ORIGINS OF THE BARYON SPECTRUM

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Abstract

I begin with a key problem of light and strange baryon spectroscopy which suggests a clue for our understanding of underlying dynamics. Then I discuss spontaneous breaking of chiral symmetry in QCD, which implies that at low momenta there must be quasiparticles - constituent quarks with dynamical mass, which should be coupled to other quasiparticles - Goldstone bosons. Then it is natural to assume that in the low-energy regime the underlying dynamics in baryons is due to Goldstone boson exchange (GBE) between constituent quarks. Using as a prototype of the microscopical quark-gluon degrees of freedom the instanton-induced 't Hooft interaction I show why the GBE is so important. When the 't Hooft interaction is iterated in the qq t-channel it inevitably leads to a pole which corresponds to GBE. This is a typical antiscreening behavior: the interaction is represented by a bare vertex at large momenta, but it blows up at small momenta in the channel with GBE quantum numbers, explaining thus a distinguished role of the latter interaction in the low-energy regime. I show how the explicitly flavour-dependent short-range part of the GBE interaction between quarks, perhaps in combination with the vector-meson exchange interaction, solves a key problem of baryon spectroscopy and present spectra obtained in a simple analytical calculation as well as in exact semirelativistic three-body approach.

1 Where is a key problem?

If one considers a model with an effective confining interaction between quarks in light and strange baryons, which is flavour- and spin-independent and assuming that there are no residual interactions, then the spectrum of lowest baryons should be arranged into successive bands of positive and negative parity, see Fig. 1. In Nature, however, the lowest levels in the spectra of nucleon, Δ-resonance and Λ-hyperon, which are shown on Fig. 2, look pretty different. One can immediately conclude that a picture, where all other possible interactions are treated as only residual and weak is certainly wrong.

\footnote{The Thomas precession, which is a kinematical effect, and which produces a strong spin-orbit force, certainly presents in heavy quark systems, where the heavy quark constantly sits on the end of the string. A relativistic rotation of the string implies the Thomas precession. In the light quark systems, where it costs no energy to break a string and the light quark permanently fluctuates into other quark and the quark-antiquark pair, this kinematical effect should be strongly suppressed. That is why there are no strong spin-orbit splittings in light baryon and meson spectra.}
Typically models pay an attention to the octet-decuplet splittings. Within a quark picture one needs a spin-spin force between valence quarks with a proper sign. Then, adjusting a strength of this spin-spin force one can explain why $\Delta$ is heavier than nucleon, or why $\Sigma$ is heavier than $\Lambda$ [1]. When QCD appeared, it has been immediately suggested that such a spin-spin force is supplied by the colour-magnetic component of the one gluon exchange (OGE) [2, 3, 4], in analogy with the magnetic hyperfine interaction from the one photon exchange in quantum electrodynamics. At the price of a very large strong coupling constant, $\alpha_s \sim 1$, one can then fit $\Delta - N$ mass difference. Clearly that such a picture is self-contradictory, because a big value of $\alpha_s$ is not compatible with the perturbative treatment of QCD.

The crucial point, however, is that the perturbative gluon exchange (does not matter, one gluon exchange or one thousand gluon exchange) is sensitive only to spin (and colour) degrees of freedom of quarks and there is no sensitivity at the operator level to the flavour of quarks (in the u,d,s quark sector there is only a very weak sensitivity via different masses of quarks which, however, completely vanishes in the chiral limit). The spin structure of all baryons in $N$ and $\Lambda$ spectra, depicted in Fig. 2, is the same, it is described by the mixed permutational symmetry. This means that the contribution of the colour-magnetic interaction to leading order is the same in all these baryons (up to some small difference in baryon orbital wave functions), which is in apparent conflict with the opposite orderings of the lowest levels in $N$ and $\Lambda$ spectra. The only difference between $N$ and $\Lambda$ system is that one light quark is substituted by a strange one. It immediately hints that the physics, responsible for Fig. 2, should be explicitly flavour dependent. In addition, a colour magnetic interaction cannot shift the $N = 2$ states $N(1440)$ and $\Lambda(1600)$ below the $N = 1$ states $N(1535) - N(1520)$ and $\Lambda(1670) - \Lambda(1690)$, respectively, because to leading order its contribution is the same in all these states. In the $\Delta$ spectrum the situation is even more dramatic as the colour - magnetic interaction shifts the $N = 2$ state $\Delta(1600)$ up, but not down, with respect to the $N = 1$ pair $\Delta(1620) - \Delta(1700)$.

These facts rule out perturbative gluon exchange picture as a source of the hyperfine interactions in the light and strange baryons.

The other possible source of the hyperfine interactions, the ’t Hooft instanton induced interaction [5] between valence quarks, could, generally speaking, generate the octet-decuplet splittings [6, 7, 8] when its strength is adjusted. However, it is easy to see from its operator structure that it also fails to explain Fig. 2 as far as $N$ and $\Lambda$ parts are
concerned. But the most convincing evidence comes from the $\Delta$ spectrum, where the ’t Hooft interaction between valence quarks is identically zero (it is absent in flavour - symmetric states). So according to this scenario the $\Delta$ spectrum should be exclusively due to confining interaction, which is ruled out by comparison of Figs. 1 and 2.

Thus a key problem is to explain at the same time both the octet-decuplet splittings and the pattern of Fig. 2.

2 Spontaneous breaking of chiral symmetry and its implications

The $SU(3)_L \times SU(3)_R$ chiral symmetry of QCD Lagrangian is spontaneously broken down to $SU(3)_V$ by the QCD vacuum (in the large $N_c$ limit it would be $U(3)_L \times U(3)_R \rightarrow U(3)_V$). There are two important generic consequences of the spontaneous breaking of chiral symmetry (SBCS). The first one is an appearance of the octet of pseudoscalar mesons of low mass, $\pi, K, \eta$, which represent the associated approximate Goldstone bosons (in the large $N_c$ limit the flavor singlet state $\eta'$ should be added). The second one is that valence (practically massless) quarks acquire a dynamical mass, which has been called historically constituent mass. Indeed, the nonzero value of the quark condensate, $<\bar qq> \sim -(250\text{MeV})^3$, itself implies at the formal level that there must be at low momenta rather big dynamical mass, which should be a momentum-dependent quantity. Such a dynamical mass is now directly observed on the lattice [9]. Thus the constituent quarks should be considered as quasiparticles whose dynamical mass at low momenta comes from the nonperturbative gluon and quark-antiquark dressing. The flavour-octet axial current conservation in the chiral limit tells that the constituent quarks and Goldstone bosons should be coupled with the strength $g = g_M f_\pi / M$ [10], which is a quark analog of the famous Goldberger-Treiman relation.

We have recently suggested that in the low-energy regime, below the chiral symmetry breaking scale, $\Lambda_\chi \sim 1 \text{ GeV}$, the low-lying light and strange baryons should be predominantly viewed as systems of 3 constituent quarks with an effective confining interaction
and a chiral interaction mediated by a Goldstone boson exchange (GBE) between the constituent quarks [13].

### 3 Why the Goldstone boson exchange is so important?

Consider as example of a microscopical QCD nonperturbative interaction the instanton-induced 't Hooft interaction for two light flavours (I consider for simplicity a chiral limit)

$$H = -G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2 - (\bar{\psi}\tau\psi)^2 - (\bar{\psi}i\gamma_5\psi)^2].$$

This interaction is known to lead to chiral symmetry breaking, i.e. to creation of the quark condensate and dynamical (constituent) mass $m$ of quarks. It happens because of the first term in (1), which represents a scalar part of the interaction. The interquark interaction in the pseudoscalar-isovector $\bar{q}q$ systems is driven by the second term, which is attractive and so strong that when it is iterated it exactly compensates the $2m$ energy supplied by the first term, and thus there appear $T=1, J^P=0^-$ mesons with zero mass - Nambu-Goldstone bosons. The first two terms in the Hamiltonian above form a classical Nambu and Jona-Lasinio model [14]. The fourth term in (1), which is repulsive, contributes only in the flavour-singlet $\bar{q}q$ pair ($\eta'$), making this meson heavy - contrary to $\pi$ - and solving thus the $U(1)_A$ problem (note that the perturbative gluon exchange force cannot solve it). There is no interaction term which can contribute in vector mesons. This means that the masses of vector mesons, $\rho$ and $\omega$, should be approximately $2m$, which is well satisfied empirically. The interaction (1), extended to the $u,d,s$ sector, also naturally explains completely different mixing between the octet and singlet components in the pseudoscalar and vector mesons [15].

Having mentioned all the positive features of the Hamiltonian (1) in mesons, I shall now discuss its implications in baryons [16]. As I said, a direct application of this instanton-induced interaction between valence quarks in baryons does not solve problems. But what happens when this interaction is iterated in $qq$ t-channel, see Fig. 3? Specifically, the second term in (1) will imply the following amplitude

$$T_P(q^2) = 2G + 2GJ_P(q^2)2G + ... = \frac{2G}{1 - 2GJ_P(q^2)},$$

where $J_P(q^2)$ is a bubble with a pseudoscalar vertex (vacuum polarization in the pseudoscalar channel). The denominator in (2) has a pole in the chiral limit at $q^2 = 0$, which can be identified as a pion-exchange (beyond the chiral limit it is shifted to a physical
pion mass $q^2 = \mu^2$. The coupling constant of pion to constituent quark can be obtained as a residue of (2) at the pole. The eq. (2) defines a "running amplitude" and a negative sign in the denominator implies its antiscreening behavior. In essence this antiscreening is some kind of asymptotic freedom: at sufficiently large space-like momenta the interaction is represented by a pure 't Hooft vertex (i.e. it has a strength $2G$), but at $q^2 \rightarrow 0$ it becomes infinitely enhanced in the channel with GBE quantum numbers. So, if a typical momentum transfer is not large, which is the case in baryons in the low-energy regime, the pole contribution dominates. It explains why the GBE is so crucially important both in baryons and baryon-baryon systems. Thus the GBE interaction between constituent quarks is an effective representation of the pole contribution in (2), which is provided by the original quark-gluon degrees of freedom.

In fact any pairwise gluonic interaction between quarks in the local approximation will necessarily contain the first and second terms of (1) with fixed relative strength. This is because of chiral invariance. Thus all our conclusions on $\pi - \rho$ mass splitting and Goldstone boson exchange interaction in baryons are rather general and do not rely necessarily on 't Hooft interaction.

4 The Goldstone boson exchange interaction

The coupling of the constituent quarks and the pseudoscalar Goldstone bosons will (in the $SU(3)_F$ symmetric approximation) have the form $g/(2m)\bar{\psi}\gamma_\mu \gamma_5 \vec{X}^F \cdot \psi \partial^\mu \bar{\phi}$ within the nonlinear realization of chiral symmetry (it would be $ig\bar{\psi}\gamma_5 \vec{X}^F \cdot \bar{\phi} \psi$ within the linear chiral symmetry representation). A coupling of this form, in a nonrelativistic reduction for the constituent quark spinors, will – to lowest order – give rise the $\sim \vec{\sigma} \cdot \vec{q} \vec{\lambda}$ structure of the meson-quark vertex, where $\vec{q}$ is meson momentum. This type of vertex implies spin-spin and tensor interactions between constituent quarks, mediated by Goldstone bosons. The spin-spin force has a traditional long-range Yukawa part, which is important for nuclear force. But at short range the spin-spin force is much stronger and its sign is opposite. This short-range interaction has a form [13]

$$H_\chi \sim - \sum_{i<j} \frac{V(\vec{r}_{ij})}{m_i m_j} \vec{X}_i^F \cdot \vec{X}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j,$$

where a radial behavior of this short-range interaction is unknown. It is this short-range part of the GBE interaction between the constituent quarks that is of crucial importance for baryons: it has a sign appropriate to reproduce the level splittings and strongly dominates over the Yukawa tail towards short distances. Note that this spin-spin force is explicitly flavour-dependent, which reflects the fact that the GBE interaction is a flavour-exchange one. It is also significant that this short-range part of the interaction appears at the leading order within the chiral perturbation theory (i.e. in the chiral limit) [17], while the Yukawa part of the interaction vanishes in this limit. This simple observation has by far-going consequences: while the physics of baryons does not change much in the chiral limit (e.g. the $\Delta - N$ mass splitting persists), the long-range nuclear spin-spin force vanishes. This means that in some sense the short-range part of the pion exchange interaction is "more fundamental" than its Yukawa part.
5 The vector- and scalar-exchange interactions

Already in ref. [13] it has been pointed out that the vector-like meson exchange interactions could be also important. This possibility is taken seriously in refs. [18, 19]. Both the vector- and scalar-meson exchange interaction can be also considered as a representation of the correlated two GBE interaction [20] as it has a vector meson pole in t-channel. A phenomenological motivation to include these interactions in addition to one GBE is as follows. The spin-spin component of the vector-meson exchange interaction at short range has exactly the same flavor-spin structure (3) as one GBE, but their tensor force components are just of opposite sign and cancel each other to a big extent. This could explain an empirical fact that the tensor force component of the interaction between quarks in baryons should not be large. Otherwise it would cause small, but empirically counterindicated spin-orbit splittings in L=1 baryons. The small net tensor force component should be, however, important for the mixing in baryon wave functions, while the baryon mass is weakly sensitive to this small residual tensor force. The present uncertainties in the coupling constants and unknown short-range behavior of these effective interactions make it very difficult to determine a precise amount (and even sign) of this weak net tensor force from the low-lying baryon spectroscopy. Other datum, e.g. mixing angles extracted from strong and electromagnetic decays should be used to determine the precise relative contributions of the effective ps- and vector-exchanges.

The scalar- and vector-meson exchanges have spin-orbit force components. These spin-orbit forces are known to be very important in \(NN\) system, where both \(\rho\)- and \(\omega\)-exchange provide spin-orbit force with the same sign in P-wave. In baryons the relative sign of these spin-orbit components becomes opposite in P-wave (because of additional colour degree of freedom) and the \(\rho\)-exchange spin-orbit force becomes strongly enhanced [18]. This explains a weak net spin-orbit force in baryons, while it is big and empirically very important in baryon-baryon systems.

6 The flavour-spin hyperfine interaction and the structure of the baryon spectrum

Summarizing previous sections one concludes that the pseudoscalar- and vector-meson exchange interactions produce strong flavour-spin interaction (3) at short range while the net tensor and spin-orbit forces are rather weak. That the net spin-orbit and tensor interactions between constituent quarks in baryons should be weak also follows from the typically small splittings in LS-multiplets, which are of the order 10-30 MeV. These small splittings should be compared with the hyperfine splittings produced by spin-spin force, which are of the order of \(\Delta - N\) splitting. Thus, indeed, in baryons it is the spin-spin interaction (3) between constituent quarks that is of crucial importance.

Consider first, for the purposes of illustration, a schematic model which neglects the radial dependence of the potential function \(V(r)\) in (3), and assume a harmonic confinement among quarks as well as \(m_u = m_d = m_s\). In this model

\[
H_x = - \sum_{i<j} C_x \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j.
\]
The Hamiltonian (4) reduces the $SU(6)_{FS}$ symmetry down to $SU(3)_{F} \times SU(2)_{S}$. Let us now see how the pure confinement spectrum of Fig. 1 becomes modified when the Hamiltonian (4) is switched on. The leading $SU(6)$ wave functions are known for all low-lying baryons and we thus can evaluate analytically the expectation values of the operator (4) [13].

For the octet states $N, \Lambda, \Sigma, \Xi$ ($N = 0$ shell) as well as for their first radial excitations of positive parity $N(1440), \Lambda(1600), \Sigma(1660), \Xi(?)$ ($N = 2$ shell) the expectation value of the Hamiltonian (4) is $-14C_{\chi}$. For the decuplet states $\Delta, \Sigma(1385), \Xi(1530), \Omega$ ($N = 0$ shell) and their lowest radial excitations of positive parity $\Delta(1600)$ ($N = 2$) the corresponding matrix element is $-4C_{\chi}$. In the negative parity excitations ($N = 1$ shell) in the $N, \Lambda$ and $\Sigma$ spectra ($N(1535) - N(1520), \Lambda(1670) - \Lambda(1690)$ and $\Sigma(1750) - \Sigma(?)$) the contribution of the interaction (4) is $-2C_{\chi}$. The first negative parity excitations in the $\Delta$ spectrum $\Delta(1620)$ and $\Delta(1700)$ ($N = 1$) produce the matrix element $4C_{\chi}$. The first negative parity excitation in the $\Lambda$ spectrum ($N = 1$ shell) $\Lambda(1405) - \Lambda(1520)$ is flavor singlet and, in this case, the corresponding matrix element is $-8C_{\chi}$. The latter state is unique and is absent in other spectra due to its flavour-singlet nature.

These matrix elements alone suffice to prove that the ordering of the lowest positive and negative parity states in the baryon spectrum will be correctly predicted by the chiral boson exchange interaction (4). The constant $C_{\chi}$ may be extracted from the $N - \Delta$ splitting to be $29.3$ MeV. The oscillator parameter $\hbar\omega$, which characterizes the effective confining interaction with this schematic model, may be determined as one half of the mass differences between the first excited $\frac{1}{2}^+$ states and the ground states of the baryons, which have the same flavour-spin, flavour and spin symmetries (e.g. $N(1440) - N, \Lambda(1600) - \Lambda, \Sigma(1660) - \Sigma$), to be $\hbar\omega \simeq 250$ MeV. Thus the two free parameters of this simple model are fixed and we can make now predictions.

In the $N, \Lambda$ and $\Sigma$ sectors the mass difference between the lowest excited $\frac{1}{2}^+$ states ($N(1440), \Lambda(1600), \Sigma(1660)$) and the $\frac{1}{2}^- - \frac{3}{2}^-$ negative parity pairs ($N(1535) - N(1520), \Lambda(1670) - \Lambda(1690), \Sigma(1750) - \Sigma(?)$, respectively) will then be

$$m(\frac{1}{2}^+) - m(\frac{1}{2}^- - \frac{3}{2}^-) = 250 \text{ MeV} - C_{\chi}(14 - 2) = -102 \text{ MeV}, \quad (5)$$

whereas for the lowest state $\frac{1}{2}^+$ in the $\Lambda$ system, $\Lambda(1600)$, and the lowest negative parity pair $\frac{1}{2}^- - \frac{3}{2}^- (\Lambda(1405) - \Lambda(1520))$ it should be

$$m(\frac{1}{2}^+) - m(\frac{1}{2}^- - \frac{3}{2}^-) = 250 \text{ MeV} - C_{\chi}(14 - 8) = 74 \text{ MeV}. \quad (6)$$

At last, the lowest positive parity state $\frac{3}{2}^+$ in the $\Delta$ spectrum $\Delta(1600)$ should be approximately degenerate with the lowest negative parity $\frac{1}{2}^- - \frac{3}{2}^-$ excitations $\Delta(1620) - \Delta(1700)$

$$m(\frac{3}{2}^+) - m(\frac{1}{2}^- - \frac{3}{2}^-) = 250 \text{ MeV} - C_{\chi}(4 + 4) = 15 \text{ MeV}. \quad (7)$$

Note that the experimental uncertainties for the mass of this baryon are 1550 - 1700 MeV.
One recovers precisely the spectrum shown in Fig. 2. It is astonishing that such a crude model predicts not only a general structure of the low-lying spectrum, but also the absolute values for splittings.

This simple example shows how the chiral interaction provides different ordering of the lowest positive and negative parity excited states in the spectra of the nucleon and the Λ-hyperon. This is a direct consequence of the symmetry properties of the boson-exchange interaction [13]. Namely, completely symmetric FS state in the N(1440), Λ(1600) and Σ(1660) positive parity resonances from the \( N = 2 \) band feels a much stronger attractive interaction than the mixed symmetry FS state in the N(1535) - N(1520), Λ(1670) - Λ(1690) and Σ(1750) - Σ(?) resonances of negative parity (\( N = 1 \) shell). Consequently the masses of the positive parity states N(1440), Λ(1600) and Σ(1660) are shifted down relative to the other ones, which explains the reversal of the otherwise expected "normal ordering" of Fig. 1. The situation is different for Λ(1405) - Λ(1520) and Λ(1600), as the flavour state of Λ(1405) - Λ(1520) is totally antisymmetric. Because of this the Λ(1405) - Λ(1520) gains an attractive energy, which is comparable to that of the Λ(1600), and thus the ordering suggested by the confining oscillator interaction is maintained.

If one goes beyond the schematic - but analytical - calculation above, one should parameterize the short range parts of the interaction (the long range parts are fixed by meson masses), extract approximate meson-quark coupling constants from the known meson-baryon ones and solve with computer 3 - body equations. Such a program, with a semirelativistic Hamiltonian (i.e. kinetic energy is taken in a relativistic form) and with the linear confinement, has been realized in refs. [21, 19]. In the former case [21] only the spin-spin force of GBE interaction is included, while in the latter one [19] ps-, vector- and scalar-exchanges are considered with spin-spin, tensor and central force components. The spectra in both cases look pretty much the same, which is achieved by a slight readjustment of the cut-off parameters in the latter case, see Fig. 4.

It is clear that the higher Fock components \( QQ\pi, QQK, \ldots \) (including meson continuum) cannot be completely integrated out in favor of the meson-exchange \( Q - Q \) potentials for some states above or near the corresponding meson thresholds. Such components, in addition to the main one \( QQ \), could explain e.g. an exceptionally large splitting of the flavour singlet states \( \Lambda(1405) - \Lambda(1520) \), since the \( \Lambda(1405) \) lies below the \( KN \) threshold and can be presented as \( KN \) bound system [22]. Note, that in the case of the present approach this old idea is completely natural and does not contradict a flavour-singlet \( QQ \) nature of \( \Lambda(1405) \) (it simply means that both \( QQ \) and \( QQK \) components are significant in the present case) while it would be in conflict with naive constituent quark model where no room for mesons in baryons. The alternative explanation of the latter extraordinary big LS splitting would be that there is some rather large spin-orbit force specific to the flavour-singlet state only, which is also not ruled out.

An admixture of higher Fock components will be important in order to understand strong decays of some excited states, especially in the case where the threshold in the decay channel is close to the resonance energy. While technically inclusion of such components in addition to the main one \( QQ \) in a coupled-channel approach is rather difficult task, it should be considered as one of the most important future directions.
Figure 4: Energy levels of the low-lying light- and strange-baryon states with total angular momentum and parity $J^P$ (solid lines). The shadowed boxes represent the experimental values with their uncertainties.
7 Instead of a conclusion

Similar conclusions, that it is a GBE force which is responsible for \( \Delta - N \) splitting have been obtained in a recent lattice study [23]. A phenomenological analysis of the \( L=1 \) negative parity spectra [24] as well as \( 1/N_c \) expansion studies of \( L=1 \) nonstrange spectra and of mixing angles obtained in strong and electromagnetic decays [25], also give a credibility to the interaction (3).

Finally, it is worth to mention, that this quark-quark interaction in the baryon-baryon systems provides a strong short-range repulsive core [26, 27].

References


\[ \Delta(1600) - \Delta(1700) \quad \Lambda(1405) - \Lambda(1520) \]

\[ \Lambda(1600) - \Lambda(1670) - \Lambda(1690) \]

\[ + \quad N(1440) \]

\[ - \quad N(1535) - N(1520) \]

\[ + \quad N \]

\[ - \quad \Delta(1620) - \Delta(1700) \quad \Delta(1600) \]

\[ + \quad \Delta \]

\[ - \quad \Lambda(1600) \]

\[ + \quad \Lambda(1405) - \Lambda(1520) \]

\[ - \quad \Lambda(1670) - \Lambda(1690) \]

\[ + \quad \Lambda \]

\[ + \quad \Delta \]

\[ + \quad \Lambda \]