Spectra and M1 Decay Widths of Heavy-Light Mesons

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Abstract

The M1 decay widths of charm and beauty mesons and their excited states are sensitive to the relativistic aspect of the quark current operators and obtain a significant contribution from the interaction current that is associated with the scalar confining interaction between heavy quarks ($Q$) and light antiquarks ($\bar{q}$). Consequently they provide direct evidence on the latter. The spectra and the M1 transition widths of the $D$, $D_s$ and $B$, $B_s$ mesons and their orbital excitations are calculated here within the framework of the Blankenbecler-Sugar equation, which allows a covariant treatment, while retaining a formal similarity to the nonrelativistic approach. The hyperfine interaction in the $Q\bar{q}$ mesons is modeled by screened gluon exchange, which is compared to the instanton induced interaction. The quality of the model is tested by calculation of the spectra and M1 decay widths of charmonium.
1 Introduction

A dynamical description of the spectrum of the heavy flavor mesons $Q\bar{q}$ has a priori to be relativistic as the confined light quarks and antiquarks have velocities close to that of light. Combination of this requirement, with the non-relativistic instantaneous aspect of the effective linear [1] (or near linear [2]) confining interaction suggests that a 3-dimensional quasipotential reduction of the Bethe-Salpeter equation should provide the most obvious framework for a realistic description. This is a fortiori the case as the Bethe-Salpeter equation with instantaneous interactions, while commonly employed, [3], fails to provide satisfactory results for the M1 transitions of the analogous heavy quarkonia [4].

Among the many choices of quasipotential equations the Blankenbecler-Sugar equation [5, 6] suggests itself because of its transparent formal similarity to the nonrelativistic Lippmann-Schwinger equation, to which it reduces in the adiabatic limit. Moreover it admits direct employment of the conventional (relativistic) models for the hyperfine interaction between quarks and antiquarks, as well as the linear confining interaction along with its obvious relativistic modifications. The key advantage of the Blankenbecler-Sugar equation over the nonrelativistic or “relativized” versions of the nonrelativistic wave equation is the unapproximated treatment of the relativistic two-particle kinetic energy operator of the $Q\bar{q}$ system. The relativistic modifications to the interaction are in comparison modest, and may be treated in an approximate fashion, when convenient. This feature is shared with the Gross [7] reduction of the Bethe-Salpeter equation, which has been applied to heavy flavor mesons in ref.[8].

Given this situation, several different versions of the Blankenbecler-Sugar equation have been employed in the literature [9]. We shall here use the symmetrical version of ref. [10] to calculate both the spectra and the M1 decay widths of the $D$ and $D_s$ as well as the $B$ and $B_s$ mesons and their orbital excitations. The formalism then differs somewhat from that used to describe the $Q\bar{q}$ system in refs. [11]. The main advantage of the version used here is that it leads to a Lippmann-Schwinger type wave-equation. It differs most obviously from that used in refs.[11] in the appearance of the “minimal relativity” [12] square root factors $\sqrt{m/E}$ in the effective interaction potential in the wave equation.

The radiative transitions of the ground state $Q\bar{q}$ mesons may be described phenomenologically by effective field theory methods [13, 14, 15]. A complete dynamical model is however required for any comprehensive description of the decays of the full spectrum of $Q\bar{q}$ states. The static quark model is insufficient here, as it overpredicts the (upper limit for the) width of the M1 transition for $D^{++} \to D^+\gamma$ by a large factor. This overprediction is similar
to that corresponding large overprediction of the width for the M1 decay
$J/\psi \to \eta_c \gamma$, and which is resolved only by employment of the relativistic
magnetic moment operator for the heavy quarks along with the interaction
current operator that is associated with the scalar confining interaction [16].

In comparison to heavy quarkonia the heavy-light $Q\bar{q}$ mesons are more
complex systems in view of the still unsettled form of the hyperfine interac-
tion for the light quarks and concomitantly their empirically still only poorly
known spectrum. While the branching ratios of the M1 decays of the charmed
vector mesons are known, their still unknown total decay widths prevent an
absolute empirical determination of their M1 decay widths. In spite of – and
because of – this situation it remains a challenge to decode the structure and
the dynamics of these systems, as this is a main key to the understanding of
the nature of the confining interaction and the hyperfine interaction between
constituent quarks.

There are good theoretical [1, 17] and phenomenological [16] reasons to
expect the confining interaction to be linear with a scalar spinor structure, al-
though mixtures of combinations of scalar and vector coupling terms continue
to be employed [3, 11]. For the heavy light $Q\bar{q}$ systems the hyperfine inter-
action is likely to be formed as a combination of the screened perturbative
gluon exchange interaction [18] and the direct instanton induced interaction
[19], which share many qualitative features. The latter appears in the instan-
ton liquid model description of the QCD vacuum [20], which is supported by
numerical lattice calculations [21, 22]. In the case of heavy light quarks the
relativistic corrections to the gluon exchange interaction cause an apprecia-
ble damping of the that interaction, besides the screening of the the color
hyperfine coupling strength.

We here consider the M1 decay rates of the $Q\bar{q}$ systems, because these
are widely overpredicted by the static quark model, and are likely to provide
decisive information on the nature of the confining interaction. We show
that the overprediction by the static quark model of the decay rate of the
M1 transition $D^* \to D \gamma$ may be solved by (1) employment of the relativis-
tic magnetic moment operator for the quarks in combination with (2) the
interaction current, that appears if the confining interaction couples as a
scalar. This solution falls in line with that for the corresponding M1 decay
rate problem of heavy quarkonia [16]. For $q\bar{q}$ systems (i.e. systems with
equal quark and anti-quark masses) neither the gluon exchange interaction
nor the instanton induced interaction gives rise two any two-body current
operator with a spin-flip part. The spin-flip component of those exchange
current operators that appear in the case of $Q\bar{q}$ systems have very small ma-
trix elements compared to the matrix elements of the two-body current that
is associated with the scalar confining interaction.

The effective interaction potential between the heavy quark and the light
antiquark is accordingly described by a scalar linear confining interaction
with a hyperfine interaction formed of the perturbative gluon exchange interaction in combination with the instanton induced interaction. The parameters of this model are, to the extent possible, fixed to values obtained by lattice calculations as well as to the known part of the $D$ and $D_s$ meson spectra, and then appropriately scaled to the beauty mesons. Finally the quality of the model is tested by a calculation of the spectrum and the M1 decay widths of charmonium.

With the framework of the Blankenbecler-Sugar equation and the $Q\bar{q}$ interaction models above it is possible to obtain spectra for both charmonium and the heavy-light mesons, which agree overall with the empirical spectra, where these are known. Similarly the description of the (known) M1 decay rates of the $J/\psi$ and the $\psi'$ are well described. The M1 decay rates for the $D$ and $B$ mesons are predictions, as absolute empirical determination of these are yet to made. The present result that it is possible to achieve a satisfactory description of both the spectra and M1 decay rates of heavy quarkonia and heavy light mesons differs from that of ref.[23]. The key point here is the employment of an unapproximated quark current operator.

This report falls into 7 sections. Section 2 contains a description of the application of the Blankenbecler–Sugar equation to the $Q\bar{q}$ and $Q\bar{Q}$ systems. Section 3 contains a description of the interaction parts of the Hamiltonian and the current operators. The calculated $D, D_s$ and $B, B_s$ meson as well as the $c\bar{c}$ spectra are described in section 4. Section 5 describes the quark current operators, including the two-quark current that arises along with a linear scalar confining interaction. The calculated M1 transition rates are reported in section 6. Section 7 contains a concluding discussion.

2 The Blankenbecler–Sugar equation applied to $Q\bar{q}$ systems

The Blankenbecler–Sugar equation for a $Q\bar{q}$ system may be expressed as an eigenvalue equation of the form

$$\left(\frac{\vec{p}^2}{2m_r} + V\right) \psi = \epsilon \psi. \quad (1)$$

Here $m_r$ is the reduced mass of the heavy quark ($Q$) mass $M$ and the light the antiquark ($\bar{q}$) mass $m$: $(m_r = Mm/(M + m))$. The the eigenvalue $\epsilon$ is related to the energy $E$ of the $Q\bar{q}$ meson system as

$$\epsilon = \frac{[E^2 - (M - m)^2][E^2 - (M + m)^2]}{8m_rE^2}. \quad (2)$$
The interaction operator $V$, which need not be a local operator, may be obtained from the $Q\bar{q}$ irreducible quasipotential $\mathcal{V}$ (in momentum space) as [24]:

$$V(p'', p') = \sqrt{\frac{M + m}{W(p'')}} V(p'', p') \sqrt{\frac{M + m}{W(p')}}.$$  \hspace{1cm} (3)

Here the function $W$ is defined as

$$W(p) = E_Q(p) + E_{\bar{q}}(p),$$  \hspace{1cm} (4)

with $E_Q(p) = \sqrt{M^2 + p^2}$ and $E_{\bar{q}}(p) = \sqrt{m^2 + p^2}$ respectively. In the Born approximation the quasipotential $\mathcal{V}$ equals the $Q\bar{q}$ invariant scattering amplitude $T$, and thus a constructive relation to field theory obtains.

Numerical solution of the wave equation (1) yields a value for the eigenvalue $\epsilon$. The energy (rest mass) of the $Q\bar{q}$ meson is then obtained as

$$E = \sqrt{M^2 + m^2 + 4m\epsilon + 2\sqrt{M^2m^2 + 2m^2(M^2 + m^2)\epsilon} + 4m^2\epsilon^2}. \hspace{1cm} (5)$$

In the equal mass case $M = m$ this expression reduces to $E = \sqrt{4m(m + \epsilon)}$.

The relation between the (total) energy $E$ of the $Q\bar{q}$ system and the eigenvalue $\epsilon$ in the equation (1) shows that the relativistic treatment of the kinetic energy term in the Hamiltonian leads to a lowering of the former in comparison to what a nonrelativistic treatment would have implied. This is a consequence of the effective weakening of the repulsive kinetic energy operator, that results when the quadratic $p^2$ terms are replaced by square roots of $p^2$. Despite the formal similarity between the Blankenbecler-Sugar equation and the Schrödinger equation, the former involves a quadratic mass operator.

The main component of the interaction between the $Q$ and $\bar{q}$ quarks is the linear confining interaction, which is well defined only in the position space representation, which we shall therefore adhere to. While the Blankenbecler–Sugar equation takes full account of the relativistic two-quark kinetic energy operator, the relativistic aspects of the interaction operators will be treated perturbatively. This treatment is more than adequate for the heavy quark, but less so for the light one. It does, however, allow retention of the conventional operator structure of the interaction operator between the quarks.
3 The Interaction Hamiltonian for $Q\bar{q}$ systems

3.1 The confining interaction

The linear confining interaction will here – on the basis of compelling indications – be assumed be a scalar in the spinor representation [16, 17]. To second order in the inverse quark masses this interaction, as calculated from the invariant amplitude according to (3), takes the form

$$V_C = cr \left( 1 - \frac{3}{2} \frac{\vec{p}^2}{m_2^2} \right) + \frac{c}{4Mmr} \frac{c M^2 + m^2}{r} \vec{S} \cdot \vec{L}$$

$$+ c \frac{M^2 - m^2}{r} \frac{c r}{8M^2m^2} \left( \vec{\sigma}_Q - \vec{\sigma}_{\bar{q}} \right) \cdot \vec{L}. \quad (6)$$

Here $c$ is the string tension parameter, the value of which is $\sim 1$ GeV/fm. The mass coefficient $m_2$ is defined as

$$\frac{1}{m_2} = \sqrt{\frac{M^2 + m^2 + Mm}{3Mm}}. \quad (7)$$

The spin–independent term that is proportional to $1/r$ is a consequence of the square root factors in (3). Without those factors the factor $3/2$ in the momentum dependent term in first bracket on the r.h.s. of (6) would be 1. The terms of second order in the inverse quark masses in (6) are implied by scalar coupling for the confining interaction. As the antisymmetric spin-orbit interaction has no diagonal matrix elements between any of the states in the $S$- and $P$-shells, we shall not consider it further.

The term of second order in $\vec{p}$ in the spin-independent term in (6) is unrealistically large for light quarks, but is substantially moderated by inclusion of the next term in the (asymptotic) $(v/c)^2$ expansion. Similarly the spin-orbit interaction in (6) is unrealistically large for light quarks. Both of these explicitly velocity dependent terms are strongly moderated by the higher order terms in the (asymptotic) expansion in $\vec{v}^2$. This is seen directly by comparison to the corresponding terms in the quartic correction term to (6), which takes the form

$$V_C^{(4)} = -\frac{c\pi}{16m^2} \delta^{(3)}(r) - \frac{c}{r} \frac{\vec{p}^2}{16m^4} - \frac{c}{r} \frac{(\vec{p}^2 - (\hat{r} \cdot \vec{p})^2)}{32m^4}$$

$$+ \frac{cr^3}{8m^4} \left[ \frac{1}{32r^3} \left( \frac{3}{M^3} + \frac{2}{m} \right) + \frac{1}{m^3} \left( \frac{3}{m} + \frac{2}{M} \right) \right] \vec{S} \cdot \vec{L}$$

$$+ \frac{c}{8r} \frac{\vec{p}^2}{M^2m^2} \left( 5 + \frac{3}{2} \left( \frac{M}{m} \right)^2 + \frac{3}{2} \left( \frac{m}{M} \right)^2 \right) \vec{S} \cdot \vec{L}$$

$$+ \frac{c}{32r^3} \left[ \frac{1}{M^3} + \frac{2}{m} \right] + \frac{1}{m^3} \left( \frac{3}{m} + \frac{2}{M} \right) \vec{S} \cdot \vec{L}, \quad (4)$$
where $Q_{12}$ is the quadratic spin-orbit interaction operator

$$Q_{12} = (\vec{\sigma}_Q \cdot \vec{L} \vec{\sigma}_q \cdot \vec{L} + \vec{\sigma}_q \cdot \vec{L} \vec{\sigma}_Q \cdot \vec{L})/2.$$  

Above the mass coefficients $m_{4j}$ are defined as

$$\frac{1}{m_{4a}} = \left[ \frac{(M + m)^2}{M^4m^3} - \frac{1}{M^2m^2} \right],$$

$$\frac{1}{m_{4b}} = \left[ \frac{3(M + m)^4}{M^4m^4} - \frac{8(M + m)^2}{M^3m^3} \right],$$

$$\frac{1}{m_{4c}} = \left[ \frac{5(M + m)^4}{M^4m^4} - \frac{16(M + m)^2}{M^3m^3} + \frac{2}{M^2m^2} \right],$$

$$\frac{1}{m_{4d}} = \left[ \frac{(M + m)^4}{M^4m^4} - \frac{3(M + m)^2}{M^3m^3} + \frac{1}{M^2m^2} \right].$$

The scalar structure of the confining interaction implies functional relations between the different spin components of the potential. Thus e.g. in the static limit the spin-orbit component may be calculated from the central component as

$$v_{LS}(r) = -\frac{1}{4} \left( \frac{1}{M^2} + \frac{1}{m^2} \right) \frac{1}{r} \frac{\partial}{\partial r} v_c(r).$$

Here $v_c(r)$ is the central confining potential and $v_{LS}(r)$ is the coefficient function for the spin-orbit coupling operator $\vec{S} \cdot \vec{L}$. When the static limit is not invoked, the corresponding relation is more complicated. To a good approximation it may however be cast in the form

$$v_{LS}(r) = -\frac{2c}{\pi \sigma} \frac{\partial}{\partial r} \int_0^\infty dr' r'^3 \int_0^\infty dk k^2 j_0(kr) j_0(kr') \left\{ \frac{1}{(e_Q + M)^2} + \frac{1}{(e_q + m)^2} \right\},$$

where $e_Q = \sqrt{M^2 + k^2/4}$ and $e_q = \sqrt{m^2 + k^2/4}$. This relation implies that the static approximation to the spin-orbit interaction in (6) represents a considerable overestimate of the net spin-orbit interaction in the case of light constituent quarks.

A similar more accurate representation of the quadratic spin-orbit component of the confining interaction obtains with the replacement

$$\frac{1}{16M^2m^2} \frac{c}{r^3} Q_{12} \rightarrow -\frac{2c}{\pi \sigma} \frac{1}{r} \frac{\partial}{\partial r} \int_0^\infty dr' r'^3 \int_0^\infty dk k^2 j_0(kr) j_0(kr') \frac{j_0(kr) j_0(kr')}{(e_Q + M)^2(e_q + m)^2}.$$
The $\vec{p}^2$ expansion of the interaction is an asymptotic series, which has to be truncated. As in the case of constituent quarks the successive terms have increasing amplitudes of alternating signs, this series is but a poor representation of the full spinorial structure of the interaction, the velocity dependence of which is well behaved. We shall therefore truncate the interaction to second order and retain only the local spin-orbit and quadratic spin-orbit interaction terms of quartic order.

To this order the most realistic treatment of the $\vec{p}^2$ correction to the linear potential $e r$ in (6) is to take it into account as a mass shift,

$$m^* \rightarrow m^* + \frac{3}{2} cr\left(\frac{m^*}{m_2}\right)^2,$$

in the kinetic term in (6). If this mass shift is not taken into account explicitly it has to be taken into account in the form of somewhat larger constituent quark masses than used in nonrelativistic quark models, and in the subtraction of a constant from the kinetic term, which otherwise is unrealistically large. This becomes evident below.

### 3.2 The one-gluon exchange interaction

In the case of heavy quarkonia the perturbative gluon exchange interaction forms an important component of the hyperfine interaction between quarks [18]. To second order in the inverse quark masses (with exception for the quadratic spin-orbit interaction, which is quartic) that interaction takes the form:

$$V_G = -\frac{4}{3} \alpha_s \left\{ \frac{1}{r} - \frac{3\pi}{2m_2^2} \delta^{(3)}(r) + \frac{\vec{p}^2}{2Mmr} \right\}$$

$$+ \frac{8\pi}{9} \frac{\alpha_s}{Mm} \delta^{(3)}(r)\vec{\sigma}_Q \cdot \vec{\sigma}_\bar{q} + \frac{\alpha_s}{3Mm^3} S_{12}$$

$$+ \frac{2\alpha_s}{3r^3} \left\{ \frac{M^2 + m^2}{2M^2m^2} + \frac{2}{Mm} \right\} \vec{S} \cdot \vec{L} + \frac{\alpha_s}{4r^5M^2m^2} Q_{12}. \quad (18)$$

Here the mass coefficient $m_2$ is defined in (7). Note that without the square root factors in (3) the numerical coefficients in the last two terms in the first bracket on the r.h.s. would be $-\pi$ and 1 instead of $-3\pi/2$ and $-1/2$ respectively.

For light quarkonia the static gluon exchange interaction (18) has only qualitative value in view of the slow convergence of the asymptotic expansion in $v/c$. Moreover the quadratic spin-orbit interaction is ill-behaved in this limit. A more realistic version of this interaction is obtained by replacing the main $1/r$ (and the accompanying $\delta$ function term) by the unapproximated
Here the function \( f_0(r) \) is defined as
\[
f_0(r) = \frac{2}{\pi} \int_0^\infty dk \frac{\sin(kr)}{k} \frac{M}{e_Q} \frac{m}{e_Q + e_\bar{q}} \alpha_s(k^2).
\] (20)

The factors \( e_Q \) and \( e_\bar{q} \) are defined as
\[
e_Q = \sqrt{M^2 + \frac{k^2}{4}}, \quad e_\bar{q} = \sqrt{m^2 + \frac{k^2}{4}}.
\] (21)

Here \( \alpha_s(k^2) \) is the running coupling constant of QCD, for which we use the screened parametrization [27]
\[
\alpha_s(k^2) = \frac{12\pi}{27} \frac{1}{\ln[(k^2 + 4m^2_{\bar{q}})/\Lambda^2_0]}.
\] (22)

The corresponding modification of the \( \vec{p}^2/r \) term in (18) is obtained by the replacement
\[
-\frac{4}{3} \alpha_s \frac{\vec{p}^2}{2Mmr} \rightarrow -\frac{4}{3} f_2(r) \frac{\vec{p}^2}{r},
\] (23)

where
\[
f_2(r) = \frac{2}{\pi} \int_0^\infty dk \frac{\sin(kr)}{k} \left( \frac{e_Q + M}{2e_Q} \right) \left( \frac{e_\bar{q} + m}{2e_\bar{q}} \right) \left( \frac{M + m}{e_Q + e_\bar{q}} \right)
\]
\[
\left\{ \frac{4}{(e_Q + M)(e_\bar{q} + m)} - \frac{1}{2e_Qe_\bar{q}} + \frac{1}{(e_Q + M)^2} \left[ 1 - \frac{M(e_Q + M)}{2e_Q^2} \right] \right. \\
+ \frac{1}{(e_\bar{q} + m)^2} \left[ 1 - \frac{m(e_\bar{q} + m)}{2e_\bar{q}^2} \right] \right\} \alpha_s(k^2).
\] (24)

It is necessary to treat this term exactly in the solution of the wave equation and not as a first order perturbation. The unapproximated version of the spin-spin component of the gluon exchange interaction is introduced by the replacement of the delta function as
\[
\alpha_s \delta^{(3)}(r) \rightarrow \frac{1}{2\pi^2r} \int_0^\infty dk k \sin(kr) \frac{M + m}{e_Q + e_\bar{q}} \frac{Mm}{e_\bar{q}e_\bar{q}} \alpha_s(k^2).
\] (25)

The unapproximated forms for the tensor and spin-orbit interactions in (18) are
\[
V_G(T) = \frac{2}{9\pi} S_{12} \int_0^\infty dk k^2 j_2(kr) \left( \frac{M + m}{e_Q + e_\bar{q}} \right) \frac{\alpha_s(k^2)}{e_\bar{q}e_\bar{q}}.
\] (26)
\[ V_G(LS) = \frac{2}{3\pi r^2} \bar{S} \cdot \vec{L} \int_0^\infty dk k j_1(kr) \left( \frac{M + m}{e_\bar{q} + e_q} \right) \frac{e_Q + M e_\bar{q} + m}{e_\bar{q}} \alpha_s(k^2) \left\{ \frac{1}{(e_\bar{q} + m)^2} \left[ 1 - \frac{k^2}{4(e_Q + M)^2} \right] + \frac{1}{(e_Q + M)^2} \left[ 1 - \frac{k^2}{4(e_\bar{q} + m)^2} \right] \right\} \alpha_s(k^2). \] (27)

The integrals in the spin-spin and tensor interactions above are well defined only when the running coupling strength \( \alpha_s(k^2) \) is taken into account. The quadratic spin-orbit interaction component in (18) may similarly regulated by employment of the unapproximated form

\[ \frac{\alpha_s}{4r^5 M^2 m^2} \to \frac{2}{3\pi r^2} \int_0^\infty dk k^2 j_2(kr) \frac{\alpha_s(k^2)}{e_Q e_\bar{q}(e_Q + M)(e_\bar{q} + m)} \left( \frac{M + m}{e_\bar{q}} \right). \] (28)

This form of the quadratic spin-orbit interaction has finite matrix elements.

The parametrization (22) takes the long distance screening of the quark-gluon coupling into account through the gluon mass parameter \( m_g \). For this we use the value \( m_g = 240 \text{ MeV} \) while for the confinement scale parameter \( \Lambda_0 \) we use the value 280 MeV. These values are chosen by a fit to the heavy-light meson spectra below. They are similar to those used in refs. [27, 28].

The employment of the screened running quark-gluon coupling strength along with the relativistic corrections to the gluon exchange interaction leads to significant dampening of the gluon exchange interaction at both long and short range. This is shown in Fig. 1, where the unapproximated values for \( f_0(r) \) as given by the expression (19) are plotted for the case of D and B mesons with \( M = 1.58 \text{ GeV} \) and 4.825 GeV respectively, and \( m = 450 \text{ MeV} \). The result reveals that the approximation \( f_0(r) = \alpha_s \) is inadequate. The parametrization (22) gives the value 0.4 for \( \alpha_s \) at the charmonium scale, which is consistent with the values extracted by lattice methods in refs. [25, 26]. In the numerical results for the spectra of the heavy-light mesons below, it proved essential to use the unapproximated expressions for the functions \( f_0(r) \) and \( f_2(r) \), the reason being that the small mass of the light quarks render the static approximations misleading.

### 3.3 The instanton induced interaction

Numerical QCD lattice calculations indicate that the instanton liquid model provides an appropriate description of the dynamics in the low energy regime [2, 21, 22]. The instantons induce a pointlike interaction between heavy and light quarks, which contributes significantly to the hyperfine interaction in heavy-light mesons [19].
Figure 1: The exact function $-\frac{4}{3} \frac{f_0(r)}{r}$ calculated for several sets of heavy and light quark masses and compared to the static approximation for $\alpha_s = 0.45$

The instanton induced interaction for $Q\bar{q}$ systems contains a color dependent component, which may be the cause of the splitting between light pseudoscalar and vector mesons, for which the color magnetic interaction is screened. The form of the instanton induced interaction, which is appropriate for heavy-light mesons has been derived in ref.[19], who considered the large $N_C$ limit, where the matrix element of the color exchange operator is
\[ \langle \lambda^1 \cdot \lambda^2 \rangle = -N_c/2. \]
The spin-dependent component of this interaction couples as a \( T \) invariant \( \sigma^1_{\mu\nu}\sigma^2_{\mu\nu}/2 \), which implies that it may be viewed as a vector meson exchange like interaction with anomalous couplings to quarks.

If the large \( N_c \) limit is not invoked, the instanton induced interaction for the \( Q\bar{q} \) system takes the form [19]:

\[
H_{Q\bar{q}} = \frac{2}{3n^*} \Delta M_Q \Delta m_q \delta^{(3)}(r) + \frac{1}{3n^*} \Delta M_Q^{\text{spin}} \Delta m_q \delta^{(3)}(r) \vec{\sigma}_Q \cdot \vec{\sigma}_{\bar{q}}. \tag{29}
\]

Here \( n^* \) represents the instanton density, which typically is assigned the value \( 1 \text{ fm}^{-4} \). The mass parameters \( \Delta M_Q, \Delta m_q \) and \( \Delta M_Q^{\text{spin}} \) represent the mass shifts of the heavy (\( Q \)) and light (\( \bar{q} \)) quarks, which are caused by the instanton induced interaction. The numerical values for these mass shifts were obtained as \( \Delta M_Q = 86 \text{ MeV}, \Delta M_Q^{\text{spin}} = 3 \text{ MeV} \) and \( \Delta m_q = 420 \text{ MeV} \) respectively in ref. [19].

The expression (29) represents the approximate effective instanton induced interaction that obtains when one of the quarks is much heavier than the other one, which is appropriate here. The Lorentz structure of the main term in the interaction is a scalar, whereas the spin dependent term has a tensor coupling structure.

In the large color limit the sign of the spin-independent term in (29) is negative. In that limit the instanton induced interaction (29) plays a role akin to that of gluon exchange, with matrix elements of similar magnitude [19]. Without invocation of that limit, the spin-dependent term of the interaction (29) is not sufficiently strong to explain e.g. all of the difference between the \( D^* \) and \( D \) meson masses. As below it is found that the screened gluon exchange interaction described above provides sufficient splitting strength, the numerical results do not include the interaction (29). If the gluon mass parameter were increased, it would be necessary to include the additional spin-dependence given by the instanton induced interaction, however.

## 4 The spectra of charmonium and the heavy-light mesons

### 4.1 The charmonium spectrum

In order to test the framework of the Blankenbecler-Sugar equation as applied to \( Q\bar{q} \) systems and in order to fix the mass of the charm quark phenomenologically we first calculate the spectrum of charmonium. This spectrum is fairly well known experimentally.

The interaction model consists of the all the local terms in the scalar linear confining interaction (6),(8) and the gluon exchange interaction, with
full account of the relativistic corrections and the running coupling strength as described in section (3.2) above.

The calculated spectrum is compared to the experimental spectrum in Table 2. The parameters in the interaction model are listed in Table 1. The value for the charm quark mass (1580 MeV) was chosen to be the same as that used for the $D$ and $D_S$-mesons below. With the parameter values listed in Table 1 we are able to accurately reproduce the experimentally determined $J/\Psi$-$\eta_c$ splitting, but at the price of 30-50 MeV overpredictions of the excited states. This effect is similar to that noted in [7]. Better agreement with experiment can be achieved by lowering the confining string constant to 960 MeV/fm and raising the gluon mass $m_g$ by 20 MeV to 260 MeV. In this case the excited states agree fairly well with experiment, while the $J/\Psi$-$\eta_c$ splitting is underpredicted by 15 MeV. A lower charm quark mass for the charmonium system would also improve the spectra, but as the charmonium spectrum is calculated primarily for testing purposes we keep the charm quark mass equal to that used for the $D$ and $D_S$-mesons.

$$\begin{array}{|c|c|c|c|}
\hline
 & c\bar{c} & D_S & D \\
\hline
c & 1120 \text{ MeV/fm} & 1120 \text{ MeV/fm} & 1120 \text{ MeV/fm} \\
b & -50 \text{ MeV} & -260 \text{ MeV} & -320 \text{ MeV} \\
\Lambda_0 & 280 \text{ MeV} & 280 \text{ MeV} & 280 \text{ MeV} \\
m_g & 240 \text{ MeV} & 240 \text{ MeV} & 240 \text{ MeV} \\
m_c & 1580 \text{ MeV} & 1580 \text{ MeV} & 1580 \text{ MeV} \\
m_s & - & 560 \text{ MeV} & - \\
m_{u,d} & - & - & 450 \text{ MeV} \\
\hline
\end{array}$$

Table 1: Model parameters used for the charmonium, $D$ and $D_S$ meson spectra.

The quality of the spectrum is similar to that obtained with an effective interaction constructed by means of lattice methods [1], as well as by completely nonrelativistic phenomenology [29, 16]. The numerical values for the calculated energies of the $c\bar{c}$ are listed in Table 2, along with the experimental values [30].

4.2 The $D$ and $D_S$ -meson spectra

In the calculation of the spectra of the $D$ and $D_S$ mesons, the constituent masses of the light and strange quarks are treated as phenomenological parameters to be fitted against the known splittings in the spectrum. While a reasonable overall description of the $D$ meson spectrum is readily obtainable with the interaction models described above, a quantitative description of e.g.
the spin-orbit splitting of the P-shell $D_j$ ($j = 0, 1, 2$) $D$ meson resonances demands a real effort.

The main difficulty with the spin orbit splittings may be ascribed to the small mass of the light quarks, which makes the matrix elements of the spin-orbit components of both the confining and hyperfine interactions large. That is the reason for taking into account the relativistic damping of these interaction components that is described in section 3.
\[ M_{\text{ud}} = 450 \text{ MeV} \]

|     | \[1^{1}S_{0}\] | \[1^{3}S_{1}\] | \[2^{1}S_{0}\] | \[2^{3}S_{1}\] | \[3^{1}S_{0}\] | \[3^{3}S_{1}\] | \[4^{1}S_{0}\] | \[4^{3}S_{1}\] | \[1^{1}P_{1}\] | \[1^{3}P_{0}\] | \[1^{3}P_{1}\] | \[1^{3}P_{2}\] | \[2^{1}P_{1}\] | \[2^{3}P_{0}\] | \[2^{3}P_{1}\] | \[2^{3}P_{2}\] | \[3^{1}P_{1}\] | \[3^{3}P_{0}\] | \[3^{3}P_{1}\] | \[3^{3}P_{2}\] | \[1^{1}D_{2}\] | \[1^{3}D_{1}\] | \[1^{3}D_{2}\] | \[1^{3}D_{3}\] | \[2^{1}D_{2}\] | \[2^{3}D_{1}\] | \[2^{3}D_{2}\] | \[2^{3}D_{3}\] |
|-----|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|     | 1874          | 2006          | 2540          | 2601          | 2904          | 2947          | 3175          | 3208          | 2389          | 2341          | 2407          | 2477          | 2792          | 2758          | 2802          | 2860          | 3082          | 3050          | 3085          | 3142          | 2689          | 2750          | 2727          | 2688          | 2997          | 3052          | 3029          | 2999          |
|     |               |               |               |               |               |               |               |               | 1864.6±0.5    | 2006.7±0.5    | 2637 ?        | 2637 ?        |               |               |               |               | 2422.2±1.8    | 2458.9±2.0    | 2459 ± 4      |               |               |               |               |               |               |               |               |
|     |               |               |               |               |               |               |               |               | 1869.3±0.5    | 2010.0±0.5    |               |               |               |               |               |               |               |               |               |               |               |               |               |               |               |

Table 3: Calculated and experimental \( D \)-meson states.
The $D$ Spectrum (State energies in MeV)

Figure 2: Calculated and experimental $D$-meson states.

The spectrum of the $D$ meson that is obtained with the scalar linear confining interaction and with the quark masses $m_u = m_d = 450$ MeV and $m_c = 1580$ MeV is shown in Fig. 2. The light quark mass here is about 100 MeV larger than the typical values employed in nonrelativistic phenomenology.
Reducing this mass further in the calculation would lead to an unrealistically large spin-orbit splitting of the $P-$ states.

Because of the large value for the constituent mass of the $u$ and $d$ quarks, a constant $(b)$ of 320 MeV has been subtracted from the kinetic energy term in (1) in the calculation of the spectrum. The interaction parameters used in the calculation are listed in Table 1.

The calculated energies of the $D-$ meson states are listed in Table 3 along with the known empirical energies [30].

The calculated $D$ meson spectrum has the correct $D^* - D$ ground state splitting and only slightly underpredicts the $D$ meson excitation at 2637 MeV. The $P$-shell states around 2400 MeV are also well reproduced.

The calculated energies of the $D_s$ meson states are shown in Fig. 3 and are listed in Table 4. As in the case of the $D$ meson a satisfactory description of the still very incompletely known experimental spectrum is only achievable with a fairly large value for the constituent mass of the $s$ quark ($m_s = 560$ MeV). Concomitantly a constant $(b)$ 260 MeV has been subtracted from the kinetic term in (1) in the calculation.

The overall structure of the $D_s$ meson spectrum in Fig. 3 is similar to that of the $D$ meson spectrum in Fig. 2. The ground state splitting is given correctly, as are the known $P$-state excitations.

The calculated $D$ meson spectrum is similar to that obtained by the Gross reduction of the Bethe-Salpeter equation in ref.[8]. The shell spacing at increasing energy of the latter is somewhat wider, which may reflect a difference between the different quasipotential reductions, but also the employment of static potentials in ref.[8].
The $D_s$ Spectrum (State energies in MeV)

Experimental States

$M = 1580$ MeV, $m = 560$ MeV

Figure 3: Calculated and experimental $D_s$ meson states.
4.3 The $B$ and $B_s$-meson spectra

The empirical knowledge of the spectra of the $B$ and $B_s$ mesons is still very incomplete. Only the ground state pseudoscalar and vector meson state energies are known with certainty. In addition one orbital excitation of the $B$ meson is known at 5700 MeV, which presumably belongs to the $P$–shell.

The only additional parameter required to calculate the spectra of the beauty mesons with the interaction models described above is the $b$ quark mass. For this we choose the value $m_b=4825$ MeV, with which value the

<table>
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<th>$M_s=560$ MeV</th>
<th>Experimental states (MeV)</th>
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<tbody>
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Table 4: Calculated and experimental $D_s$-meson states.
pseudoscalar-vector splitting of the beauty and strange beauty mesons is obtained correctly.

The calculated $B$ and $B_s$ meson spectra are shown in Figs. 4 and 5. The energies of these states are also listed in Tables 6 and 7, along with the empirical values that are taken from ref.[30]. The orbital excitation of the $B$ meson at 5700 MeV is obtained correctly, under the assumption that it corresponds to a $j = 1 P$-shell state.

The overall features if the calculated beauty meson spectra are similar to those obtained with the Gross reduction [7] of the Bethe-Salpeter equation in ref.[8], although as in the case of the charm mesons, we obtain somewhat smaller shell spacings at higher excitation.

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<tr>
<td>$b$</td>
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<td>-250 MeV</td>
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<td>$\Lambda_{QCD}$</td>
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</tr>
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<td>$m_{u,d}$</td>
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Table 5: Parameter values used in the calculation of the $B_S$ and $B$-meson spectra.
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Table 6: Calculated and experimental $B$-meson states
The $B$ Spectrum (State energies in MeV)

M = 4825 MeV, m = 450 MeV

Figure 4: Calculated and experimental $B$-meson spectra
The $B_s$ Spectrum (State energies in MeV)

M = 4825 MeV, m = 560 MeV

Figure 5: Calculated and experimental $B_s$-meson spectra.
5 Quark current operators

5.1 Single quark currents

In order to calculate the M1 transitions of the $Q\bar{q}$ systems we employ the full Dirac structure of the single quark current operators. This was shown to be necessary in ref.[16] for a satisfactory description of the corresponding M1 decays of heavy quarkonia. The currents density operators of the constituent
light quarks and the $c$ and $b$ quarks are then
\begin{equation}
\langle p' | \mathcal{J}(0)| p \rangle = i Q [\bar{u}(p') \gamma^\mu u(p)].
\end{equation}
Here $Q$ is the quark charge operator, which takes the values $\pm 2e/3$ and $\pm e/3$. With canonical boosts for the quark spinors $u, \bar{u}$, this current operator leads to the magnetic moment operator
\begin{equation}
\vec{\mu} = Q \left( \frac{m_p}{m_Q} \right) \frac{\vec{\sigma}}{\sqrt{1 + \vec{v}^2}} \left\{ 1 - \frac{1}{3} \left( 1 - \frac{1}{\sqrt{1 + \vec{v}^2}} \right) \right\} \mu_N,
\end{equation}
where $\vec{v}$ is the quark velocity: $\vec{v} = (\vec{p}' + \vec{p})/2m_Q$, $m_p$ is the proton mass and $\mu_N$ is the nuclear magneton.

For the $D^\pm$ mesons the spin flip part of the combination of the magnetic moments of the charm quark and the light antiquarks is
\begin{equation}
\vec{\mu}_{\text{flip}}(D^\pm) = \pm \frac{1}{6} m_p \left( \sigma_c - \sigma_{\bar{d}} \right) \left\{ \frac{2}{m_c} \frac{1}{\sqrt{1 + \vec{v}_c^2}} \left[ 1 - \frac{1}{3} \left( 1 - \frac{1}{\sqrt{1 + \vec{v}_c^2}} \right) \right] \right.
\end{equation}
\begin{equation}
\left. - \frac{1}{m_{\bar{d}}} \frac{1}{\sqrt{1 + \vec{v}_{\bar{d}}^2}} \left[ 1 - \frac{1}{3} \left( 1 - \frac{1}{\sqrt{1 + \vec{v}_{\bar{d}}^2}} \right) \right] \right\} \mu_N.
\end{equation}
For the $D^0, \bar{D}^0$ mesons the corresponding operator is
\begin{equation}
\vec{\mu}_{\text{flip}}\left( D^0, \bar{D}^0 \right) = \pm \frac{1}{3} m_p \left( \sigma_c - \sigma_{\bar{u}} \right) \left\{ \frac{1}{m_c} \frac{1}{\sqrt{1 + \vec{v}_c^2}} \left[ 1 - \frac{1}{3} \left( 1 - \frac{1}{\sqrt{1 + \vec{v}_c^2}} \right) \right] \right.
\end{equation}
\begin{equation}
\left. + \frac{1}{m_{\bar{u}}} \frac{1}{\sqrt{1 + \vec{v}_{\bar{u}}^2}} \left[ 1 - \frac{1}{3} \left( 1 - \frac{1}{\sqrt{1 + \vec{v}_{\bar{u}}^2}} \right) \right] \right\} \mu_N.
\end{equation}
Here $m$ is the mass of the light antiquark, and $\vec{v}_c$ and $\vec{v}_{\bar{q}}$ denote the velocity operators of the charm quark and the light antiquark respectively. The spin-flip magnetic moment operator for the $D^\pm_s$ meson may be obtained from the corresponding expression for the $D^\pm$ mesons by replacing $\sigma_{\bar{d}}, m_{\bar{d}}$ and $v_{\bar{d}}$ in (32) by $\sigma_{\bar{s}}, m_{\bar{s}}$ and $v_{\bar{s}}$ respectively.

The spin-flip magnetic moment operators for the beauty mesons may be constructed by similar replacements. For the $B^\pm$ mesons these operators are obtained from those for the $D^\pm$ (32) by replacement of the $c$ quark mass and velocity by the corresponding $u$ quark mass and velocities and by replacement of the $d$ quark mass and velocity by the $b$ quark mass and velocity.

The spin-flip magnetic moment operator for the $B^0$ and $\bar{B}^0$ mesons is obtained from that for the neutral $D$ mesons (33) by multiplying the operator by $-1/2$ and by replacing the $c$ and $\bar{u}$ quark masses and velocities by the corresponding masses and velocities for the $d$ and $\bar{b}$ quark. A further
replacement of the $d$ quark mass and velocity by the corresponding $s$ quark mass and velocity yields the spin-flip magnetic moment operator for the $B_s$ mesons.

Note that when matrix elements of these magnetic moment operators are evaluated with wave functions that are solutions to the Blankenbecler-Sugar equation (1), these operators should be multiplied by the factor

$$ f_{BS} = \frac{M + m}{M\sqrt{1 + v^2_Q} + m\sqrt{1 + v^2_{\bar{q}}}}. $$

(34)

5.2 Two-quark currents

Both the scalar confining interaction and the hyperfine interaction may excite virtual quark-antiquark states, which are deexcited by the electromagnetic field. This process is illustrated by the Feynman diagrams in Fig.6. These operators may be derived by the methods described in ref.[16].

Figure 6: Exchange current operators associated with the effective scalar confining and gluon exchange interactions, with intermediate virtual $q\bar{q}$ excitations.

For the $D^\pm$ system the two-quark current that is implied by the scalar confining interaction to lowest order in $v/c$ takes the form

$$ \bar{J}_2(C) = \mp i e c r \frac{m_p}{12} \left( \frac{2}{m_c^2} - \frac{1}{m_d^2} \right) (\vec{\sigma}_c - \vec{\sigma}_d) \times \vec{q}. $$

(35)
This current operator may be viewed as a direct renormalization of the corresponding spin-flip combination of the single $c$ and $\bar{d}$ current operators:

$$\vec{j}_{c\bar{d}} = \pm ie \frac{m_p}{12} \left( \frac{2}{m_c^2} - \frac{1}{m_{\bar{d}}^2} \right) (\vec{\sigma}_c - \vec{\sigma}_{\bar{d}}) \times \vec{q}. \quad (36)$$

The two-quark magnetic moment operator that is implied by this current operator is then accordingly

$$\vec{\mu}_2(C) = \mp cr \frac{m_p}{6} \left\{ \frac{2}{m_c^2} - \frac{1}{m_{\bar{d}}^2} \right\} (\sigma_c - \sigma_{\bar{d}}) \mu_N. \quad (37)$$

For the $D^0, \bar{D}^0$ mesons the corresponding two-quark magnetic moment is then

$$\vec{\mu}_2(C) = \mp cr \frac{m_p}{3} \left\{ \frac{1}{m_c^2} - \frac{1}{m_\bar{u}^2} \right\} (\sigma_c - \sigma_{\bar{u}}) \mu_N. \quad (38)$$

The exchange magnetic moment operator for the $D_{s\pm}$ mesons is obtained from that for the $D_{\pm}$ mesons (37), by replacing $\vec{\sigma}_{\bar{d}}$ and $m_{\bar{d}}$ by $\vec{\sigma}_s$ and $m_s$ respectively.

The general rule is that the exchange current that is associated with the confining interaction has opposite sign to the corresponding combination of single quark current operators, and may be obtained directly from the former by multiplication by the factor $-cr$ and by squaring the quark masses in the denominators. The corresponding operators for the beauty mesons are readily constructed with the help of this rule.

The gluon exchange interaction (18) also implies a two-quark exchange current operator. The matrix elements of this two-body current are however much weaker than those of the exchange current operator that is associated with the scalar confining operator [16]. Because of the cancellations between the matrix elements of the single quark current operators and those of the exchange current operator that is associated with the confining interaction, the gluon exchange current operator does give a significant contribution to the calculated M1 transition rates between the ground state heavy-light mesons.

For the $D_{\pm}$ mesons this two-quark current, to lowest order in the inverse quark masses, has the form

$$\vec{j}_2(G) [D^\pm] = \pm e \frac{8\pi}{9} \left\{ \frac{2\alpha_s(k_\bar{d}^2)}{3k_\bar{d}^2} \left[ \vec{p}_\bar{d} + \vec{p}_{\bar{d}}' \right] \right. \\
+ i \left( \frac{\vec{\sigma}_c}{m_c^2} + \frac{\vec{\sigma}_{\bar{d}}}{m_{\bar{d}}^2} \right) \times \vec{k}_\bar{d} \\
\left. + \frac{\alpha_s(k_c^2)}{3k_c^2} \left[ \vec{p}_c + \vec{p}_c' \right] \right. \\
\left. + i \left( \frac{\vec{\sigma}_c}{m_c^2} + \frac{\vec{\sigma}_{\bar{d}}}{m_{\bar{d}}^2} \right) \times \vec{k}_c \right\}. \quad (39)$$

Here the momentum operators $\vec{k}_\bar{d}$ and $\vec{k}_c$ denote the fractional momenta imparted to the $\bar{d}$ and $c$ (or $d$ and $\bar{c}$) quarks respectively.
The spin-flip component of the magnetic moment operator, that is obtained from this current operator, is then

$$\vec{\mu}_2(G) \left[ D^\pm \right] = \pm \frac{f_0(r)}{27} \frac{m_p}{r} \left( \frac{2}{m_c^2} - \frac{1}{m_c m_d} - \frac{1}{m_d^2} \right) (\vec{\sigma}_c - \vec{\sigma}_d) \mu_N.$$  \hfill (40)

In the case of the $D^0$ and $\bar{D}^0$ mesons the gluon exchange current operator takes the form

$$\bar{j}_2(G) \left[ D^0, \bar{D}^0 \right] = \pm e \frac{16\pi}{27} \left\{ \frac{\alpha_s(k_0^2)}{k_0^2} \left[ \vec{p}_{u'} + \vec{p}_{u} \right] + i \left[ \frac{\vec{\sigma}_c}{m_c m_u} + \frac{\vec{\sigma}_u}{m_c m_u} \right] \times \vec{k}_u \right\}. \hfill (41)$$

The corresponding spin-flip magnetic moment operator is

$$\bar{\mu}_2(G) \left[ D^0, \bar{D}^0 \right] = \pm 2 \frac{f_0(r)}{27} \frac{m_p}{r} \left( \frac{1}{m_c^2} - \frac{1}{m_u^2} \right) (\vec{\sigma}_c - \vec{\sigma}_u) \mu_N.$$ \hfill (42)

The gluon exchange spin-flip magnetic moment for the $D^+_s$ mesons is obtained from that for the $D^\pm$ mesons (40) by replacing $\vec{\sigma}_d$ and $m_d$ by $\vec{\sigma}_s$ and $m_s$ respectively.

The gluon exchange spin-flip magnetic moment operator for the $B^-$ and $B^+$ mesons is obtained from the corresponding expressions (42) for the $D^0$ and $\bar{D}^0$ mesons by replacing the $c$ quark mass and spin operator by the $b$ quark mass and spin operator. Similarly by replacing the $c$ quark mass and spin operator in the expressions (40) for the $D^+$ and $D^-$ gluon exchange spin-flip magnetic moment operator by the $b$ quark mass and spin operator one obtains the corresponding operator for the $\bar{B}^0$ and $B^0$ mesons. Finally the expressions for the gluon exchange spin-flip magnetic moment for the $\bar{B}^0_s$ and $B^0_s$ mesons are obtained from those for the $D^+_s$ and $D^-_s$ mesons by the replacement of the $c$ quark mass and spin operator by the $b$ quark mass and spin operator.

### 6 M1 transitions

#### 6.1 M1 transition matrix elements for $D$ mesons

The spin-flip component of all the magnetic moment operators derived above may be written in the general form

$$\vec{\mu} = \mathcal{M} (\vec{\sigma}_Q - \vec{\sigma}_q).$$ \hfill (43)
Here $\mathcal{M}$ is the matrix element of the orbital part of the magnetic moment operator between the initial and final meson states. The transition width for M1 transitions of the type $Q\bar{q}(J = 1) \rightarrow Q\bar{q}(J = 0)\gamma$ may be expressed as

$$\Gamma = \frac{4}{3} \alpha_{em} \frac{M_f}{M_i} \mathcal{M}^2 q^3.$$  \hfill (44)$$

Here $\alpha_{em}$ is the fine structure constant and $q$ is the photon momentum in the laboratory frame. The masses of the initial and final meson states are denoted $M_i$ and $M_f$ respectively.

The amplitude $\mathcal{M}$ is formed as a sum of matrix elements of single quark and exchange magnetic moment operators. In the case of the $D^{\pm}$ mesons, the matrix element between $S$-states of the single quark operator (32) may be written in the form

$$\mu_{IA}(D^\pm) = \pm \frac{2}{3} \int_0^\infty dp p^2 \int_1^\infty dz f_{BS}(p) \int_0^\infty dr' r'^2 \int_0^\infty dr r^2 \varphi_f(r') \left\{ \frac{2}{mc\sqrt{1 + v_c^2}} \left[ 1 - \frac{1}{3} \left( 1 - \frac{1}{\sqrt{1 + v_c^2}} \right) \right] \right. \left. - \frac{1}{md\sqrt{1 + v_d^2}} \left[ 1 - \frac{1}{3} \left( 1 - \frac{1}{\sqrt{1 + v_d^2}} \right) \right] \right\} j_0(r' \sqrt{p^2 + pqz/2 + q^2/16}) j_0(r \sqrt{p^2 - pqz/2 + q^2/16}) \varphi_i(r). \hfill (45)$$

Here $v_c = p/m_c$, $v_d = p/m_d$, and the factor $f_{BS}(p)$ is the “minimal relativ-ity” factor (34). The expression for corresponding matrix elements in the case of the $D^0$ and $\bar{D}^0$ systems is readily inferred from eqn. (33) by comparison. Here $\varphi_i(r)$ and $\varphi_f(r')$ denote the initial and final orbital wavefunctions respectively.

It is instructive to compare these expressions to those of the static quark model ($v_c, v_d \rightarrow 0$). In the case of the $D^\pm$ mesons that expression is simply

$$\mu_{IA}(D^\pm) = \pm \frac{2}{3} \int_0^\infty dr r^2 \varphi_f^*(r) \left( \frac{2}{mc} - \frac{1}{md} \right) j_0 \left( \frac{qr}{2} \right) \varphi_i(r), \hfill (46)$$

whereas for the $D^0, \bar{D}^0$ systems it is

$$\mu_{IA}(D^0, \bar{D}^0) = \pm \frac{4}{3} \int_0^\infty dr r^2 \varphi_f^*(r) \left( \frac{1}{mc} + \frac{1}{md} \right) j_0 \left( \frac{qr}{2} \right) \varphi_i(r). \hfill (47)$$

The expressions (46) and (46) apply to $D_s^\pm$ mesons with the substitutions $m_d \rightarrow m_s$ and $v_d \rightarrow v_s$.  

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The matrix elements of the exchange magnetic moment that is associated with the confining interaction (6) is in the case of the $D^{\pm}$ mesons

$$
\mu_2(C) = \mp \frac{2}{3} c \int_0^\infty dr r^3 \varphi_i^*(r) \left( \frac{2}{m_c^2} - \frac{1}{m_d^2} \right) j_0 \left( \frac{qr}{2} \right) \varphi_i(r).
$$

This expression applies to the $D_s^{\pm}$ mesons as well, with the substitution $m_d \rightarrow m_s$. In the case of the $D_0, \bar{D}_0$ mesons the corresponding expression is

$$
\mu_2(C) = \mp \frac{4}{3} c \int_0^\infty dr r^3 \varphi_i^*(r) \left( \frac{1}{m_c^2} + \frac{1}{m_u^2} \right) j_0 \left( \frac{qr}{2} \right) \varphi_i(r).
$$

The matrix element of the gluon exchange spin-flip magnetic moment operator is in the case of the $D^{\pm}$ mesons (cf.(40)): 

$$
\mu_2(G) = \mp \frac{4}{27} \left( \frac{2}{m_c^2} - \frac{1}{m_c m_d} - \frac{1}{m_d^2} \right) \int_0^\infty dr r \varphi_i^*(r) f_0(r) j_0 \left( \frac{qr}{2} \right) \varphi_i(r).
$$

In the case of the $D^0, \bar{D}^0$ mesons this matrix element is

$$
\mu_2(G) = \pm \frac{8}{27} \left( \frac{1}{m_c^2} + \frac{1}{m_d^2} \right) \int_0^\infty dr r \varphi_i^*(r) f_0(r) j_0 \left( \frac{qr}{2} \right) \varphi_i(r).
$$

The rule for constructing the corresponding matrix element expressions for the other heavy-light meson charge states may be inferred from these expressions.

### 6.2 M1 transitions of charm mesons

At present the only experimental information on the M1 transition rates of $D$-meson states are the M1 branching ratios for $D^*(2010)^\pm \rightarrow D^\pm \gamma$: $1.1^{+2.1}_{-1.0}\%$ and for $D^*(2007)^0 \rightarrow D^0 \gamma$: $38.1 \pm 2.9\%$ [30]. As the upper limit for the total width of the former transition is 0.131 MeV and for the latter 2.1 MeV, the upper limits for the M1 decay widths are accordingly 1.44 keV for $D^*(2010)^\pm \rightarrow D^\pm \gamma$ and 800 keV for $D^*(2007)^0 \rightarrow D^0 \gamma$. These upper limits suggest that the M1 decay widths should be of the order 1 keV.
This is indeed what we find by explicit calculation here. The calculated widths for the transitions $D^*(2010)^\pm \to D^{\pm}\gamma$ and $D^*(2007)^0 \to D^0\gamma$, as well as for the transitions between their respective excited states, are listed in Tables 9 and 8 respectively. The first column gives the nonrelativistic impulse approximation results (NRIA), which are obtained with static quark current operators. The experience from the corresponding decays in charmonium suggests that these are not realistic. The results obtained in the relativistic impulse approximation, with the full Dirac magnetic moment operators are listed in the second column (RIA). These values should also, on the basis of the experience with heavy quarkonia, be unrealistic. Finally the column RIA + c.f. gives the results that are obtained when the exchange current contribution that is associated with the scalar confining interaction is taken into account. The last column (RIA + c.f. + ex.) also takes into account the contribution of the gluon exchange current. The contribution of the latter is significant mainly for the M1 transitions between the ground states.

In Tables 9 and 8 the M1 transition between the excited $D^*$ and $D$ mesons and the ground states are given as well. In the case of the forbidden transitions between the orbitally excites states the two-body current contributions are dominant. The large difference between the calculated M1 transition rates of the charged and neutral $D$-mesons is due to the very different quark mass dependence of the associated spin-flip magnetic moment operators. In the derivation of these results we determined the value of the photon momentum $q$ from the differences between the empirical values for the $D$ meson states as given in ref.[30]. The difference is only notable in the case of the excited $D'$ and $D^*$ mesons, the latter of which is underpredicted.
We also have calculated the M1 transition rates for $J/\psi \to \eta_c \gamma$ and $\psi' \to \eta_c \gamma$ with the present framework and parameters. In this case the gluon exchange current does not contribute at all because of the equal masses of the charm quarks and antiquarks. The empirical width for the $J/\psi \to \psi_c \gamma$ decay is known to be $1.14 \pm 0.39$ keV. For this we find the value $0.75$ keV when the two-body contribution is included, while the relativistic impulse approximation value is $1.47$ keV. The static quark model prediction is $2.51$ keV, which amounts to an overprediction of about a factor 2. The empirical width of the forbidden M1 transition $\psi' \to \eta_c \gamma$ is $0.78 \pm 0.24$ keV. For this we obtain the value $1.49$ keV when the two-body current is taken into account. This is much closer to the empirical value than the value $0.17$ keV given by the static quark model or the value $8.48$ keV that is obtained in the relativistic impulse approximation.

The M1 calculated transitions between the $D_s^*$ and $D_s$ mesons are listed in Table 10. These transition rates should a priori be expected to be qualitatively similar to those of the $D_s^{*\pm}$ and $D_s^{\pm}$ mesons. This is borne out by the calculation, although the numerical values show marked differences. This is another illustration of the sensitivity of the calculated M1 transition rates to the parameters of the model and in this case to the difference between the constituent masses of the $s$ and $d$ quarks.

### 6.3 M1 transitions of beauty mesons

The calculated M1 transition widths of the beauty mesons are listed in Tables 11 and 12. The experimental decay widths for these transitions are yet to be measured. The large masses of the beauty mesons make these transition widths much smaller than those of the corresponding transitions of charm mesons.

The overall features of these results are however similar to those of the charm mesons. The nonrelativistic impulse approximation leads to considerable overestimates. In the case of the forbidden transitions the two-body exchange currents are again dominant.

The calculated M1 transition rates for the strange beauty mesons are

<table>
<thead>
<tr>
<th>Transition</th>
<th>NRIA</th>
<th>RIA</th>
<th>RIA + cf.</th>
<th>RIA + cf. + ex.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_s^* \to D_s$</td>
<td>178 eV</td>
<td>7.10 meV</td>
<td>736 eV</td>
<td>337 eV</td>
</tr>
<tr>
<td>$D_s^* \to D_s$</td>
<td>7.88 eV</td>
<td>1.22 keV</td>
<td>3.26 keV</td>
<td>4.47 keV</td>
</tr>
<tr>
<td>$D_s^* \to D_s^*$</td>
<td>17.1 eV</td>
<td>246 meV</td>
<td>258 eV</td>
<td>200 eV</td>
</tr>
<tr>
<td>$D_s^* \to D_s^*$</td>
<td>22.4 eV</td>
<td>654 eV</td>
<td>2.53 keV</td>
<td>3.13 keV</td>
</tr>
</tbody>
</table>

Table 10: The M1 decay widths of the $D_s (cs\bar{s})$ mesons by about 35 MeV.
Table 11: The M1 decay widths of the $B_0$ ($b\bar{d}$) mesons

<table>
<thead>
<tr>
<th>Decay</th>
<th>NRIA (eV)</th>
<th>RIA (eV)</th>
<th>RIA + cf. (eV)</th>
<th>RIA + cf. + ex. (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^{*0} \rightarrow B^0$</td>
<td>152</td>
<td>51.9</td>
<td>25.1</td>
<td>9.55</td>
</tr>
<tr>
<td>$B^{*0} \rightarrow B^0$</td>
<td>149</td>
<td>5.39</td>
<td>9.73</td>
<td>12.2</td>
</tr>
<tr>
<td>$B^{*0} \rightarrow B^0$</td>
<td>27.7</td>
<td>9.59</td>
<td>48.3</td>
<td>40.6</td>
</tr>
<tr>
<td>$B^{*0} \rightarrow B^0$</td>
<td>34.6</td>
<td>4.31</td>
<td>7.95</td>
<td>9.72</td>
</tr>
</tbody>
</table>

Table 12: The M1 decay widths of the $B^\pm$ ($b\bar{u}$) mesons

<table>
<thead>
<tr>
<th>Decay</th>
<th>NRIA (eV)</th>
<th>RIA (eV)</th>
<th>RIA + cf. (eV)</th>
<th>RIA + cf. + ex. (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^{*\pm} \rightarrow B^\pm$</td>
<td>462</td>
<td>132</td>
<td>160</td>
<td>67.4</td>
</tr>
<tr>
<td>$B^{*\pm} \rightarrow B^\pm$</td>
<td>455</td>
<td>20.1</td>
<td>39.2</td>
<td>50.9</td>
</tr>
<tr>
<td>$B^{*\pm} \rightarrow B^\pm$</td>
<td>84.3</td>
<td>24.6</td>
<td>221</td>
<td>183</td>
</tr>
<tr>
<td>$B^\pm \rightarrow B^{*\pm}$</td>
<td>105</td>
<td>15.8</td>
<td>32.4</td>
<td>40.7</td>
</tr>
</tbody>
</table>

Table 13: The M1 decay widths of the $B_s$ ($b\bar{s}$) mesons

<table>
<thead>
<tr>
<th>Decay</th>
<th>NRIA (eV)</th>
<th>RIA (eV)</th>
<th>RIA + cf. (eV)</th>
<th>RIA + cf. + ex. (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^{*}_s \rightarrow B_s$</td>
<td>116</td>
<td>46.4</td>
<td>945</td>
<td>148</td>
</tr>
<tr>
<td>$B^{*}_s \rightarrow B_s$</td>
<td>2.39</td>
<td>3.87</td>
<td>2.29</td>
<td>3.22</td>
</tr>
<tr>
<td>$B^{*}_s \rightarrow B_s$</td>
<td>20.9</td>
<td>8.26</td>
<td>12.4</td>
<td>9.76</td>
</tr>
<tr>
<td>$B^<em>_s \rightarrow B^</em>_s$</td>
<td>10.6</td>
<td>3.12</td>
<td>1.84</td>
<td>2.50</td>
</tr>
</tbody>
</table>

7 Discussion

The main result of the present study is that it is possible to achieve a satisfactory description of the heavy-light meson spectra, as well as credible predictions for their radiative spin-flip transitions with the framework of the Blankenbecler-Sugar equation with a scalar confining interaction. This conclusion differs from that in ref.[11]. The calculated results are however very sensitive to the parameters in the model, and to the relativistic damping of the hyperfine interaction. This suggests that that model for the hyperfine interaction (relativistic gluon exchange) may not provide a sufficiently complete description of the hyperfine interaction. It may be conjectured that screening the gluon exchange interaction at shorter distances and compen-
sating with the instanton induced interaction may lead to less parameter sensitive results.

Both the Blankenbecler-Sugar and Gross quasipotential reductions of the Bethe-Salpeter equation lead to two-body exchange current operators, which connect to negative energy intermediate states. If the confining interaction couples as a scalar to the quarks, the accompanying two-quark magnetic moment is a pure spin-flip operator for mesons with equal quark and antiquark masses. The contribution of this operator is essential for obtaining realistic M1 transition rates for heavy quarkonia [16]. In the equal mass case the analog two-quark operator that arises with a vector confining interaction has no spin-flip component [31].

For mesons with unequal quark and antiquark masses the gluon exchange current operator has a spin-flip component. It therefore also contributes to the M1 decay rates of the heavy-light mesons. Although this contribution is much smaller than that of the confining interaction, it is significant in the case of the M1 transitions between the ground state vector and pseudoscalar heavy-light mesons, because of the partial cancellation between the matrix elements of the single quark current operators and those of the exchange current associated with the confining interaction. 

Strong conclusions concerning the M1 transition rates here have to await experimental determination of the total width of the heavy-flavor vector mesons. As the relative branching ratios for M1 decays of these mesons are known, the key missing empirical information is their total widths for strong decay.

Acknowledgments

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References


