to apply when $\nu \gg Q^2$ in which case $N$ must be an increasing function of $Q^2$ \textsuperscript{35}) (see Fig. 4). Harari's reasoning then fails if the level density of resonances increases sufficiently rapidly. This happens, for example, with the Veneziano type amplitude of Landshoff and Polkinghorne\textsuperscript{36}) and the 'ordinary' trajectories give scale invariant contributions.

The difference between Harari's and other models is illustrated in Fig. 4. Harari predicts that at sub-asymptotic energies $\nu$ at which

\[ \Delta(q^2 = 0) = \sigma_T^P(\nu,q^2 = 0) - \sigma_T^N(\nu,q^2 = 0) \]

is not zero (between lines I and II) $\Delta(Q^2)$ will vanish rapidly if $Q^2$ increases at fixed $\nu$, since it is due to $\rho$ exchange (and $\beta_\rho(Q^2) \sim (Q^2)^{-n}$ with $n \sim 4$ possibly). In other models the boundary (I) of the Regge region is soon reached with increasing $Q^2$ at fixed $\nu$ and the Regge model tells us nothing.

Harari's model also differs from the parton model in which all the constituents behave as free bare particles and all contributions to the amplitude are scale invariant. Pomeron dominance can be arranged in the parton model so that $\nu W_2^P = \nu W_2^N$ at $\omega = \infty$ but if $Q^2$ increases with $\nu$ fixed (so that $\omega$ decreases) we move out of the Regge region and find $\nu W_2^P \neq \nu W_2^N$. 


**Veneziano-type models**

Many authors have attempted to construct Veneziano-type amplitudes for Compton scattering of virtual photons: \( \gamma(q_1^2) + P \rightarrow \gamma(q_2^2) + P' \). A recent and fairly successful attempt is that of Landshoff and Polkinghorne\(^{36}\)). Their model incorporates many desirable features (poles in \( q_1^2, q_2^2, s \) and \( t \); factorization at the parent level; the Fubini-Dashen, Gell-Mann sum rule is satisfied, the form factor in the sum rule being the same one obtained by taking the residue at the nucleon pole; etc). The main deficiencies of the model are that the Pomeron and the \( \Delta \) trajectory are ignored.

When the imaginary part of the forward amplitude is calculated in the Bjorken limit it turns out that \( \nu W_2 \) is scale invariant; whether this follows uniquely from the properties required for the amplitude or whether it is only a property of the particular ansatz taken is not clear. [However, it seems hard to find an ansatz which does not give scale invariance\(^{37}\)]. The shape of the \( \nu W_2 \) obtained is fairly close to experiment. If this is regarded as significant it follows that the \( \Delta \) trajectory and the Pomeron are relatively unimportant in the deep inelastic region.

**Formal developments**

We shall briefly discuss some formal developments without going into details.

Bjorken's pioneering work \([\text{mentioned above}^5]\) showed that scale invariance follows from the assumption that certain functions do not oscillate in the infinite \( Q^2 \) limit and that

\[
\lim_{Q^2 \to \infty} Q^2 \int_0^\infty \frac{d\nu}{\nu} W_2(\nu, Q^2) < \infty.
\]

(The \( \nu \) integration converges if Regge behaviour is assumed; the possibility that the scale functions \( F_i \) \([\text{Eq. (10)}]\) are zero is not excluded.)

Callan and Gross\(^{38}\)) showed that if scale invariance is assumed the ratio of \( \sigma_T/\sigma_L \) depends on the tensor structure of

\[
\lim_{|\vec{P}| \to \infty} \frac{M}{|\vec{P}|^2} \int d^4x \delta(x_0) \langle \vec{F} | J_i(x), J_j(0) | \vec{F} \rangle.
\]

(23)
The tensor structure of the commutator can be studied in the quark model since \( \psi \) (and hence \( \hat{J} \)) is known in terms of \( \psi \) from the Dirac equation. Assuming a vector (the 'gluon' model) pseudoscalar or scalar interaction between quarks it turns out the \( \sigma_L/\sigma_T = 0 \) while in the algebra of fields model \( \sigma_L/\sigma_T = \infty \). These results are intelligible in terms of simple models: the spin \( \frac{1}{2} \) parton model, in which the current is constructed as in the quark model, also gives \( \sigma_L = 0 \) and VMD, which is closely connected with the algebra of fields, gives \( \sigma_T = 0 \).

Although \( \sigma_L/\sigma_T = \infty \) is excluded by experiment we cannot conclude that the algebra of fields is wrong. It turns out that the formal arguments fail in perturbation theory\(^{39}\). If scale invariance is not a chimera, however, the real world must be smoother than perturbation theory which does not give scale invariance (in second order of the coupling in the gluon model \( v W_2 \) diverges as \( \log(Q^2/M^2) \) in the \( \omega \) fixed limit).

Recently, many authors\(^{32,46}\) have studied the implications of scale invariance for the behaviour of the commutator function near the light cone. These studies give the restrictions which theories with scale invariant limits must satisfy if they are formulated in configuration space.

2. HIGHLY INELASTIC NEUTRINO REACTIONS

2.1 Kinematics

The process to be considered is represented by the same diagram as the electron scattering process (page 2), except that the incoming electron is replaced by a (anti) neutrino and the outgoing electron by the corresponding electron or muon. The cross-section is given by an expression analogous to Eq. (1):

\[
\sigma^\nu,\bar{\nu} = \mu^2 \sum_F \sum_{J_\mu} \langle F | J^\dagger_\mu | P \rangle |^2 \delta(P + k - P_F - k')
\]

where \( j_\mu = \bar{u}(k) \gamma_\mu (1 \mp \gamma_5) u(k) \) is the well-known lepton current and \( J^\dagger_\mu \) is the Cabibbo current which may be written
\[ J^+ \mu = \bar{p} \gamma_\mu (1 - \gamma_5) (n \cos \theta_c + \lambda \sin \theta) \]
\[ J^- \mu = (J^+ \mu)^\dagger \]  
(25)

in terms of quark fields \( p, n \) and \( \lambda \). The lepton tensor is easily calculated

\[ m_{\mu \nu} = \text{Tr} \, \bar{w} \gamma_\mu (1 - \gamma_5) (w' + m_\mu) \gamma_\nu (1 - \gamma_5) \]
\[ = 8 (k_\mu k'_\nu + k_\nu k'_\mu - k_\mu k'_\nu g_{\mu \nu} + i \varepsilon_{\mu \nu \alpha \beta} k_\alpha k'_\beta) \]

\( \bar{w}_{\mu \nu} \) may be written (with the spin average implied):

\[ \bar{w}^\nu_{\mu \nu} = \int \frac{d^n q}{4\pi^2} e^{iq \cdot x} \left\langle p \left| J^\nu_\mu (x), J_\mu^+(0) \right| p \right\rangle \]
\[ = \frac{\text{Im}}{4\pi^2} \int d^n x \, e^{iq \cdot x} \theta(x_0) \left\langle p \left| J^\nu_\mu (x), J_\mu^+(0) \right| p \right\rangle \]
\[ = - \left( g_{\mu \nu} - \frac{q_\mu q_\nu}{q^2} \right) \bar{w}_{1, \nu} \bar{\nu} + \left( p_\mu - \frac{q_\mu q^*}{q^2} \right) \left( p_\nu - \frac{q_\nu q^*}{q^2} \right) \bar{w}_{2, \nu} \bar{\nu} - \frac{i \varepsilon_{\mu \nu \alpha \beta} \frac{p_\alpha q_\beta}{2M^2}}{2M^2} \bar{w}_{3, \nu} \bar{\nu} \]
\[ + \frac{q_\mu q_\nu}{M^2} \bar{w}_{4, \nu} \bar{\nu} + \frac{\left( p_\mu q_\nu + p_\nu q_\mu \right)}{M^2} \bar{w}_{5, \nu} \bar{\nu} + \frac{i \left( p_\mu q_\nu - p_\nu q_\mu \right)}{M^2} \bar{w}_{6, \nu} \bar{\nu}. \]
(26)

In contrast to the electromagnetic case there are no constraints due to current or parity conservation and \( \bar{w}_{\mu \nu} \) depends on all the independent tensors available. It follows directly from Eq. (26) that \( (\bar{w}^\nu_{\mu \nu})^* = \bar{w}^\nu_{\mu \nu} \bar{w}_{\mu \nu} \) so that the \( \bar{w}_i \) are real. If we assume that the currents and states have the usual properties under time reversal it follows that \( \bar{w}_{\mu \nu}(q, P) = \bar{w}_{\mu \nu}(\tilde{q}, \tilde{P}) \)
where \( q_\mu = q_\mu, \tilde{q}_\mu = -q_\mu \) and similarly for \( P \). This implies that \( \bar{w}_{1,2,3,4,5} \) are real and \( \bar{w}_6 \) imaginary and hence \( \bar{w}_6 \equiv 0 \).

\( \bar{w}_{4,5,6} \) do not contribute if the lepton mass is neglected, as it will be henceforth, since \( q_\mu j_\mu = 0(m_\mu, e) \). In this case the cross-section is
\[ \frac{\pi}{M E E^{'}} \frac{d^2 \sigma^{\nu, \nu}}{d \Omega d E^{'}} = \frac{d^2 \sigma^{\nu, \nu}}{d q^2 d \nu} = \frac{E' G^2}{2 M^2 E \pi} \left( \cos^2 \theta/2 \, \bar{W}_2^{\nu, \nu} \right. \\
+ 2 \sin^2 \theta/2 \, \bar{W}_1^{\nu, \nu} + \frac{E + E'}{M} \sin^2 \theta/2 \, \bar{W}_3^{\nu, \nu} \right). \] (27)

By considering \((W_{\mu \nu}^\nu)^*\) we find the crossing property

\[ \bar{W}_i^{\nu} (\nu, q^2) = - \bar{W}_i^{\nu} (-\nu, q^2). \]

Another useful property is obtained by noting that under the isospin rotation which turns neutrons \(\leftrightarrow\) protons, \(J^+_{\mu} \leftrightarrow J^+_{\mu}\) in the approximation \(\theta_c = 0\). Hence

\[ \langle P | [J^+_{\mu}, J^-_{\nu}] | P \rangle = \langle n | [J^+_{\mu}, J^-_{\nu}] | n \rangle \\
= \bar{W}_i^{\nu p} = \bar{W}_i^{\nu n} \quad \text{for} \quad \theta_c = 0. \] (28)

We can introduce normalized polarization vectors for the current which satisfy \(\varepsilon_{\mu} q^\mu = 0\), as in the electromagnetic case:

\[ \varepsilon_{\mu}^S = \frac{1}{\sqrt{Q^2}} (q_3, 0, 0, q_0) \]
\[ \varepsilon_{\mu}^R = \frac{1}{\sqrt{2}} (0, 1, i, 0) \]
\[ \varepsilon_{\mu}^L = \frac{1}{\sqrt{2}} (0, i, 1, 0) \]

(we use \(S\) for scalar, rather than \(L\) for longitudinal, in order to keep \(R\) and \(L\) for right and left). The current-nucleon absorption cross-sections are easily calculated:
\[
\sigma_R = \frac{\pi}{\nu + \frac{q^2}{2}} \left( W_1 + \frac{1}{2} \sqrt{\nu^2 - \frac{q^2}{M^2}} W_3 \right)
\]

\[
\sigma_L = \frac{\pi}{\nu + \frac{q^2}{2}} \left( W_1 - \frac{1}{2} \sqrt{\nu^2 - \frac{q^2}{M^2}} W_3 \right)
\]

\[
\sigma_S = \frac{\pi}{\nu + \frac{q^2}{2}} \left[ W_2 \left( \frac{\nu^2}{q^2 M^2} + 1 \right) - W_1 \right]
\]

(29)

where we have followed the convention of Ref. 41) and put the current coupling constant = 1 and [as in Eq. (8)] used the flux factor for a zero mass current with the same value of S = (q + P)^2. Since W_3 \propto \sigma_R - \sigma_L we see why the contributions of W_3 to the \nu and \bar{\nu} cross-sections have opposite signs [Eq. (27)] when \nu \leftrightarrow \bar{\nu}, j_{\mu} \leftrightarrow j_{\mu}^{L, 42}). The fact that \sigma_{R,L,S} > 0 gives the inequalities^43)

\[
0 \leq \frac{1}{2M^2} \sqrt{\nu^2 - M^2 q^2} \left| W_3^{\nu, \bar{\nu}} \right| \leq W_1^{\nu, \bar{\nu}} \leq \left( 1 + \frac{v^2}{M^2 Q^2} \right) W_2^{\nu, \bar{\nu}}.
\]

(30)

2.2 Scale invariance and the data

Bjorken argued^5) that the limits

\[
W_1(\nu, q^2) = \tilde{F}_1(\omega, q^2) \xrightarrow{\omega \text{ fixed}} F_1(\omega)
\]

\[
\frac{\nu}{M^2} W_2(\nu, q^2) = \tilde{F}_2(\omega, q^2) \xrightarrow{\omega \text{ fixed}} F_2(\omega)
\]

\[
\frac{\nu}{M^2} W_3(\nu, q^2) = \tilde{F}_3(\omega, q^2) \xrightarrow{\omega \text{ fixed}} F_3(\omega)
\]

(31)

exist for the \nu(\bar{\nu}) scattering structure constants. The cross-section may be written in terms of the \tilde{F}_i:
\[ \frac{d^2 \sigma}{dxd\omega} \left( 1 - x - \frac{M x}{2\omega E} \right) \frac{F_2^\nu \bar{\nu}}{\omega} + \frac{x^2}{\omega} \frac{F_1^\nu \bar{\nu}}{\omega} \bar{\nu}^\nu \left( 1 - \frac{x}{2} \right) \frac{F_3^\nu \bar{\nu}}{\omega} \]

\[ x = \frac{\nu}{ME} \]  

(32)

If Eq. (31) is correct, this gives a total cross-section which increases linearly with energy:

\[ \lim_{E \to \infty} \sigma_{\nu, \bar{\nu}} = \frac{G^2 \text{ME}}{\pi} \int_0^\infty d\omega \left( \frac{F_2^\nu \bar{\nu}(\omega)}{2\omega^2} + \frac{F_1^\nu \bar{\nu}(\omega)}{3\omega^3} \bar{\nu}^\nu + \frac{F_3^\nu \bar{\nu}(\omega)}{3\omega^3} \right). \]  

(33)

Note that whenever we talk about the E \to \infty limit for neutrino interactions we are indulging the hope that there is a region which is asymptotic in the sense that our formulae apply but is nevertheless below the energy at which the effective current-current interaction fails.

The CERN neutrino experiment\(^{45}\) will be discussed by Franzinetti at this meeting. We summarize a few points about the data, which are based on 740 events with E_{\nu} > 1 \text{ GeV} in propane:

1) The total cross-section is shown in Fig. 5. The data suggest that \sigma rises linearly, in agreement with Eq. (33). A linear fit to the events with E_{\nu} > 2 \text{ GeV} gives

\[ \sigma_T = (0.51 \pm 0.13) \frac{G^2}{\pi} \frac{\text{ME}}{\text{nucleon}} \]  

(34)

where the errors are only statistical\(^{46,47}\).

2) The distribution of events in \rho = \omega/2 is shown in Fig. 6. It suggests that Eq. (31) holds with F_1(\omega)/\omega, F_2(\omega) and F_3(\omega)/\omega constant; it must be noted, however, that the large \omega points correspond to very small Q^2 and can hardly be considered asymptotic.

3) The x distribution is shown in Fig. 7 where \( K_1 = \frac{2F_1(\omega)}{\omega}, K_2 = F_2(\omega) \) and \( K_3 = -\frac{F_3(\omega)}{\omega} \) are assumed constant. The number of events with E_{\nu} > 2 \text{ GeV}, \rho > 1.5 and x > \frac{1}{2} (so that the missing mass, denoted M^0 in Fig. 7, is > 2 \text{ GeV}) gives\(^{47}\):

\[ K_2 + \frac{7}{6} K_1 + \frac{11}{6} K_3 = 2.0 \pm 0.6 \]  

(35)
or, assuming \( K_3 = 0 \) (which is true in Regge models as \( \rho \to \omega \); see below)

\[
K_1 = K_2 = 0.9 \pm 0.3 .
\]  

(36)

Note that the \( x \) distribution excludes \( K_1 = K_3 = \sigma_R = \sigma_L = 0 \).

In summary, the statistics are too small to separate \( W_1, W_2, W_3 \) as functions of \( \nu \) and \( q^2 \) but the data are easily understood if scale invariance is assumed.

2.3 Models for highly inelastic neutrino scattering

In this section we put the Cabibbo angle \( \theta_C = 0 \) for simplicity; the results therefore really apply to the cross-sections for producing non-strange final states divided by \( \cos^2 \theta_C \). [See footnote (72) for results with \( \theta_C \neq 0 \).]

The parton model

It is simple to extend the parton model to \( \nu(\bar{\nu}) \) scattering\(^{48,49,41,17} \). As in the electromagnetic case the assumption that the partons have spin 0 gives \( \sigma_R = \sigma_L = 0 \), in contradiction to the data. Assuming that the only partons with weak and electromagnetic couplings are quarks [and allowing any number of other partons and arbitrary \( P_N \) and \( f_N^i(x) \)] the following relations hold\(^{49} \):

\[
2F_1 = \omega F_2 \ (\sigma_S = 0) \quad \text{[Callan Gross relation]} \]

\[
\int \frac{d\omega}{\omega^2} (F_{\nu n}^\nu - F_{\nu \nu}^\nu) = 1
\]  

(37)

(Bjorken sum rule\(^{50} \) or Adler sum rule\(^{51} \) using \( 2F_1 = \omega F_2 \))

\[
- \int \frac{d\omega}{\omega^2} (F_{2}^{\nu \nu} + F_{2}^{\nu}) = 6
\]

[Reference 48] \[
12(F_{1}^{\nu \nu} - F_{1}^{\nu}) = F_{3}^{\nu \nu} - F_{3}^{\nu}
\]

\[
F_{1}^{\nu \nu} + F_{1}^{\nu} \approx \frac{5}{18} (F_{1}^{\nu \nu} + F_{1}^{\nu})
\]

\[
\left\{ \begin{array}{c}
F_{1}^{\nu \nu} \frac{d\omega}{\omega^2} \geq \frac{1}{2}, \\
F_{1}^{\nu} \frac{d\omega}{\omega^3} \geq \frac{1}{3}, \\
F_{1}^{\nu \nu} \frac{d\omega}{\omega^2} \geq 1, \\
F_{1}^{\nu} \frac{d\omega}{\omega^2} \geq 2.
\end{array} \right.
\]
Most of these results can be obtained without the parton model and this is also true of an integral form of two of the remaining relations\(^{49}\) (see Table 1). The model is therefore hard to substantiate in general.

Three results follow from the fact that \( W_3 \) is negative for a quark and positive for an antiquark\(^{52}\):

1) In reasonable models \( F_3 < 0 \) so that \( \sigma^\nu > \sigma^{\bar{\nu}} \).

2) Baryon and antibaryons contributions cancel in \( F_3 \) while they add in \( F_1 \) and \( F_2 \). \( F_3/F_1 \) is therefore, roughly speaking, a measure of the average effective baryon number\(^{53}\).

3) In most models\(^{54}\) the asymptotic behaviour of \( W_3 \) will be that of a bare quark \((W_3 \underset{\nu \to \infty}{\to} 0)\). The 'Pomeron'-like behaviour \( W_3 \to \text{const.} \) cannot be obtained from an infinite Q-\( \bar{Q} \) sea (as it was for \( \nu W_2 \)) as the Q and \( \bar{Q} \) contributions cancel \([\text{the Pomeron should not contribute to } W_3 \text{ anyway (see below) so this is satisfactory}]\).

In particular parton models some additional results hold for all \( P_i^N \) and \( f_i^N \). These results are given in Table 1 for the model discussed previously (model A: nucleon \( \sim 3 \) valence quarks + \( Q\bar{Q} \) sea + gluons) and for another model (model B: nucleon \( \sim Q\bar{Q} \) valence pair + 3Q SU(3) scalar + \( + Q\bar{Q} \) sea + gluons). \([\text{Note that in Table 1 it is not assumed that the different types of quark and antiquark have the same momentum distributions - this would lead to much more specific results - see footnote (71).}]

When highly inelastic \( e^-n \) scattering data are available the models will make more definite predictions for \( \nu(\bar{\nu}) \) structure functions.

Some predictions can be made already using only \( e^-p \) data; we consider some predictions for the sum of neutron and proton cross-sections\(^{55}\):

\[
\sigma^{\nu p} + \sigma^{\nu n} + \sigma^{\bar{\nu} p} + \sigma^{\bar{\nu} n} < 1.72 \frac{G^2_{\text{ME}}}{\pi} \text{ (all parton models)}
\]
\[
= 1.44 \frac{G^2_{\text{ME}}}{\pi} \text{ (model A)}
\]
\[
< 1.44 \frac{G^2_{\text{ME}}}{\pi} \text{ (model B).}
\]

Using the experimental result of Eq. (34) we find\(^{56}\)
\[ \sigma^{VP} + \sigma^{VN} - \sigma^{\bar{V}P} - \sigma^{\bar{V}n} \geq (0.32 \pm 0.56) \frac{G^2_{ME}}{\pi} \quad \text{(all parton models)} \]
\[ = (0.6 \pm 0.56) \frac{G^2_{ME}}{\pi} \quad \text{(model A)} \quad (39) \]
\[ < (0.6 \pm 0.56) \frac{G^2_{ME}}{\pi} \quad \text{(model B)} \]

and hence
\[ \frac{1}{3} \leq \frac{\sigma^{\bar{V}P} + \sigma^{\bar{V}n}}{\sigma^{VP} + \sigma^{VN}} = x < 0.89 \quad \text{(models A and B)} \quad (40) \]

[Note that this result is very sensitive to the errors in Eq. (34). Neglecting the errors \( x < 0.68 \) (all models), \( x = 0.41 \) (model A).]

The field theory parton model of Drell et al.\(^{17}\) can also be applied to \( V \) and \( \bar{V} \) scattering. One striking prediction of the model is that in the region \( \omega \gg 1 \) the proton behaves like a bare nucleon; it follows that the inequalities between \( W_1, W_2 \) and \( W_3 \) [Eq. (30)] are saturated and
\[ \frac{(d\sigma^{\bar{V}}/d\omega)}{(d\sigma^V/d\omega)} = \frac{1}{3} \quad (\omega \gg 1) \quad (41) \]

(in contrast to the value 1 when \( \omega \gg 1 \) if Regge behaviour is assumed, see below). Various results similar to those in Eq. (37) can be derived which do not depend on the choice of parameters in the model\(^{57}\).

**Vector meson dominance models**

Stodolsky and Piketty have applied VMD to highly inelastic \( V \) scattering\(^{58}\). It is technically a little more complicated than the electromagnetic case because of the non-conservation of the axial current and the necessity to incorporate PCAC. In view of the great difficulties of VMD in describing inelastic electron scattering (discussed above) it does not seem worth-while to discuss it further here.

**Diffraction models**

The discussion of \( W_1 \) and \( W_2 \) in Section A also applies in the case of the weak current. Assuming that \( \sigma_{R,L} \sim \alpha(\omega)^{-1} \) in the Regge limit (or
making a t-channel analysis as in footnote 25) we find \( W_3 \sim \nu^{A(n)-1} \). \( W_3 \) is clearly the vector-axial vector current interference term \(^{52}\); since the vector current resembles the \( \rho \) meson in its internal quantum numbers and the axial current resembles the \( A_1 \), \( W_3 \) is like the imaginary part of the forward amplitude for the process:

\[
\rho \rightarrow A_1
\]

It follows that odd \( G \) parity is exchanged in the \( t \) channel and the Pomeron cannot contribute to \( W_3 \). In fact the leading trajectories are the \( \omega \) and \( \phi \) with \( \alpha(0) = \frac{1}{2} \). Hence \( W_3 \sim \nu^{-\frac{1}{2}} \) and, if the leading Regge term continues to dominate in the Bjorken limit and \( \nu W_3 = F_3 \) is scale invariant, \( F_3(\omega) \propto \omega^{\frac{1}{2}} \). Therefore [using Eq. (33)] \( \sigma^\nu(\omega) = \sigma^\nu(\omega) \) when \( \gg 1 \) in this model.

An ansatz which has this behaviour and satisfies the sum rule for \( F_3^\nu P + F_3^\nu N \) [Eq. (37)] can be used to maximize \( \sigma^\nu P + \sigma^\nu N - \sigma^\nu P - \sigma^\nu N \) \( + \delta \). This gives the "empirical bound":

\[
\sigma^\nu P + \sigma^\nu N - \sigma^\nu P - \sigma^\nu N \approx 0.5 \frac{G^2M}{\pi} .
\]  

**Harari's model**

In Harari's model only the Pomeron contributions are scale invariant and \( \nu W_3 \) therefore does not scale and goes rapidly to zero with increasing \( Q^2 \). In the Pomeron dominated region \( \sigma^\nu = \sigma^\nu \). Since as \( E_\nu \rightarrow \infty \) the Pomeron region tends to 100% of the physical region in this model, \( \sigma^\nu - \sigma^\nu \rightarrow 0 \) as the neutrino energy \( \rightarrow \infty \). This is quite different from the result found with parton models [Eq. (39)].

**Veneziano-type models**

A Veneziano-type model has been constructed for \( \nu(\bar{\nu}) \) scattering by Landshoff\(^{59}\). It turns out that the amplitude for \( \nu W_3 \) is scale invariant in the appropriate limit, as is \( \nu W_2 \). The comments made in Section A about the model for electroproduction apply again.
Formal developments

The celebrated Adler sum rule was one of the first results obtained for high-energy neutrino scattering\(^{31}\). It depends on assuming the local commutation relations of the quark model and reads:

\[
\int \left[ \mathcal{W}^n_2(\nu, q^2) - \mathcal{W}^P_2(\nu, q^2) \right] \frac{d\nu}{M^2} = 2
\]

which implies:

\[
\lim_{E \to \infty} \frac{d\sigma^n}{dq^2} - \frac{d\sigma^P}{dq^2} = \frac{G^2}{\pi}.
\]

[The scale invariant limit of this sum rule holds in all quark models \{Eq. (37)\}.] This sum rule is interesting not only because it provides a test of the commutation relations but also because the right-hand side of Eq. (44) is 'point like', i.e. independent of \(q^2\), which suggests parton behaviour\(^{12}\). Note that this sum rule fails in Harari's model, except at \(q^2 = 0\) where it is the Adler-Weisberger relation (technically a 'no subtraction' assumption must fail due to a fixed pole). [Again the Veneziano model\(^{59}\) provides a counter example to Harari's arguments as the Adler sum rule holds.]

Several other sum rules for neutrino scattering have subsequently been obtained which we list in Table 1. Hopefully, some of these will be tested soon. Note in particular that the results of Eqs. (38), (39) and (40) which are true in all parton models also follow from the integral inequality which is true in the (weaker) gluon model; a slightly weaker result follows from an assumption about 'asymptotic chiral symmetry'\(^{41}\).

3. RELATED PROCESSES

It may soon be technically possible to measure the study \(e^- p \to e^- + \ldots\) using a polarized electron beam and a polarized target. Bjorken has shown\(^{60}\) that the quark model commutation relations imply a mean polarization of greater than 20% throughout most of the highly inelastic region. This result is also true for parton models [see footnote 61] where it is discussed at length], but Harari's model (for example) suggests small polarizations; the experiment would therefore be of great interest.
Bjorken and Paschos\textsuperscript{13}) have shown that the parton model can be fairly plausibly extended to the inelastic Compton scattering process $\gamma p \rightarrow \gamma^+\ldots$. They argue that the dominant process is one in which the two photons interact with the same parton. Calculation proceeds as before by adding up the Compton cross-sections from each elementary parton. It turns out that the functions $P_N$ and $f_N^i(x)$ occur in the same combinations as previously so that, independently of them, the ratio of cross-sections for $\gamma p \rightarrow \gamma^+\ldots$ to $e^- p \rightarrow e^- + \ldots \sim \alpha \times$ (known kinematic factor) $\times < \sum_i Q_i^4 > / < \sum_i Q_i^2 >$. If this very difficult experiment ($\gamma p \rightarrow \gamma^+\ldots$) can be performed it will therefore discriminate clearly between fractionally and integrally charged partons, if the model is correct.

It is natural to ask whether the models developed for the process $e^- p \rightarrow e^- + \ldots$ can also be applied to processes involving a virtual timelike photon such as $p p \rightarrow e^+ e^- + \ldots$ or $e^+ e^- \rightarrow p + \ldots$ . At first glance an intimate connection might be expected and, in particular, it looks as if $e^- p \rightarrow e^- + \ldots$ and $e^+ e^- \rightarrow p + \ldots$ might be connected by crossing. However, the exploitation of this idea meets formidable difficulties:

1) $W_{\mu\nu}$ defined in Eqs. (2) and (3) is not directly related to the process $\gamma(q^2) p + \ldots$ when $q^2$ is timelike (let alone to other processes). This is because $W_{\mu\nu}$ has the structure:

![Diagram](image)

(a) + (b) + (c) + (d)

(the x indicates that the diagram is connected, i.e. no lines entering the bubble are totally disconnected from the others). When $q^2$ is spacelike the terms b, c and d vanish [since $\gamma(q^2 < 0)$ is impossible] leaving a, which is what is measured in the process $\gamma p \rightarrow \ldots$.

2) $W_{\mu\nu}$ can be redefined by changing $< F|J|P > \rightarrow < F|J|P > c$, where c means connected, in Eq. (2), line 1. The new $\tilde{W}_{\mu\nu}$ describes $\gamma p + \ldots$ for all $q^2$ but it is $W_{\mu\nu}$ which is well studied; we are theoretically ignorant of $\tilde{W}_{\mu\nu}$ for $q^2$ timelike.
3) Formally $\tilde{W}_{\mu\nu}$ describes $\gamma \rightarrow p + ...$ as well as $\gamma p + ...$ by crossing. However, this formal statement is contentless since the structure functions are not analytic (by definition they $\propto \langle F|J|P \rangle \propto F|J|P \rangle^*$) in $q^2$ and their behaviour may be entirely unconnected in the different physical regions.

4) In its simple form the parton model treats: photon + free parton + free parton. This is kinematically impossible for timelike photons.

These difficulties may not be insurmountable. For example it may be that the dominant terms in the structure functions of the connected amplitude are analytic; this is the case in the model of Drell et al.\cite{ref17,ref62} which predicts (1) scaling in $e^+ e^- + p + ...$, (2) $\sigma_T(e^+ e^- + ...) \propto 1/E^2$, (3) the secondaries in $e^+ e^-$ collisions will be divided into two jets of particles with small transverse momenta\cite{ref63}. Pestieau and Roy\cite{ref64} have also discussed the possibility of scaling in $e^+ e^- + p + ...$; they noticed that the term d in the diagrammatic representation of $\tilde{W}_{\mu\nu}$ above is directly related to $\gamma + p + ...$. The assumption of Bjorken's representation for $\tilde{W}_{\mu\nu}$ and that the terms a, b, c have certain analytic properties as $s \rightarrow \infty$ (this requires a cut-off in the rest mass of intermediate states) gives them scaling.

The unproven conjecture that $e^+ e^-$ annihilation processes are scale invariant is extremely interesting because it implies that the cross-section falls slowly with energy, which would make life easier for experimentalists (clearly if no scale is defined $\sigma \propto 1/E^2$; with a scale $m$, $\sigma \propto m^2/E^{2+n}$ but in simple models $n \geq 0$ might be expected).

An experiment on the possibly related process $pp + \mu^+ \mu^- + ...$ has already been performed\cite{ref65}. It seems that a very simple model assuming elementary point-like constituents (partons) can give a qualitatively correct description of the data\cite{ref66}. The meaning of this agreement is somewhat obscure since (1) the model is applied for large $q^2$; theoretically, it applies as $q \rightarrow 0$; (2) it also seems that a vector dominance model can describe the data qualitatively\cite{ref67,ref68}.

Finally, we mention possible connections with hadron reactions. Kinematically, the processes $e^- p + e^- + ...$ and $p + p + p + ...$ are clearly related. A dynamical connection might be expected (by analogy with the Chou-Yang model which connects $e^- p \rightarrow e^- p$ and $pp \rightarrow pp$); this
seems implausible, however, as both the incident particles generally
shatter in the second case. The general ideas of constituent structure
and scaling, however, have been used by Feynman\textsuperscript{11) and Bjorken\textsuperscript{69}) to make
plausible speculations about hadron reactions.

4. SUMMARY

Despite the length of this paper, several interesting topics and
models have not been discussed, e.g. the implications for the p–n mass
difference \[\text{[see the review in Ref. 10]}\], the eikonal model developed by
Griffith\textsuperscript{70}), the recent model due to West\textsuperscript{71}), the effect of the W meson
on the $\nu(\bar{\nu})$ predictions\textsuperscript{41}) and $\Delta S = 1, \nu(\bar{\nu})$ reactions\textsuperscript{72})

The main features of the various models which we have discussed are
collected for comparison in Table 2. Sum rules are collected in Table 1.
Apart from the fact that the vector meson dominance model is in trouble,
not much can be said about the comparative success of the models at pre-
sent. When e – n data are available it may rule out either the parton model
or Harari’s model (or both). In any case, e – n data can be used as input
to make predictions for $\nu(\bar{\nu})$ scattering and these experiments will pro-
vide a test of the speculative ideas discussed here.

Acknowledgements

I wish to thank Professors J.S. Bell and J. Prentki for reading the
manuscript and P. Landshoff and M. Nauenberg for discussions of their
work. Such clarity as this paper may have is due to the patient and per-
sistent criticisms of A. de Rujula whom I thank for many useful discussions.
Table 1
Sum rules in the Bjorken limit

<table>
<thead>
<tr>
<th>Input</th>
<th>Sum rule</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark model commutators</td>
<td>$\int \frac{d\omega}{\omega} (F_2^{VP} - F_2^{VP}) = 2$</td>
<td>51</td>
</tr>
<tr>
<td>Quark model commutators</td>
<td>$\int \frac{d\omega}{\omega^2} (F_0^{VP} - F_0^{VP}) = 1$</td>
<td>50</td>
</tr>
<tr>
<td>Quark model commutators</td>
<td>$\int \frac{d\omega}{\omega} (F_0^{VP} + F_0^{VP}) \geq \frac{1}{2}$</td>
<td>73</td>
</tr>
<tr>
<td>Quark model commutators</td>
<td>$-\int \frac{d\omega}{\omega^2} (F_0^{VP} + F_0^{VP}) = 6$</td>
<td>48</td>
</tr>
<tr>
<td>Gluon model</td>
<td>$2F_1 = \omega F_2$</td>
<td>38, 48</td>
</tr>
<tr>
<td>&quot; &quot;</td>
<td>$12 \int \frac{d\omega}{\omega^3} (F_1^{VP} - F_1^{VP}) = \int \frac{d\omega}{\omega^3} (F_3^{VP} - F_3^{VP})$</td>
<td>49</td>
</tr>
<tr>
<td>&quot; &quot;</td>
<td>$\int \frac{d\omega}{\omega^3} (F_1^{VP} + F_1^{VP}) \geq \frac{5}{18} \int \frac{d\omega}{\omega^3} (F_1^{VP} + F_1^{VP})$</td>
<td>49</td>
</tr>
<tr>
<td>Asymptotic chiral symmetry</td>
<td>$\int \frac{d\omega}{\omega^3} (F_1^{VP} + F_1^{VP}) \geq \frac{1}{4} \int \frac{d\omega}{\omega^3} (F_1^{VP} + F_1^{VP})$</td>
<td>41</td>
</tr>
<tr>
<td>All quark parton models</td>
<td>$12(F_1^{VP} - F_1^{VP}) = (F_3^{VP} - F_3^{VP})$</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>$F_1^{VP} + F_1^{VP} \geq \frac{5}{18}(F_1^{VP} + F_1^{VP})$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\int \frac{d\omega}{\omega^3} F_1^{VP} \geq \int \frac{d\omega}{\omega^3} F_1^{VP}$</td>
<td></td>
</tr>
<tr>
<td>Parton model A (see p. 23)</td>
<td>$F_1^{VP} - F_1^{VP} = 6(F_1^{VP} - F_1^{VP})$</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>$= \frac{F_3^{VP} - F_3^{VP}}{2} = -\frac{F_3^{VP}}{4}$</td>
<td></td>
</tr>
<tr>
<td>Parton model B (see p. 23)</td>
<td>$F_1^{VP} - F_1^{VP} = 6(F_1^{VP} - F_1^{VP})$</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{2}(F_3^{VP} - F_3^{VP})$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_1^{VP} + F_1^{VP} = 2(F_1^{VP} + 2F_1^{VP})$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\int \frac{d\omega}{\omega^2} F_3^{VP} = -2$</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>Scaling</td>
<td>$\sigma_L/\sigma_T$</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>---------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>Formal field-theoretical considerations</td>
<td>Possible</td>
<td>0 Quarks $\approx$ Alg. of Fields</td>
</tr>
<tr>
<td>Partons</td>
<td>Yes</td>
<td>0 spin $\frac{1}{2}$ $\approx$ spin 0</td>
</tr>
<tr>
<td>Drell's partons</td>
<td>Yes</td>
<td>0 if $\omega &gt; &gt; 1$</td>
</tr>
<tr>
<td>Diffraction</td>
<td></td>
<td>const.</td>
</tr>
<tr>
<td>Harari</td>
<td></td>
<td>const.</td>
</tr>
<tr>
<td>Veneziano-like</td>
<td>Yes</td>
<td>0</td>
</tr>
<tr>
<td>Vector meson dominance</td>
<td>Yes for $\nu W_2$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
FOOTNOTES AND REFERENCES

1) Experiments which investigate the hadronic debris have been proposed, or begun, at SLAC, DESY, NINA and Cornell. The neutrino experiment was done in a bubble chamber, so that the final hadron state was necessarily observed.

2) We employ the metric \( q^2 = q_0^2 - \gamma^2 \) and the conventions for \( \gamma \) matrices, etc. used by Bjorken and Drell (Relativistic Quantum Mechanics, McGraw Hill), except that fermion states, as well as boson states, are covariantly normalized to \( 2E \) and \( \bar{\psi} \psi = 2m \).

3) In the definition of \( W_{\mu \nu} \) it is understood that the connected part should be taken; i.e. strictly, we should write \( \langle P | \ldots | P \rangle - \langle 0 | \ldots | 0 \rangle \) (where \( | 0 \rangle \) is the vacuum state) in place of \( \langle P | \ldots | P \rangle \). This removes the unwanted disconnected term:

\[ \begin{array}{c}
\end{array} \]

4) This can also be derived directly from Eqs. (2) and (4) by noting that \( \eta^*_\mu W_{\mu \nu} \eta_{\nu} > 0 \) for an arbitrary vector \( \eta_\mu \). Actually, appealing to the positivity of \( \sigma_T \) and \( \sigma_L \) is slightly fraudulent as they are not physical quantities, so their positivity should be derived (note that \( \epsilon^T_\mu \) is timelike and has a 'ghost' normalization).


8) DESY preprint 69/46.


14) Naively, it appears that \( p_i^\mu \approx x_i p_\mu \) implies \( m_i \approx x_i M \ll M \), i.e. that partons are light! However, the next order of expansion

\[
p_{i,0} = x_i p_0 + \left[ \frac{m_i^2 + p_i^{T^2} - x_i^2 m_i^2}{2x_i p_0} \right] + O \left( \frac{1}{x_i^2 p_0^2} \right)
\]

must be kept for consistency if \( p_i^2 \) is calculated, so that any \( m_i \) \((< x_i p_0)\) is possible.

15) For a bare particle with charge \( Q \):

\[
\langle \hat{P} | J_\mu | \hat{P}' \rangle = Q(p + p')_\mu + \text{spin dependent terms}.
\]

Putting this into Eq. (12) with single particle intermediate states

\[
\left( \int \frac{d^4 P_F}{(2\pi)^3} \delta(p^2_F - m^2) | \hat{P}_F \rangle \right)
\]

gives

\[
W^{\text{Free particle}}_{\mu\nu} = 2Q \left( p_\mu - \frac{q \cdot p_\mu}{q^2} \right) \left( p_\nu - \frac{q \cdot p_\nu}{q^2} \right) \delta(q^2 + 2q \cdot P) + \text{spin dependent terms}.
\]

The spin dependent terms vanish as \( p' = p \) and the coefficient of \( p_\mu p_\nu \) is \( W_2 \) for a free particle normalized to 2E per unit volume. To obtain \( W_2 \) from this we let \( Q \to Q_1 \) and \( p_\mu \to x_i p_\mu \) and divide by \( x_i \) in order to normalize to one parton per proton, which is required in Eq. (11) (with the covariant normalization, number of partons per proton = \( 2E_1/2E = x_i \)).
16) It is easy to obtain these results by direct calculation. Footnote 15) shows that \( W_1 = 0 \) and hence \( \sigma_T = 0 \) for a spinless particle. For a free spin \( \frac{1}{2} \) particle \( W_{\mu\nu} \) can be calculated using Eq. (2) or, equivalently, Eq. (3) and taking the imaginary part of the Feynman amplitude for

\[
\text{Im} \left( \frac{1}{p - M + i\varepsilon} \right) = -\pi(p + M) \delta(p^2 - M^2); \]

\[
W_{\mu\nu} = \frac{1}{4} \text{Tr}(\gamma_\mu(p + M) \gamma_\nu(p + q + M) \delta[(p + q)^2 - M^2])
\]

\[
= 2\delta(q^2 + 2q \cdot P) \left( \frac{q_\mu q_\nu}{q^2} - \frac{q_\mu q_\nu}{q^2} \right) - \frac{q \cdot P}{2} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right)
\]

i.e.

\[
\frac{v}{M^2} W_2 = \frac{1}{2} W_1 = 2v \delta(q^2 + 2q \cdot P)
\]

\[
\frac{\sigma_L}{\sigma_T} = \frac{M^2}{2v} \xrightarrow{v \to \infty} 0.
\]


S.D. Drell, D.J. Levy and Tung Mow Yan, SLAC Pub. 606, 645 and 685 (to be published in Phys. Rev.).


S.D. Drell, Lectures at Erice Summer School 1969 (SLAC Publication 689 to be published).


19) s (and not \( \nu \)) should be kept fixed or else the final states involved change with \( Q^2 \) (so that a change of \( Q^2 \) could lead in or out of the resonance region, for example) and 'smoothness' would be unreasonable.


24) Etim Etim (Naples preprint: to be published in Nuovo Cimento) has proposed a criterion for choosing a form factor but the motivation is not entirely clear and it seems to lead to \( \sigma_L / \sigma_T \propto Q^2 \), in contradiction to experiment (7).

25) A proper derivation of this result relies on a t-channel analysis of the reaction \( \gamma P \to \gamma P \). This is usually made in a standard way using helicity amplitudes; however, the complications of this method can be avoided by using a trick due to Fubini\(^{26}\).

Note that

\[
G_{\mu \nu} F(\nu, q^2) = \left( p_\mu - \frac{q_{\mu \nu}}{q^2} \right) \left( p_\nu - \frac{q_{\mu \nu}}{q^2} \right) \frac{\partial^2 F(\nu, q^2)}{\partial \nu^2}
\]

\[
G_{\mu \nu} = \left( g_{\mu \alpha} - \frac{q_{\mu q_{\alpha}}}{q^2} \right) \left( g_{\nu \beta} - \frac{q_{\mu q_{\beta}}}{q^2} \right) \frac{\partial}{\partial q_{\alpha}} \frac{\partial}{\partial q_{\beta}}.
\]

We put \( W_2 = \frac{\partial^2 F}{\partial \nu^2} \) so that

\[
W_{\mu \nu} = - \left( g_{\mu \nu} - \frac{q_{\mu q_{\nu}}}{q^2} \right) W_1 + G_{\mu \nu} F.
\]

The tensors in this equation depend only on the photon variables and are therefore invariant if (considering the reaction \( \gamma P \to \gamma P \) in the t-channel centre-of-mass) the proton-antiproton system is rotated with the two photon system fixed, i.e. \( W_1 \) and \( F \) are scalar amplitudes and have the Regge behaviour of the invariant amplitudes for scattering of scalar particles at \( t = 0 \), which is \( \nu \alpha(0) \).

This gives Eq. (21).

27) D.O. Caldwell et al., Santa Barbara preprint "Total hadronic (γp) and (γn) cross-sections from 4 to 18 GeV" (submitted to Phys. Rev. Letters).

28) Only $W_1$ contributes to the photoabsorption cross-section but

$W_1 \sim -\nu^2 W_2/M^2 q^2$ as $q^2 \rightarrow 0$. $W_2 = \frac{2F}{\nu^2}$ [cf. footnote 25] and if

$F_2(\nu,0) + c \nu^{\alpha(0)}$, $W_2 + c \alpha(0) \left[\alpha(0) - 1\right] \nu^{\alpha(0)-2}$ and the Pomeron with $\alpha(0) = 1$ decouples. However, the analytically continued partial wave amplitude $a(J,q^2,t)$ has a fixed pole at $J = 1$ \cite{29} as well as a moving pole at $\alpha(t)$:

\[ a(J,q^2,t) \sim \frac{\beta(q^2,t)}{(J-1)(J-\alpha(t))} \]

\[ F(\nu,q^2,t) \sim \int_c a(J,q^2,t) P_J(\nu) \xi(J) dJ \]

\[ \sim \frac{\beta(q^2,t)}{\alpha(t) - 1} \nu^{\alpha(t)} \xi(\alpha(t)) + \frac{\beta(q^2,t)\nu}{1 - \alpha(t)} \xi(1) \]

When $W_2 = \frac{2F}{\nu^2}$ is constructed, the contribution of the fixed pole vanishes and the pole in the Pomeron contribution cancels the factor $\alpha(t)-1$ from the derivative.


31) H. Harari, Phys. Rev. Letters 22, 1078 (1969); and Phys. Rev. Letters 24, 286 (1970). In the latter paper Harari discusses his model in parton language. However, the valence quarks do not behave as partons (i.e. as free particles) in his model - otherwise their contribution would be scale invariant. Harari's model and a pure parton model are therefore radically different (cf. Table 2).
32) R.A. Brandt has shown that this connection of the Bjorken and Regge limits follows from certain smoothness assumptions about the spectral functions in the D.G.S. representation. Phys. Rev. Letters 22, 1149 (1969) and Rockefeller University Preprint "Electroproduction Structure Functions, Integral Representations and Light Cone Commu-
tators".


35) H.T. Nieh, Stony Brook Preprint,"When will electro and photopro-
duction show diffractive features?"

(see also Ref. 61).

37) P. Landshoff, private communication.


39) For a review of the failure of formal arguments using the Bjorken limit see R. Jackiw, CERN preprint TH.1065.

40) L.S. Brown, Boulder Lectures (1969) to be published.

41) J.D. Bjorken and E.A. Paschos, SLAC Pub. 678 (to be published in
Phys. Rev.).

42) $W_3$ is the only term which contributes a pseudotensor to $W_{\mu\nu}$ and is therefore the vector (V)-axial vector (A) interference term.
Since the cross-section is a scalar, the contribution of $W_3$ to $m_{\mu\nu}W_{\mu\nu}$ must come from the V-A interference term in the lepton tensor. This term changes sign when $\nu \leftrightarrow \bar{\nu}$ as $V_{\mu} = \bar{u} \gamma_{\mu} u \leftrightarrow V_{\mu}$

$$A_{\mu} = -u \gamma_{\mu} \gamma_{5} u \leftrightarrow -A_{\mu}.$$  

43) See footnote 4). The extra inequalities for $W_4$, $W_5$ and $W_6$ which follow from the positive semi-definite nature of $W_{\mu\nu}$ are (Rdo.Dr. M.G. Doncel S.J., private communication):
\[ q^2 W_4 + 2uW_5 < 0 \]

\[ W_5^2 \leq \left( \frac{4\sigma}{\sigma} W_4 + \frac{2u}{\sigma} W_5 \right) \left( \frac{u}{\sigma} W_1 + \frac{\sigma}{q^2} W_2 \right) - W_5^2 
\]

\[ \left[ \sigma = \sqrt{v^2 - M^2 q^2} \right] \]

44) As \( E \to \infty \) the region in which Eq. (36) is supposed to be valid tends to 100% of the physical region.


46) The flux of high-energy neutrinos (coming from the K's in the hadron beam) was not well known so systematic errors could be considerable.

47) Note that Eq. (4) in Ref. 45 is wrong and does not follow from Eq. (3) (W. Venus - private communication).


52) On going from baryon \( \leftrightarrow \) antibaryon, \( 1 - \gamma_5 \leftrightarrow 1 + \gamma_5 \). The V-A interference term \( W_3 \) therefore changes sign.

53) It should not be concluded that if a large Q\( \bar{Q} \) sea is present \( F_3/F_1 \) is small for all \( \omega \). See, for example, Eq. (23), Ref. 48; the importance of the 'sea' contribution is \( \omega \) dependent.

54) This is not necessarily true if \( f_4^N(x) \) is different for different types of quark (antiquark); \( F_3^{\bar{N}} = F_3^N = 0 \) in all models provided \( \int d\omega \sum_N = \int d\omega \) (we have assumed this when stating that the sum rule for \( F_3^{\bar{P}} + F_3^{\bar{N}} \) holds in all models). This is not necessarily the case in Drell's model - see footnote 57).

55) This is interesting because the available data are from a C_1H_8 target\(^{15}\)) which is approximately an equal mixture of protons and neutrons.
56) In the crude model of p.10[with $\varepsilon \times (N - 3)$ of the N parton configuration in the gluon state]:

$$\sigma^{VP} + \sigma^{VN} - \sigma^{VP} - \sigma^{VN} = \frac{G_{ME}^2}{\pi} 4 \left\langle \frac{1}{N} \right\rangle < 0.72 \frac{G_{ME}^2}{\pi}.$$

57) In the simplest form of the model the Gell-Mann commutators are not satisfied (and the sum rules are therefore changed) because there is no axial partner of the pion. An interesting feature of the model is that a sum rule for $\int d\omega/\omega^2 (F^{VP}_3 + F^{VN}_3)$ is supposed to hold generally but $F^{VP}_3 + F^{VN}_3 \rightarrow \omega^\xi$, where $\xi$ is an arbitrary parameter. However, $\xi > 1$ would violate the sum rule (note that $F^{V}_2 \sim \omega^{5-1}$ and Pomeron dominance would require $\xi = 1$); which implies that an interchange of summation and integration fails in this case.


59) P. Landshoff, CERN preprint TH. 1180.

60) J.D. Bjorken, SLAC publication 670 (to be published).

61) Bjorken's result is actually:

$$\lim_{q^2 \rightarrow -\infty} \lim_{E \rightarrow \infty} \int \frac{d\nu}{\nu} \left( \frac{d^2 \sigma^{++}}{dq^2 d\nu} - \frac{d^2 \sigma^{++}}{dq^2 d\nu} \right) = -\frac{8\pi a^2 Z}{q^2 E}$$

where $\sigma^{++}$ is the cross-section when the electron and proton spins are parallel and along the direction of the electron beam and $\sigma^{++}$ the cross-section when they are antiparallel. $Z$ depends on the target and is model dependent but $Z_p - Z_n = \frac{1}{3}(g_A/g_V)$ with the $U(6) \times U(6)$ algebra; Bjorken used this to show that the polarization is $\geq 20\%$ for either a neutron or a proton target. Here we will examine this result in the parton model which will then be used to bound the polarization for a proton target. In the parton model

$$\frac{d\sigma^{++}}{dq^2 d\nu} - \frac{d\sigma^{++}}{dq^2 d\nu} = \frac{4\pi a^2}{E q^2 \nu} \sum_i \sum_N \frac{f_i(x)(n_{iN}^{++} - v_{iN}^{++})Q_i^2}{P_N}$$
in an obvious notation. Hence

\[ \sum_{\nu} \frac{d^2 \sigma^{\uparrow \downarrow}}{d^2 \nu} \left( \frac{d^2 \sigma^{\uparrow \downarrow}}{d^2 \nu} \right) = \frac{8\pi\alpha_s^2}{E_\nu} \sum_{N} P_N \sum_{i} (n_{N_i}^+ - n_{N_i}^-) Q_i^2. \]

In order to reach Bjorken's result we must apply the parton model at
\[ q^2 = 0 \] (where it is not really supposed to hold) to calculate \( g_A^z / g_V^z \). Calculating \( < P | \bar{\psi} I_3 \gamma \mu \gamma_5 \psi | P > = \frac{1}{2} < P | \bar{\psi} I_+ \gamma_\mu \gamma_5 \psi | n > \) we find

\[ \frac{g_A^z}{g_V^z} = \sum_{N} P_N \left[ (n_{N_i}^+ - n_{N_i}^-) - (n_{N_i}^+ - n_{N_i}^-) \right] \]

where \( n_{Nq}^\uparrow \) = the sum of the number of q and \( \bar{q} \) quarks with spin-up in the N parton configuration of the proton. Combined with the preceding equation this gives Bjorken's result \( Z_P^z - Z_n^z = \frac{1}{2}(g_A^z / g_V^z) \).

(We repeat that this is not really a derivation even in the framework of the parton model.)

Since in all quark parton models the proton contains an odd number of neutron quarks we get the bound

\[ |Z_P^z| = |\sum_{N} P_N \sum_{i} (n_{N_i}^+ - n_{N_i}^-) Q_i^2|_{\text{proton}} \geq \frac{1}{9} \]

Hence (using Bjorken's 20% estimate), the polarization with a proton target is \( > 6\% \) in all quark parton models. This argument can be used to show that \( (g_A^z / g_V^z) \gg 1 \) in these models. (Unfortunately, the argument cannot be extended to neutron targets).

Polarizations may be calculable for neutrino scattering from polarized targets but such experiments are hardly feasible at present.

(The considerations in this footnote were formulated in collaboration with Dr. A. de Rújula).


63) An opposite extreme occurs in the statistical model considered by J.D. Bjorken and S. Brodsky (SLAC Publication 665, 1969). All the secondaries have a momentum cut-off (\( \sim 500 \) MeV) in the centre-of-mass which forces the multiplicity to grow as \( < n > \sim E_{\text{c.m.}} \).
65) L. Lederman, invited talk at the Daresbury conference.
68) R.F. Kögler and R.M. Muradyan (Dubna preprint E2-4791) have discussed \( \gamma + p + a + \ldots \) and \( p + \bar{a} + \gamma + \ldots \) following Bjorken's type of analysis but the assumptions used are rather dubious.
(Wisconsin publication.)
72) If we do not put \( \theta_c = 0 \) the results in the text follow provided we make the substitution:

\[
\begin{align*}
\bar{W}^{\gamma,\bar{\nu}}_i &\rightarrow \cos^2 \theta_c \bar{W}^{\gamma,\bar{\nu}}_i + \sin^2 \theta_c \omega^{\gamma,\bar{\nu}}_i \\
F^{\gamma,\bar{\nu}}_i &\rightarrow \cos^2 \theta_c F^{\gamma,\bar{\nu}}_i + \sin^2 \theta_c f^{\gamma,\bar{\nu}}_i
\end{align*}
\]

in Eqs. (26), (27), (31), (32) and (33). The \( \bar{W}_i (F_i) \) describe \( \Delta S = 0 \) reactions, and the \( \omega_i (f_i) \), \( \Delta S = 1 \) reactions. Adler has derived a sum rule for \( \omega^\gamma_2 - \omega^\gamma_{21} \) and there is a sum rule for \( f^\gamma_3 + f^\gamma_{\bar{3}} \).

Using the methods of Ref. 49 the following results can be derived in the gluon model:

\[
\begin{align*}
\int \frac{d\omega}{\omega^3} (f_1^{\gamma P} + f_1^{\bar{\nu} P}) &= \int \frac{d\omega}{\omega^3} (12F_1^{\gamma P} + 6F_1^{\gamma n} - 2F_1^{\gamma P} - 2F_1^{\gamma n}) \\
\int \frac{d\omega}{\omega^3} (f_1^{\gamma n} + f_1^{\bar{\nu} n}) &= \int \frac{d\omega}{\omega^3} (12F_1^{\gamma n} + 6F_1^{\gamma P} - 2F_1^{\gamma P} - 2F_1^{\gamma n}) \\
\int \frac{d\omega}{\omega^3} (f_3^{\gamma P} - f_3^{\bar{\nu} P}) &= \int \frac{d\omega}{\omega^3} (-12F_1^{\gamma P} - 24F_1^{\gamma n} + 6F_1^{\gamma P} + 6F_1^{\gamma n}) \\
\int \frac{d\omega}{\omega^3} (f_3^{\gamma n} - f_3^{\bar{\nu} n}) &= \int \frac{d\omega}{\omega^3} (-12F_1^{\gamma n} - 24F_1^{\gamma P} + 6F_1^{\gamma P} + 6F_1^{\gamma n})
\end{align*}
\]
In all quark parton models these results hold in unintegrated form and in addition, the following results hold:

\[ f_1^{VP} - f_1^{Vn} = \frac{1}{2} (f_3^{VP} - f_3^{Vn}) = \frac{1}{2} (f_1^{VP} - f_1^{Vn}) + 3 (f_1^{\gamma P} - f_1^{\gamma n}) \]

\[ f_1^{\bar{V}P} - f_1^{\bar{V}n} = \frac{1}{2} (f_3^{\bar{V}P} - f_3^{\bar{V}n}) = -\frac{1}{2} (f_1^{\bar{V}P} - f_1^{\bar{V}n}) + 3 (f_1^{\gamma P} - f_1^{\gamma n}) . \]

These results are hard to test. More specific models make stronger predictions; e.g. in model A (p. 23) with the additional assumption that all types of quark and antiquark have the same \( f_N(x) \), everything can be predicted in terms of \( e^+p \) and \( e^-n \) data:

\[ f_1^{VP} = f_1^{\bar{V}n} = 3f_1^{\gamma n} \]

\[ f_1^{Vn} = f_1^{\bar{V}P} = 6f_1^{\gamma P} - 3f_1^{\gamma n} \]

\[ f_3^{VP} = f_3^{Vn} = 12f_1^{\gamma n} - 12f_1^{\gamma P} \]

\[ f_3^{Vn} = f_3^{\bar{V}P} = -24f_1^{\gamma P} + 24f_1^{\gamma n} \]

\[ f_1^{\bar{V}P} = f_1^{\bar{V}n} = 9f_1^{\gamma n} - 6f_1^{\gamma P} \]

\[ f_3^{\bar{V}P} = f_3^{\bar{V}n} = 0 . \]

These predictions are sufficiently precise to be destroyed, not only by \( \Delta S = 0 \) reactions, but also by \( \Delta S = 1 \) reactions.

Boundary of physical region for inelastic scattering.

I  Elastic scattering  \( q^2 + 2\nu = 0 \).

II  Threshold for pion production  \( q^2 + 2\nu = 2M\pi + M_\pi^2 \).

III  Line of fixed missing mass \( M^* \):  \( q^2 + 2\nu + M^2 = M'^2 \).

IV  'Experimental' line of fixed incident energy and fixed scattering angle  \( -q^2 = 4E(E - \nu) \sin^2 \theta/2 \).
Fig. 2 \((\sigma/\sigma_{\text{Mott}})^{\gamma p} = W_2 + 2W_1 \tan^2 \theta/2\) plotted against \(|q^2|\) at fixed missing mass \(W\).
Fig. 3 $v W^p_2$ (deduced from $\theta = 10^\circ$ data assuming (a) $\sigma_L/\sigma_T = \infty$,
(b) $\sigma_L/\sigma_T = 0$) plotted against $\omega = 2v/q^2$. 
Fig. 4

Boundary of inelastic region.

I  Regge pole dominance to the right of this line $\nu = N(q^2)$.

II  Pomeron dominance to the right of this line.
Fig. 5  Total neutrino-nucleon cross-sections as a function of neutrino energy E. The freon cross-sections have been multiplied by 1.35 to normalize them to the propane cross-sections for E > 2 GeV. The errors shown are statistical only.
Fig. 6 Distribution of number of events in $\rho = \sqrt{Q^2} = \omega/2$ for neutrino events with $E_\nu > 4$ GeV.
Fig. 7 Distribution in $x = \nu/ME$ of events with $E_\nu > 2$ GeV and $\rho = \nu/Q^2 > 1.5$. The contributions of the different structure constants are shown assuming $F_1(\rho)/\rho$, $F_2(\rho)$, $F_3(\rho)/\rho$ are constants $K_1$, $K_2$ and $-2K_3$ and putting $K_1 = K_2 = K_3$. 