The non-zero baryon number formulation of QCD
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We discuss the non-zero baryon number formulation of QCD in the quenched limit at finite temperature. This describes the thermodynamics of gluons in the background of static quark sources. Although a sign problem remains in this theory, our simulation results show that it can be handled quite well numerically. The transition region gets shifted to smaller temperatures and the transition region broadens with increasing baryon number. Although the action is in our formulation explicitly $Z(3)$ symmetric the Polyakov loop expectation value becomes non-zero already in the low temperature phase and the heavy quark potential gets screened at non-vanishing number density already this phase.

1. Introduction

An important aim of lattice QCD is the understanding of the QCD phase diagram and its dependence on the temperature $T$ and the baryon density $n_B$. Especially the region of non-zero density is important, as it describes the behaviour of dense matter created in heavy ion collisions and plays an important role in the cosmological context. Due to the well known problem of the complex fermion determinant [1] when a non-zero chemical potential $\mu$ is introduced [2,3], only qualitative features of the phase diagram at non-zero density can be understood in terms of models and approximations.

Introducing a chemical potential $\mu$ [2] leads to the grand canonical partition function of finite density QCD. An alternative formulation is given in terms of the canonical partition function at fixed non-zero baryon number [4]. This is achieved by introducing an imaginary chemical potential [4,5] in the grand canonical partition function and performing a Fourier integration to project onto the canonical partition function for a given sector of fixed baryon number [4]

$$Z(B, T, V) = \frac{1}{2\pi} \int_{0}^{2\pi} d\phi e^{-iB\phi} Z(i\phi, T, V). \quad (1)$$

This formulation still suffers from a sign problem, but leads to a quite natural and useful formulation of the quenched limit of QCD at non-zero baryon number density, where the sign problem can be handled quite well [6].

2. The quenched limit of QCD at non-zero density

The static limit of QCD at non-zero chemical potential $\mu$ in the grand canonical approach has been formulated in [7] and [8]. The numerical results indicate that in this case the first order deconfinement transition of the $SU(3)$ gauge theory turns into a crossover for arbitrarily small, non-zero values of the chemical potential [8]. The quenched partition function in the canonical approach can be written as

$$Z(B, T, V) = \int \prod_{x,\nu} dU_{x,\nu} \hat{f}_B e^{-S_G} \quad (2)$$

where the constraint on the baryon number is encoded in the function $\hat{f}_B$ which is a function of Polyakov loops, $B$ counts the number of quarks, i.e. $B/3$ is the baryon number. For $B = 3$ and one flavour of Wilson fermions $\hat{f}_B$ is, for instance, given by

$$\hat{f}_{B=3} = (2\kappa)^3 N_c (V^3 \frac{4}{3} [L_{1,0}]^3$$

$$+ V^2 (8[L_{1,0}][L_{2,0}] - 4[L_{1,0}][L_{0,1}])$$

$$+ V (12 + \frac{2}{3} [L_{4,0} - 2[L_{1,1}]])) \quad (3)$$

with $[L_{i,j}] = V^{-1} \sum_{x} (Tr L_x)^i (Tr L_x^2)^j$. For a more detailed description of $\hat{f}_B$ see [6].
\[ \text{Figure 1.} \langle \text{sgn(Re } \hat{f}_B) \rangle \text{ for } B = 6 \text{ and 12 calculated on lattices of size } N_\sigma^3 \times N_\tau. \]

\[ S_G \] is the gluonic action, which is \( Z(3) \) symmetric. The partition function \( Z(B, T, V) \), is non-zero only if \( B \) is a multiple of 3, because \( \hat{f}_B \) is invariant under \( Z(3) \) transformations only if \( B \) is a multiple of 3. In general it changes by a factor \( e^{2\pi i B/3} \) under a global \( Z(3) \) transformation of time-like link variables. \( \hat{f}_B \) is still a complex function, but upon integration over the gauge fields the imaginary part of the partition function vanishes. The remaining sign problem can be handled by using the absolute value of \( \text{Re } \hat{f}_B \) and including the sign in the calculation of observables [9].

Our simulations are performed on \( N_\sigma^3 \times N_\tau \) lattices with \( N_\sigma = 8, 10, 12, 16 \) and \( N_\tau = 2, 4 \) using the standard Wilson action and one flavour of Wilson fermions with quark number values of \( B = 6 \) and 12 at fixed \( n_B/T^3 = (1/3)B(N_\tau/N_\sigma)^3 \). Fig. 1 shows the average sign \( \langle \text{sgn(Re } \hat{f}_B) \rangle \) as a function of the coupling \( \beta \). For large values of the temperature the sign is almost always positive, but also for the smallest temperature in our analysis the sign can be well determined. It depends on the spatial volume \( N_\sigma^3 \) but varies little with \( B \). The Polyakov loop expectation values in Fig. 2 show a clear signal for a first order transition for the \( B = 0 \) case, while for all \( B > 0 \) the transition is continuous. The transition region is shifted towards smaller \( \beta \)-values and it broadens with increasing \( B \). Note that by changing the gauge coupling \( \beta \) we vary the lattice cut-off and through this also the baryon number density continuously. The broadening of the transition region may indicate the presence of a region of coexisting phases. The Polyakov loop susceptibility (Fig. 3) reflects the existence of a transition region that becomes broader with increasing \( n_B \), but does not show indications for a discontinuity. The Polyakov loop expectation value becomes
non-zero already in the low temperature phase. This indicates that the heavy quark potential stays finite at large distances. We validate this by calculating the potential using Polyakov loop correlations (Fig. 4). For zero baryon number it shows the usual linearly rising behaviour for the quenched case. For $B = 6$ the potential stays finite at large distances due to the screening of the static quark anti-quark sources by already present static quarks. This behaviour is comparable to heavy quark potentials in full QCD [10].

3. Conclusions

We have analyzed the quenched limit of QCD at non-zero baryon number. The sign problem in this theory can be handled quite well numerically. We find indications for a region of coexisting phases, which broadens with increasing baryon number density and is shifted towards smaller temperatures. Further analyses are needed to see if this is a signal of the existence of a first order phase transition or a smooth crossover at non-zero density. We also see evidence that the heavy quark potential for non-zero baryon density stays finite for large distances already in the hadronic phase. The potential gets screened by the static quarks that induce the non-vanishing density. This will have a direct influence on heavy quark bound states at high density.

REFERENCES