SO(3) Gauge model for neutrino masses and oscillations

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Abstract

I mainly describe neutrino masses and oscillations in the gauge model with $SO(3)_F$ lepton flavor symmetry and with two Higgs triplets. It is shown how the maximal mixing between $\nu_\mu$ and $\nu_\tau$ neutrinos comes out naturally after spontaneous breaking of the symmetry. The nearly two-flavor mixing scenario is resulted naturally from an approximate permutation symmetry between the two Higgs triplets. The hierarchy between the neutrino mass-squared differences, which is needed for reconciling both solar and atmospheric neutrino data, leads to an almost maximal mixing between $\nu_e$ and $\nu_\mu$ neutrinos. Thus the model favors the intriguing bi-maximal mixing scenario. The three Majorana neutrino masses are allowed to be nearly degenerate and large enough to play a significant cosmological role. The model can also lead to interesting phenomena on lepton-flavor violations via the $SO(3)_F$ gauge interactions.

*Supported in part by Outstanding Young Scientist Research Fund of China, under grant No. of NSF of China: 19625514.
1 INTRODUCTION

The standard model (SM) has been tested by more and more precise experiments, its greatest success is the gauge symmetry structure $\text{SU}(3)_c \times \text{SU}_L(2) \times \text{U}_Y(1)$. While neutrinos are assumed to be massless in the SM. Studies on neutrino physics have resulted in the following observations: i), The Super-Kamiokande data\cite{1} on atmospheric neutrino anomaly provide a strong evidence that neutrinos are massive; ii), The current Super-Kamiokande data on solar neutrino\cite{1} cannot decisively establish whether the deficit of the measured solar neutrino flux results from MSW solutions\cite{2} with large/small mixing angles\cite{3} or vacuum oscillation solutions\cite{4}. iii), To describe all the neutrino phenomena such as the atmospheric neutrino anomaly, the solar neutrino deficit and the results from the LSND experiment, it is necessary to introduce a sterile neutrino. It indicates that with only three light neutrinos, one of the experimental data must be modified; iv), The current experimental data cannot establish whether neutrinos are Dirac-type or Majorana-type. The failure of detecting neutrinoless double beta decay only provides, for Majorana-type neutrinos, an upper bound on an ‘effective’ electron neutrino mass; v), Massive neutrinos are also regarded as the best candidate for hot dark matter and may play an essential role in the evolution of the large-scale structure of the universe\cite{5}.

To introduce neutrino masses and mixings, it is necessary to modify and go beyond the SM. As a simple extension of the standard model, it is of interest to introduce a flavor symmetry among the three families of the leptons. In the recent papers\cite{6, 7}, we have introduced the gauged $\text{SO}(3)_F$ flavor symmetry\cite{8} to describe the three lepton families. Some remarkable features have been found to be applicable to the current interesting phenomena of neutrinos. After a detailed analysis on various possible scenarios, we have shown that the nearly degenerate neutrino mass and bi-maximal mixing scenario\cite{9} is the most favorable one in our model with two Higgs triplets\cite{7} to reconcile both solar and atmospheric neutrino flux anomalies. In this talk, I will briefly review those interesting features and try to explicitly explore the naturalness for some of the features. To understand the naturalness of the scenario, we will pay attention to the spontaneous breaking of the $\text{SO}(3)_F$ flavor symmetry in the Higgs sector. As a consequence, the maximal mixing between $\nu_\mu$ and $\nu_\tau$ neutrinos, which is needed for explaining the observed atmospheric neutrino anomaly, comes out naturally after spontaneous symmetry breaking. By considering the approximate permutation symmetry between the two Higgs triplets and by using the data of the neutrinoless double $\beta$ decay or the fact of the hierarchy between the two mass-squared differences, we then arrive at the nearly degenerate neutrino mass and bi-maximal mixing scenario.
2 THE MODEL

For a less model-dependent analysis, we directly start from an $SO(3)_F \times SU(2)_L \times U(1)_Y$ invariant effective lagrangian with two $SO(3)_F$ Higgs triplets

$$\mathcal{L} = \frac{1}{2} g'_i A^k_i (\bar{L}_i \gamma^\mu (t^k)_{ij} L_j + \bar{e}_{Ri} \gamma^\mu (t^k)_{ij} e_{Rj})$$

$$+ [(c_1 \varphi_i \varphi_j + c'_1 \varphi'_i \varphi'_j + c''_1 \delta_{ij} \chi''^0)] \bar{L}_i \phi_1 e_{Rj}$$

$$+ [(c_0 \varphi_i \varphi'_j + c'_0 \varphi'_i \varphi''_j + c\delta_{ij}) \bar{L}_i \phi_2 \delta^T_j L_j + H.c.]$$

$$+ D_\mu \varphi^* D^\mu \varphi + D_\mu \varphi^* D^\mu \varphi - V_\varphi + \mathcal{L}_{SM}$$ (1)

This effective Lagrangian can be resulted from integrating out heavy particles. $\mathcal{L}_{SM}$ denotes the lagrangian of the standard model. $\bar{L}_i(x) = (\bar{\nu}_i, \bar{e}_i)_L$ (i=1,2,3) are the SU(2)$_L$ doublet leptons and $e_{Ri}$, ($i = 1, 2, 3$) are the three right-handed charged leptons. $A^k_i(x) t^i$ (i = 1, 2, 3) are the $SO(3)_F$ gauge bosons with $t^i$ the $SO(3)_F$ generators and $g'_i$ is the corresponding gauge coupling constant. Here $\phi_1(x)$ and $\phi_2(x)$ are two Higgs doublets, $\varphi^T = (\varphi_1(x), \varphi_2(x), \varphi_3(x))$ and $\varphi'^T = (\varphi'_1(x), \varphi'_2(x), \varphi'_3(x))$ are two $SO(3)_F$ Higgs triplets, $\chi(x)$, $\chi'(x)$ and $\chi''(x)$ are three singlet scalars. The couplings $c, c_a, c'_a$ (a = 0, 1) and $c''_1$ are dimensional constants. The structure of the above effective lagrangian can be obtained by imposing an additional U(1) symmetry [6].

After the symmetry $SO(3)_F \times SU(2)_L \times U(1)_Y$ is broken down to the U(1)$_{em}$ symmetry, mass matrices of the neutrinos and charged leptons get the following forms

$$(M_\nu)_{ij} = m_0 \frac{\hat{\sigma_i} \hat{\sigma_j}}{2\sigma^2} + m'_0 \frac{\hat{\sigma'_i} \hat{\sigma'_j}}{2\sigma'^2} + m''_0 \delta_{ij},$$

$$(M_e)_{ij} = m_1 \frac{\hat{\sigma_i} \hat{\sigma_j}}{\sigma^2} + m'_1 \frac{\hat{\sigma'_i} \hat{\sigma'_j}}{\sigma'^2} + m''_1 \delta_{ij},$$ (2)

where the mass matrices $M_\nu$ and $M_e$ are defined in the basis $\mathcal{L}_M = \bar{e}_L M_e e_R + \bar{\nu}_L M_\nu \nu_R'$ + H.c. The constants $\hat{\sigma}_i = < \varphi_i(x) >$ and $\hat{\sigma'_i} = < \varphi'_i(x) >$ are the vacuum expectation values (VEVs) of the two Higgs triplets with $\sigma^2 = \sum_{i=1}^{3} |\hat{\sigma}_i|^2$ and $\sigma'^2 = \sum_{i=1}^{3} |\hat{\sigma'_i}|^2$. Here $m_1, m'_1$ and $m''_1$ as well as $m_0, m'_0$ and $m''_0$ are mass parameters.

For simplicity, we only present here the Higgs potential for the $SO(3)_F$ Higgs triplets

$$V_\varphi = \frac{1}{2} \mu^2 (\varphi^\dagger \varphi) + \frac{1}{2} \mu'^2 (\varphi'^\dagger \varphi') + \frac{1}{4} \lambda (\varphi^\dagger \varphi)^2$$

$$+ \frac{1}{4} \lambda' (\varphi'^\dagger \varphi')^2 + \frac{1}{2} \kappa_1 (\varphi^\dagger \varphi) (\varphi'^\dagger \varphi')$$

$$+ \frac{1}{2} \kappa_2 (\varphi^\dagger \varphi') (\varphi'^\dagger \varphi).$$ (3)

where we have omitted terms involving other Higgs fields since those terms will not change our conclusions.

In our considerations, the $SO(3)_F$ flavor symmetry is treated to be a gauge symmetry, thus the complex $SO(3)_F$ Higgs triplet fields $\varphi_i(x)$ ($\varphi'_i(x)$) can always be expressed in
terms of the three rotational fields \( \eta_i(x) \) (\( \eta'_i(x) \)) and three amplitude fields \( \rho_i(x) \) (\( \rho'_i(x) \)) (\( i = 1, 2, 3 \)), i.e., \( \varphi(x) = O(x)\rho(x) \) and \( \varphi'(x) = O'(x)\rho'(x) \) with \( O(x) \equiv e^{i\eta(x)x} \) and \( O'(x) \equiv e^{i\eta'(x)x} \in SO(3)_F \) being the \( SO(3)_F \) rotational fields. This is analogous to SU(2) gauge symmetry, the complex SU(2) doublet scalar field can always be expressed in terms of three SU(2) ‘rotational’ fields and one amplitude field. As the \( SO(3) \) rotation matrix is real, which is unlike the SU(2) rotation matrix that is complex, one of the three amplitude fields of the complex \( SO(3) \) triplet scalar must be a pure imaginary field so that one can generate the complex \( SO(3) \) triplet scalar fields \( \varphi_i(x) \) by the \( SO(3) \) field \( O(x) = e^{i\eta(x)x} \in SO(3) \) action on the three amplitude fields. Explicitly, one has

\[
\begin{pmatrix}
\varphi_1(x) \\
\varphi_2(x) \\
\varphi_3(x)
\end{pmatrix} = e^{i\eta_1(x)x} \frac{1}{\sqrt{2}} \begin{pmatrix}
\rho_1(x) \\
\rho_2(x) \\
\rho_3(x)
\end{pmatrix}
\]

(4)

Similar form is for \( \varphi'(x) \). \( SO(3)_F \) gauge symmetry allows one to remove three degrees of freedom from six rotational fields. Making \( SO(3)_F \) gauge transformations: \( (\varphi(x), \varphi'(x)) \rightarrow O^T(x)(\varphi(x), \varphi'(x)) \), and assuming that only the amplitude fields get VEVs after spontaneous breaking of the \( SO(3)_F \) flavor symmetry, namely \( <\rho_i(x)> = \sigma_i \) and \( <\rho'_i(x)> = \sigma'_i \), we then obtain the following equations from minimizing the Higgs potential \( V_\varphi \)

\[
\omega^2 \sigma_i + \kappa_2 \sum_{j=1}^{3} (\sigma_j \sigma'_j) \sigma'_i = 0,
\]

\[
\omega'^2 \sigma'_i + \kappa_2 \sum_{j=1}^{3} (\sigma_j \sigma'_j) \sigma_i = 0
\]

(5)

with \( \omega^2 = \mu^2 + \lambda \sigma^2 + \kappa_1 \sigma'^2 \) and \( \omega'^2 = \mu'^2 + \lambda' \sigma'^2 + \kappa_1 \sigma^2 \). To find out possible constraints, it is useful to set \( \sigma'_i = \xi_i \sigma_i \) for \( \sigma_i \neq 0 \) with \( i = 1, 2, 3 \) and \( \sigma'^2 = \xi \sigma^2 \). When \( \xi_1 = \xi_2 = \xi_3 = \sqrt{\xi} \), the two \( SO(3)_F \) Higgs triplets \( \varphi(x) \) and \( \varphi'(x) \) are parallel in the model and the introduction of the second Higgs triplet becomes trivial. For the general and nontrivial case, it is easy to check that when \( \xi_i \neq \xi_k \equiv \xi_i - \xi_0 \) with \( i \neq j \neq k \), one arrives at the strong constraints from the minimum conditions in eq.(5)

\[
\sum_{i=1}^{3} \sigma_i \sigma'_i = \sum_{i=1}^{3} \xi_i \sigma_i^2 = 0,
\]

\[
\omega^2 = \mu^2 + \lambda \sigma^2 + \kappa_1 \sigma'^2 = 0,
\]

\[
\omega'^2 = \mu'^2 + \lambda' \sigma'^2 + \kappa_1 \sigma^2.
\]

(6)

For convenience of discussions, we make, without losing generality, the convention that \( \xi_1 = \xi_2 \neq \xi_3 \equiv \xi_1 - \xi_0 \). Thus from the above constraints, we obtain the solutions

\[
\xi = \xi_1(\xi_0 - \xi_1), \quad \xi - \xi_1 \tan^2 \theta_2 = 0
\]

(7)

with \( \tan^2 \theta_2 = \sigma_{12}^2/\sigma_3^2 \) and \( \sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 \). Furthermore, one must check the minimum conditions directly from the Higgs potential at the minimizing point. It is easy to see that

\[
V_\varphi|_{\text{min}} = -\sigma^4(\lambda + \lambda' \xi^2 + 2\kappa_1 \xi)/4
\]

(8)
This implies that to have a minimum potential energy $V_{\phi}|_{\text{min}}$ for varying $\xi$, the value of $\xi$ is required to be maximal for positive coupling constants $\lambda$'s and $\kappa_1$. From such a requirement, it is seen that for the given $\xi_0$ in eq.(7), the maximum condition for $\xi$ lead to the solution $\xi_1 = \xi_0/2 = \sqrt{2} = \xi_2 = -\xi_3$, namely

$$
\begin{align*}
\sigma_1' &= \sqrt{\xi}\sigma_1, \quad \sigma_2' = \sqrt{\xi}\sigma_2, \quad \sigma_3' = -\sqrt{\xi}\sigma_3, \\
\sigma_2^2 &= \sigma_1^2 + \sigma_3^2 \text{ or } \theta_2 = \pi/4
\end{align*}
$$

where $\xi$ is given as a function of the coupling constants in the Higgs potential, $\xi = (-\lambda\mu^2 + \kappa_1\mu^2)/(-\lambda'\mu^2 + \kappa_1\mu^2)$. Thus the VEVs are completely determined by the Higgs potential.

3 NEUTRINO MASSES AND OSCILLATIONS

It is interesting to note that with the above relations the mass matrices of the neutrinos and charged leptons are greatly simplified to the following nice forms

$$
M_e = \frac{m_1}{2} \begin{pmatrix}
\frac{s_1^2}{s_1} & \frac{ic_1s_1}{ic_1} & s_1 \\
\frac{ic_1s_1}{s_1} & \frac{-ic_1^2}{ic_1} & s_1 \\
s_1 & ic_1 & 1
\end{pmatrix} + \frac{m_1'}{2} \begin{pmatrix}
s_1^2 & \frac{ic_1s_1}{s_1} & -s_1 \\
\frac{ic_1s_1}{s_1} & \frac{-c_1^2}{ic_1} & -ic_1 \\
-s_1 & -ic_1 & 1
\end{pmatrix} + \frac{m_1''}{2} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

and

$$
M_\nu = m_\nu \begin{pmatrix}
1 + \delta_+ s_1^2 & 0 & \delta_- s_1 \\
0 & 1 + \delta_+ c_1^2 & 0 \\
\delta_- s_1 & 0 & 1 + \delta_+
\end{pmatrix}
$$

with $\delta_\pm = (m_0 \pm m'_0)/2m_\nu$.

It is remarkable that the two nondiagonal mass matrices in $M_e$ can be diagonalized by a unitary bi-maximal mixing matrix $U_e$ via $M_e' = U_e^\dagger M_e U_e^\ast$. Here

$$
M_e' = \begin{pmatrix}
0 & 0 & 0 \\
0 & m_1' & 0 \\
0 & 0 & m_1
\end{pmatrix} + m_1'' U_e^\dagger U_e^\ast
$$

and

$$
U_e^\dagger = \begin{pmatrix}
ic_1 & -s_1 & 0 \\
c_2s_1 & -\frac{1}{\sqrt{2}}c_1 & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}}s_1 & -\frac{1}{\sqrt{2}}c_1 & \frac{1}{\sqrt{2}}
\end{pmatrix}
$$
where $U_e^\dagger U_e^*$ has the following explicit form

$$
U_e^\dagger U_e^* = \begin{pmatrix}
0 & \sqrt{2}i c_1 s_1 & \sqrt{2}i c_1 s_1 \\
\sqrt{2}i c_1 s_1 & \frac{1}{2} & -\frac{1}{2} \\
\sqrt{2}i c_1 s_1 & -\frac{1}{2} & \frac{1}{2}
\end{pmatrix} 
+ (s_1^2 - c_1^2) \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{2} & \frac{1}{2} \\
0 & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}
$$

(14)

The hierarchical structure of the charged lepton mass implies that $m_{\nu i}'' \ll m_{\nu i}' \ll m_i$, it is then not difficult to see that the matrix $M_{\nu}'$ will be further diagonalized by a unitary matrix $U_{\nu}'$ via $D_{\nu} = U_{\nu}'\dagger M_{\nu}' U_{\nu}'\star = U_{\nu}'\dagger U_{\nu}' M_{\nu} U_{\nu}'\star$ with

$$
D_{\nu} = \begin{pmatrix}
m_e & 0 & 0 \\
0 & m_\mu & 0 \\
0 & 0 & m_\tau
\end{pmatrix}
$$

(15)

and

$$
U_{\nu}' = \begin{pmatrix}
1 + \epsilon_2 & i \epsilon_2 & i \epsilon_1 \\
i \epsilon_2 & 1 + \epsilon_2 & \epsilon_1 \\
i \epsilon_1 & \epsilon_1 & 1 + \epsilon_1
\end{pmatrix}
$$

(16)

with $\epsilon_1 = O(\sqrt{m_e m_\mu}/m_\tau)$ and $\epsilon_2 = O(\sqrt{m_e m_\mu})$. Where $m_e \simeq m_{\nu i}''/m_{\nu i}'$, $m_\mu = m_{\nu i}' + O(m_{\nu i}''/m_{\nu i})$ and $m_\tau = m_i + O(m_{\nu i}''/m_{\nu i})$ define the three charged lepton masses. This indicates that the unitary matrix $U_{\nu}'$ does not significantly differ from the unit matrix.

The neutrino mass matrix can be easily diagonalized by an orthogonal matrix $O_{\nu}$ via $O_{\nu}^T M_{\nu} O_{\nu}$

$$
O_{\nu} = \begin{pmatrix}c_\nu & 0 & s_\nu \\
0 & 1 & 0 \\
-s_\nu & 0 & c_\nu
\end{pmatrix}
$$

(17)

with $s_\nu \equiv \sin \theta_\nu$ and $\tan 2\theta_\nu = 2s_\nu/(\delta_+c_1^2)$

When going to the physical mass basis of the neutrinos and charged leptons, we then obtain the CKM-type lepton mixing matrix $U_{\nu L E P}$ appearing in the interactions of the charged weak gauge bosons and leptons, i.e., $L_W = \bar{\nu_L} \gamma^\nu U_{\nu L E P} \nu L W^- + H.c.$ Explicitly, we have $U_{\nu L E P} = U_e^\dagger U_{\nu L E P}^\dagger O_{\nu}$ with

$$
U_{\nu L E P} = U_e \begin{pmatrix}i c_1 c_\nu & -s_1 & 0 \\
1/\sqrt{2} s_1 c_\nu & -i/\sqrt{2} c_1 & -1/\sqrt{2} c_\nu \\
1/\sqrt{2} s_1 c_\nu & i/\sqrt{2} c_1 & 1/\sqrt{2} c_\nu
\end{pmatrix}
+ U_e \begin{pmatrix}0 & 0 & i c_1 \\
1/\sqrt{2} & 0 & 1/\sqrt{2} s_1 \\
-1/\sqrt{2} & 0 & 1/\sqrt{2} s_1
\end{pmatrix} s_\nu
$$

(18)
The three neutrino masses are found to be

\[
m_{\nu e} = m_\nu [1 + \delta_+ s_1^2 - \delta_+ c_1^2 t_\nu^2/(1 - t_\nu^2)]
\]

\[
m_{\nu\mu} = m_\nu [1 + \delta_+ c_1^2]
\]

\[
m_{\nu\tau} = m_\nu [1 + \delta_+ + \delta_+ c_1^2 t_\nu^2/(1 - t_\nu^2)]
\]

with \(t_\nu \equiv s_\nu/c_\nu\).

The similarity between the Higgs triplets \(\varphi(x)\) and \(\varphi'(x)\) naturally motivates us to consider an approximate (and softly broken) permutation symmetry between them. This implies that \(m_0 \simeq m'_0\). As a consequence, one has \(|\delta_-| \ll 1\). To a good approximation, the mass-squared differences are given by

\[
\Delta m_{\mu e}^2 \simeq 2m_\nu^2\delta_+(c_1^2 - s_1^2 + (\delta_- s_1/\delta_+ c_1)^2),
\]

\[
\Delta m_{\tau\mu}^2 \simeq 2m_\nu^2\delta_+ s_1^2
\]

with \(\Delta m_{ij}^2 \equiv m_{\nu i}^2 - m_{\nu j}^2\).

On the other hand, from the fact that the failure of detecting neutrinoless double beta decay provide bounds on an effective electron neutrino mass \(<m_{\nu e}> = \sum_i m_{\nu_i}(U_{LEP})^2_{ei} < 0.2\ eV[10]\. We then obtain

\[
<m_{\nu e}> \simeq m_\nu |s_1^2 - c_1^2| < 0.2 eV
\]

Assuming that neutrino masses are large enough to play an essential role in the evolution of the large-scale structure of the universe, we may set \(m_\nu \sim 1\ eV\), thus the above constraint will result in the following bound on the mixing angle \(\theta_1\)

\[
|s_1^2 - c_1^2| < 0.2
\]

which implies that \(\Delta m_{\mu e}^2/\Delta m_{\tau\mu}^2 < 0.4\). To explain the solar neutrino data, the allowed range of the ratio is \(\Delta m_{\mu e}^2/\Delta m_{\tau\mu}^2 \sim 10^{-2} - 10^{-8}\). Here the large value is for matter-enhanced MSW solution[2] with large mixing angle[3] and the small value for the vacuum oscillation solutions[4]. As a consequence, the neutrino mixing between \(\nu_e\) and \(\nu_\mu\) becomes almost maximal

\[
\sin^2 2\theta_1 > 0.998
\]

With the hierarchical feature in \(\Delta m^2\), formulae for the oscillation probabilities in vacuum are greatly simplified to be

\[
P_{\nu_e \rightarrow \nu_\mu}|_{\text{solar}} \simeq 1 - \sin^2(\frac{\Delta m_{\mu e}^2 L}{4E})
\]

\[
P_{\nu_\mu \rightarrow \nu_\mu}|_{\text{atm.}} \simeq 1 - \sin^2(\frac{\Delta m_{\tau\mu}^2 L}{4E})
\]

\[
P_{\nu_\tau \rightarrow \nu_\mu} \simeq 4|U_{\beta3}|^2|U_{\alpha3}|^2 \sin^2(\frac{\Delta m_{\tau\mu}^2 L}{4E})
\]

\[
P_{\nu_\mu \rightarrow \nu_e}/P_{\nu_\mu \rightarrow \nu_\tau}|_{\text{atm.}} \simeq \left(\frac{\Delta m_{\mu e}^2}{\Delta m_{\tau\mu}^2}\right) < < 1.
\]
This may present the simplest scheme for reconciling both solar and atmospheric neutrino fluxes via oscillations of three neutrinos. But it needs a strong fine-tuning.

When going back to the weak gauge and charged-lepton mass basis, the neutrino mass matrix is given by $\hat{M}_\nu = \hat{U}_e^\dagger \hat{M}_\nu \hat{U}_e^\ast$. Where $\hat{M}_\nu$ has the following general and interesting form

$$\hat{M}_\nu \simeq \hat{m}_\nu \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} i & \frac{1}{\sqrt{2}} i \\ \frac{1}{\sqrt{2}} i & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} i & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} + \frac{\hat{m}_\nu \delta_-}{2} \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} i & \frac{1}{\sqrt{2}} i \\ \frac{-1}{\sqrt{2}} i & -1 & 0 \\ \frac{1}{\sqrt{2}} i & 0 & 1 \end{pmatrix}$$

$$+ \frac{\hat{m}_\nu \delta_+}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + (s_1^2 - c_1^2)M'_\nu. \tag{25}$$

with $\hat{m}_\nu = m_\nu(1 + \delta_+ c_1^2)$, $\delta_- = \delta_- s_1/(1 + \delta_+ c_1^2)$ and $\delta_+ = \delta_+ s_1^2/(1 + \delta_+ c_1^2)$. For $|s_1^2 - c_1^2| \sim 10^{-2}$, the neutrino masses can be approximately degenerate and large enough ($\hat{m}_\nu = O(1)$ eV) to play a significant cosmological role.

4 SO(3) GAUGE INTERACTIONS AND LEPTON-FLAVOR VIOLATIONS

We now come to discuss SO(3) gauge interactions. After the SO(3) gauge symmetry is spontaneously broken down, the gauge fields $A^i_\mu$ receive masses by ‘eating’ three of the rotational fields. For the SO(3) vacuum structure given above, $A^1_\mu$ and $A^3_\mu$ are not in the mass eigenstates since they mix each other.

The mass matrix of gauge fields $A^i_\mu$ is found to be

$$M^2_F = \begin{pmatrix} 1 & 0 & -\frac{s_1 \xi_-}{\xi_+} \\ 0 & 1 + c_1^2 + \alpha & 0 \\ -\frac{s_1 \xi_-}{\xi_+} & 0 & 1 + s_1^2 - \alpha \end{pmatrix}$$

with $m^2_F = \xi_+ g_3^2 s^2/8$, $\xi_\pm = (1 \pm \xi)/2$ and $\alpha = (s_1^2 - c_1^2)/2 \xi_+$. This mass matrix is diagonalized by an orthogonal matrix $O_F$ via $O_F^\dagger M^2_F O_F$. Denoting the physical gauge fields as $F^i_\mu$, we then have $A^i_\mu = O^F_F F^j_\mu$, i.e.,

$$\begin{pmatrix} A^1_\mu \\ A^2_\mu \\ A^3_\mu \end{pmatrix} = \begin{pmatrix} c_F & 0 & -s_F \\ 0 & 1 & 0 \\ s_F & 0 & c_F \end{pmatrix} \begin{pmatrix} F^1_\mu \\ F^2_\mu \\ F^3_\mu \end{pmatrix}. \tag{26}$$
with \( s_F = \sin \theta_F \) and
\[
\tan 2\theta_F = \frac{4s_1\xi_1}{2\xi_1 s_1^2 + c_1^2 - s_1^2} \tag{27}
\]

Masses of the three physical gauge bosons \( F^\mu_i \) are found to be
\[
m_{F_1}^2 = m_F^2 [(2 + s_1^2)\xi_+ + \frac{c_1^2 - s_1^2}{2} - (\xi_+ s_1^2 + \frac{c_1^2 - s_1^2}{2} \sqrt{1 + \tan^2 2\theta_F})],
\]
\[
m_{F_2}^2 = m_F^2 [2(1 + c_1^2)\xi_+ + s_1^2 - c_1^2], \tag{28}
\]
\[
m_{F_3}^2 = m_F^2 [(2 + s_1^2)\xi_+ + \frac{c_1^2 - s_1^2}{2} + (\xi_+ s_1^2 + \frac{c_1^2 - s_1^2}{2} \sqrt{1 + \tan^2 2\theta_F})].
\]

In the physical mass basis of the leptons and gauge bosons, the gauge interactions of the leptons are given by the following form
\[
\mathcal{L}_F = \frac{1}{2} g'_3 F^\mu_i \bar{L}_i O_F^\mu \gamma^\mu L + \frac{1}{2} g'_3 F^\mu_i \left( \bar{e}_L V^i e_L \gamma^\mu - \bar{e}_R V^i e_R \gamma^\mu e_R \right) \tag{29}
\]

with \( V_e^i = U^\dagger V^i U_L^e \). Here \( \hat{V}_e^i = K_e^i O_F^i, \) i.e.,
\[
\hat{V}_e^1 = \cos \theta_F K_e^1 + \sin \theta_F K_e^3, \\
\hat{V}_e^2 = K_e^2, \\
\hat{V}_e^3 = -\sin \theta_F K_e^1 + \cos \theta_F K_e^3 \tag{30}
\]

where
\[
K_e^1 = c_1 s_1 \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \\
+ i \frac{1}{\sqrt{2}} (s_1^2 - c_1^2) \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \\
K_e^2 = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} c_1 & -\frac{1}{\sqrt{2}} c_1 \\ \frac{1}{\sqrt{2}} c_1 & 0 & i s_1 \\ -\frac{1}{\sqrt{2}} c_1 & -i s_1 & 0 \end{pmatrix}, \tag{31}
\]
\[
K_e^3 = \begin{pmatrix} 0 & i \frac{1}{\sqrt{2}} s_1 & -i \frac{1}{\sqrt{2}} s_1 \\ -i \frac{1}{\sqrt{2}} s_1 & c_1 & 0 \\ i \frac{1}{\sqrt{2}} s_1 & 0 & -c_1 \end{pmatrix}.
\]
Thus the $SO(3)_F$ gauge interactions allow lepton flavor violating processes. For $\mu \rightarrow 3e$ decay, its branch ratio is found

$$Br(\mu \rightarrow 3e) = \left(\frac{v}{\sigma}\right)^4 \frac{2\xi^2}{(3\xi_+^2 - \xi_-^2)^2}$$

(32)

with $v = 246$GeV. For $\sigma \sim 10^3 v$, the branch ratio could be very close to the present experimental upper bound $Br(\mu \rightarrow 3e) < 1 \times 10^{-12}$ [11]. Thus when taking the mixing angle $\theta_F$ and the coupling constant $g'_3$ for the $SO(3)_F$ gauge bosons to be at the same order of magnitude as those for the electroweak gauge bosons, we find that masses of the $SO(3)_F$ gauge bosons are at the order of magnitudes $m_{F_i} \sim 10^3 m_W \simeq 80$ TeV.

5 CONCLUSIONS

Based on the gauge model with $SO(3)_F$ lepton flavor symmetry and two Higgs triplets, we have shown how the maximal mixing between $\nu_\mu$ and $\nu_\tau$ neutrinos comes out naturally after spontaneous breaking of the symmetry. We have also shown that a two-flavor mixing scenario can be naturally resulted from an approximate permutation symmetry between the two Higgs triplets. An almost maximal mixing between $\nu_e$ and $\nu_\mu$ neutrinos has been found to be a natural consequence of the hierarchical feature between the neutrino mass-squared differences. Thus the model favors the almost bi-maximal mixing scenario[12]. The model allows three Majorana neutrino masses to be nearly degenerate and large enough to play a significant cosmological role. The $SO(3)_F$ gauge interactions may lead to interesting phenomena on lepton-flavor violations.

References


[10] The current experimental bound is $(M_\nu)_{ee} < 0.2$ eV, see: L. Baudis et al., hep-ex/9902014.


[12] Most recently, we have shown that the intriguing nearly degenerate neutrino mass and bi-maximal mixing scenario can naturally be derived from the SO(3)$_F$ gauge model with three Higgs triplets; hep-ph/9906435.