Baryon phase-space density in heavy-ion collisions

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The baryon phase-space density at mid-rapidity from central heavy-ion collisions is estimated from proton spectra with interferometry and deuteron coalescence measurements. It is found that the mid-rapidity phase-space density of baryons is significantly lower at the SPS than the AGS, while those of total particles (pion + baryon) are comparable. Thermal and chemical equilibrium model calculations tend to over-estimate the phase-space densities at both energies.

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Baryon phase-space density affects the dynamical evolution of heavy-ion collisions from the initial stage to final freeze-out. It is one of the essential ingredients of thermal and chemical equilibrium models often used in studying heavy-ion collisions [1,2]. The studies of baryon and meson phase-space densities may provide information on particle freeze-out conditions and entropy production in heavy-ion collisions [3,4]. Experimentally, phase-space densities can be estimated from particle yields and source sizes inferred from two-particle interferometry [5–7] and, in the case of nucleons, from deuteron coalescence [3,4,8]. The values of the phase-space densities may be then compared to Bose-Einstein or Fermi-Dirac statistics to assess the degree to which thermal and chemical equilibria are established [9–11]. One unique advantage of such comparisons is that they do not require the extrapolation of data to all of phase-space [12,13].

The freeze-out particle spatial density is also of interest. However, it is known that, in high energy heavy-ion collisions, the freeze-out space point of a particle is correlated with its momentum. Because of this space-momentum correlation, the size parameters extracted from two-particle interferometry at low relative momentum do not reflect the full size of the particle-emitting source [14]. Hence, one cannot readily deduce the particle spatial density from the size parameters and particle yields which is from the entire source.

There have been several studies of the freeze-out phase-space density of pions [5,9–11]. In this paper, we study the freeze-out phase-space density of baryons. We first estimate the baryon phase-space density at mid-rapidity in heavy ion collisions at the AGS and SPS using two-proton interferometry data. We then investigate the ratio of deuteron to proton invariant cross-sections as a means to extract the baryon phase-space density [3,8]. Finally, we compare our estimates to thermal and chemical equilibrium model calculations.

We start from the definition of particle phase-space density,

\[ f(p,x) \equiv \frac{dN}{dpdx} \]  \hspace{1cm} (1)\n
which is Lorentz invariant. Consider identical particles in a small momentum cell, \((p, p + dp)\), which occupy a spatial volume \(V(p)\) in the particle rest frame. (We specify \(V(p)\) in the particle rest frame because available two-proton correlation measurements [15,16] are done in such frame.) If these particles are uniformly distributed, then the spatial averaged phase-space density is

\[ \langle f(p) \rangle = \frac{E}{m} \frac{dN}{dpV(p)} \]  \hspace{1cm} (2)\n
where \(m\) is the particle rest mass, and \(E/m\) is due to the Lorentz boost.

Instead of a uniform distribution in space, if we take a Gaussian form for the particle density profile (as often used to extract source size parameters from two-particle interferometry [17,18]),

\[ f(p,x) = \frac{E}{m} \frac{dN}{dp} \frac{1}{\sqrt{2\pi R_G(p)}} e^{-|x-x_0(p)|^2/2R_G(p)^2} \]  \hspace{1cm} (3)\n
then Eq. (2) becomes

\[ \langle f(p) \rangle = \frac{\int f^2(p,x)dx}{\int f(p,x)dx} = \frac{E}{m} \frac{dN}{dp} \frac{1}{2\sqrt{\pi R_G(p)}} \]  \hspace{1cm} (4)\n
\(R_G\) is a measure of the relative separation of two particles close in momentum in the pair rest frame when the later particle freezes out [19]. In the above equations, however, it has been assumed that the particles freeze-out at an instant time, and the phase-space density is calculated at that time. When the freeze-out process has a finite time duration, the phase-space density should be considered as a “time-averaged” quantity.

Without knowing the exact form of the space-momentum correlation, \(x_0(p)\), one cannot perform the integration over \(p\). Hence, one cannot obtain the spatial density. However, the operation can be carried out over the spatial coordinate \(x\) leading to Eq. (4). In the following discussions, we will use the Gaussian density profile and denote \(V(p) = 2\sqrt{\pi R_G(p)}\).\n
The averaged phase-space density weighted by particle number density is given as

\[ \langle f \rangle = \frac{\int \langle f(p) \rangle dN}{\int dN} = \frac{\int \frac{1}{mV(p)} \left( \frac{E}{dp} \right)^2 \frac{dp}{E}}{\int \frac{E}{dp} \frac{dN}{dp}} \]  \hspace{1cm} (5)
We are mainly interested in the baryon phase-space density. Since the transverse momentum \(p_T\) dependence of \(R_G\) for mid-rapidity proton source has not been measured, we assume a \(p_T\) independent volume \(V(y, p_T) = V(y)\) in the following discussions. Taking an exponential function in \(m_T = \sqrt{m^2 + p_T^2}\) for the invariant cross-section \(E \frac{dN}{dp_T} = \frac{dN/dy}{2\pi T} \frac{m_T}{m_T(m+T)} e^{-(m_T - m)/T}\), we obtain

\[
\langle f(y) \rangle = \frac{dN/dy}{2\pi T m^2 (T+m)^2} \int_{m}^{\infty} \frac{e^{-2(m_T - m)/T}}{V(y, p_T)} T + 2m \frac{dN/dy}{V(y)}.
\]

(6)

Bertsch obtained similar results for mesons from two-particle correlation formula [5], assuming an exponential transverse distribution in \(p_T\). Our results above are valid for both mesons and baryons, and are consistent with Bertsch’s if same assumptions are made.

Using Eq. (6) and available data [15,20–26], the average phase-space densities are estimated for freeze-out pions, protons and baryons at mid-rapidity in central Au+Au or Pb+Pb collisions at the AGS and SPS. The results are listed in Table I. The statistical and systematic errors on the phase-space densities are estimated from the corresponding errors on \(dN/dy\), \(T\), and \(R_G\). In estimating the baryon phase-space density, it is assumed that the source sizes of other baryons are the same as proton’s.

The source size parameters \(R_G\) are estimated from pion and proton interferometry measurements in invariant relative momentum [15,21,25,26]. For pions, \(R_G(p)\) is observed to decrease with \(p_T\) [25–28] due to the space-momentum correlation. However, we have only used the low \(p_T\) pion source size in our estimation. This introduces an underestimate of the pion phase-space density, for instance, by about 25% for the SPS data. For the mid-rapidity proton source size at the AGS, we have used the two-proton correlation measurement at forward rapidity [16] and estimated as \(R_G = 3.5 \pm 0.5\) (syst.) fm such that the low end of the systematic error equals to the initial nucleus size \(R_G = 3.0\) fm. This estimate is consistent with a hypothetical \(1/\sqrt{m_T}\) scaling from pion interferometry measurements. The errors introduced by assuming a \(p_T\) independent volume are not included in the systematic errors quoted in Table I.

It is known that the pion and proton transverse distributions cannot be satisfactorily described by a single \(m_T\) exponential [20,22–24]. In order to check the errors, we used the available data to calculate the phase-space density by Eq. (5) and the results are consistent with the above estimates within 15%. This is also reflected in the systematic errors on the inverse slope parameter \(T\) quoted in Table I. In fact, the values of the phase-space density extracted from this work are consistent with those obtained in [9–11].

As seen in Table I, the freeze-out pion phase-space densities are similar (~0.1) for both AGS (\(\sqrt{s} \approx 5\) AGeV) and SPS (\(\sqrt{s} \approx 20\) AGeV) energies. But the proton(baryon) phase-space density is significantly lower than the pion’s at both beam energies. On top of that, the mid-rapidity baryon phase-space density decreases as the beam energy increases. The similarity of the total phase-space density (pion + baryon) at freeze-out perhaps indicates that, in high energy collisions, the freeze-out process occurs at a fixed condition (local temperature and density) which is determined by the free cross-sections as argued by Pomeranchuk in 1951 [29]. Tevatron data seem to indicate a constant particle density in \(p\bar{p}\) annihilations [30]. Note that at the AGS energy, pion to proton ratio is about one so the baryon density is the relevant control parameter for freeze-out [31]. At the SPS energy, the pion to proton ratio is about six making mesons the dominant player at freeze-out. This will be certainly true in the upcoming RHIC collisions at \(\sqrt{s} = 200\) AGeV.

We have assumed constant \(V(p)\) in estimating \(\langle f \rangle\) above. This is because, as mentioned before, a \(p_T\) dependent two-proton correlation measurement is not presently available in high energy nucleus-nucleus collisions due to the lack of statistics. Such measurement, however, may be obtained from deuteron coalescence [32–38].

In the coalescence model with a Gaussian density profile, \(R^3_G(p) = \frac{4}{3} \pi^{3/2} \left( \frac{E_p}{m_p} \frac{dN_p}{dp_p} \right)^2 \sqrt{\int \left( \frac{E_p}{m_p} \frac{dN_p}{dp_p} \right)}\), where the factor 3/4 comes from spin consideration, and subscripts ‘p’ and ‘d’ denote proton and deuteron, respectively. From Eq. (4), we obtain

\[
\langle f(p) \rangle = \frac{4}{3(2\pi)^3} \left( \frac{E_d}{m_d} \frac{dN_d}{dp_d} \right)_{p=d=2p} \int \left( \frac{E_p}{m_p} \frac{dN_p}{dp_p} \right),
\]

(7)

where the neutron and proton differential cross sections are assumed to be identical \(^\dagger\). The average proton phase-space density [Eq. (5)] can be approximated as \(f(y) = \frac{1}{2\pi T} \int dN_d / dN_p\). Therefore, the \(p_T\) averaged proton phase-space density is

\[
\langle f(y) \rangle = \frac{1}{6(2\pi)^3} \frac{dN/dy}{dN/dy},
\]

(8)

and that averaged over the whole phase-space is

\[
\langle f \rangle = \frac{1}{6(2\pi)^3} \frac{N_d}{N_p}.
\]

(9)

\(^\dagger\)Various models [39,40] show that the neutron to proton \(dN/dy\) ratio at mid-rapidity in Au+Au (AGS) and Pb+Pb (SPS) central collisions is about 1.05.
as pointed out long ago by several authors [3,8].

Using Eq. (8) and available deuteron data [21,41–43], we estimate the phase-space density at the AGS and SPS to be \( \langle f(E) \rangle = 0.015 \pm 0.001 \) (stat.) and 0.0029 \( \pm 0.0004 \) (stat.), respectively. These results are in a good agreement with those obtained using two-proton interferometry data (Table I).

Now we compare our results to calculations by local thermal and chemical equilibrium models. In these models, temperature \((T_0)\) and baryon chemical potential \((\mu_B)\) at chemical freeze-out are extracted from particle ratios [1,2,44–46]; temperature at kinetic freeze-out is extracted from particle spectra with certain assumptions about collective flow [25,31]. These results as obtained from AGS and SPS data are reproduced in Table II.

In local thermal and chemical equilibrium models, the phase-space density is \( \langle f(E) \rangle = (2\pi)^{-3} / \left[ e^{E/T_0} - 1 \right] \) for pion and \( \langle f(E) \rangle = 2(2\pi)^{-3} / \left[ e^{(E-\mu_B)/T_0} + 1 \right] \) for proton, respectively. Here \( E \) is the particle energy, \( T_0 \) the temperature, and \( \mu_B \) the baryon chemical potential. The average particle phase-space densities at mid-rapidity, \( \langle f \rangle = \int \langle f(E) \rangle^2 dp_T^2 / \int \langle f(E) \rangle dp_T^2 \), are calculated separately for chemical and kinetic freeze-out. The results are listed in Table II. The phase-space densities at chemical freeze-out are higher by a factor of 2.7 \( \pm 1.2 \) (AGS) and 4.0 \( \pm 1.3 \) (SPS) than estimated from experimental data (Table I). Likewise, the phase-space densities calculated for kinetic freeze-out are higher by a factor of 1.5 \( \pm 0.7 \) (AGS) and 2.2 \( \pm 1.2 \) (SPS).

One important assumption in local thermal and chemical equilibrium models is that all particles freeze-out at the same time, and they occupy the same volume so that the volume is divided out in particle ratios. From this assumption and the fact that particle numbers stay constant after chemical freeze-out, an extrapolation has been made for the chemical potential from chemical to kinetic freeze-out. This is equivalent to the assumption that the system undergoes isentropic expansion after chemical freeze-out [12,47]. After chemical freeze-out, the system expands and the temperature decreases. This leads to a factor of 3 increase in the volume at both the AGS and SPS energies. Taking the average radial flow velocity as \( v_t \approx 0.5c \) [23], one may estimate the corresponding expansion time: \( (\sqrt{3} - 1)R_A/v_t \sim 6 \text{ fm/c} \) with \( R_A = 1.2A^{1/3} \).

In summary, we have studied the mid-rapidity baryon phase-space density at freeze-out through (1) proton yield and two-proton interferometry measurements, and (2) the ratio of deuteron to proton differential cross-sections. For heavy-ion collisions at both the AGS and SPS, the estimated phase-space density of baryons is significantly lower than of pions. The baryon phase-space density at the AGS is about three times that at the SPS, while the values for the pion phase-space density are similar at both energies. The thermal and chemical equilibrium model calculations are higher than experimental data at both beam energies.

Note that the present analyses suffer from large systematic uncertainties. In order to obtain more precise results for baryons, the proton interferometry or deuteron coalescence data as a function of the transverse momentum are needed. In addition, we only studied the particle phase-space density at mid-rapidity. It will be interesting to repeat the study at projectile/target rapidities and compare with thermal model predictions.

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1 In this procedure, it is explicitly assumed that local thermal and chemical equilibrium is always maintained for pions and protons during expansion of the collision system. These pions include resonance decay products.
\[\text{[38] A. Polleri \textit{et al.}, nucl-th/9904012 (1999).}\]
\[\text{[43] M. Murray (NA44 Collaboration), Quark Matter ’99 proceedings.}\]
\[\text{[44] P. Braun-Munzinger \textit{et al.}, Phys. Lett. B 344, 43 (1995). Note that Si+Pb collisions were studied in the publica-
\text{tion. We applied the same results to Au+Au collisions.}\]
\[\text{[46] P. Braun-Munzinger, private communication.}\]
TABLE I. The average phase-space density $\langle f \rangle$ estimated for freeze-out pions, protons and baryons at mid-rapidity in central Au+Au or Pb+Pb collisions at the AGS (upper section) and SPS (lower section). Input parameters are mid-rapidity $dN/dy$, inverse slope $T$, and Gaussian source size $R_G$ in the pair rest frame. Unless specified, for each quantity the first error is statistical and the second error is systematic.

<table>
<thead>
<tr>
<th>particle</th>
<th>$dN/dy$</th>
<th>$T$ (MeV)</th>
<th>$R_G$ (fm)</th>
<th>$\langle f \rangle (2\pi)^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ \approx \pi^-$</td>
<td>$62 \pm 2 \pm 8$ [20]</td>
<td>$170 \pm 2 \pm 10$ [20]</td>
<td>$6.2 \pm 0.4 \pm 0.4$ [21]</td>
<td>$0.087 \pm 0.017 \pm 0.022$</td>
</tr>
<tr>
<td>proton</td>
<td>$61 \pm 1 \pm 8$ [20]</td>
<td>$270 \pm 3 \pm 10$ [20]</td>
<td>$3.5 \pm 0.5$ (syst.)</td>
<td>$0.014 \pm 0.006$ (syst.)</td>
</tr>
<tr>
<td>baryon</td>
<td>$140 \pm 2 \pm 20$ [20]</td>
<td>$n$</td>
<td>$n$</td>
<td>$0.032 \pm 0.015$ (syst.)</td>
</tr>
<tr>
<td>$\pi^+ \approx \pi^-$</td>
<td>$180 \pm 10 \pm 20$ [22,23]</td>
<td>$206 \pm 2 \pm 20$ [22,23]</td>
<td>$7.4 \pm 0.4$ (stat.+syst.) [25,26]</td>
<td>$0.106 \pm 0.028$ (stat.+syst.)</td>
</tr>
<tr>
<td>proton</td>
<td>$27 \pm 4 \pm 5$ [22–24]</td>
<td>$285 \pm 5 \pm 30$ [22–24]</td>
<td>$3.85 \pm 0.15^{+0.66}_{-0.25}$ [15]</td>
<td>$0.0043 \pm 0.0008^{+0.0033}_{-0.0013}$</td>
</tr>
<tr>
<td>baryon</td>
<td>$68 \pm 8 \pm 10$ [22–24]</td>
<td>$n$</td>
<td>$n$</td>
<td>$0.0109 \pm 0.0018^{+0.0056}_{-0.0056}$</td>
</tr>
</tbody>
</table>

TABLE II. The average mid-rapidity phase-space densities for pions and protons at chemical ($\langle f \rangle_{ch}$) and kinetic ($\langle f \rangle_{kin}$) freeze-out from the thermal and chemical equilibrium model. Input parameters are temperatures ($T_0$ in MeV) at chemical and kinetic freeze-out, and baryon chemical potential ($\mu_B$ in MeV) at chemical freeze-out extracted from central Au+Au or Pb+Pb experimental data at the AGS (upper section) and SPS (lower section). All errors are statistical only. See text for estimation of $\mu_B$ at kinetic freeze-out.

<table>
<thead>
<tr>
<th>Chemical freeze-out</th>
<th>Kinetic freeze-out</th>
<th>particle</th>
<th>$\langle f \rangle_{ch}(2\pi)^3$</th>
<th>$\langle f \rangle_{kin}(2\pi)^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0 = 125 \pm 6$ [44–46]</td>
<td>$T_0 = 92 \pm 2$ [31]</td>
<td>$\pi^+ \approx \pi$</td>
<td>$0.172^{+0.034}_{-0.041}$</td>
<td>$0.107 \pm 0.004$</td>
</tr>
<tr>
<td>$\mu_B = 540 \pm 7$ [44–46]</td>
<td>$\mu_B = 590 \pm 5$</td>
<td>proton</td>
<td>$0.038^{+0.008}_{-0.006}$</td>
<td>$0.021 \pm 0.002$</td>
</tr>
<tr>
<td>$T_0 = 168 \pm 2.4$ [45,46]</td>
<td>$T_0 = 120 \pm 12$ [25]</td>
<td>$\pi^+ \approx \pi$</td>
<td>$0.249 \pm 0.004$</td>
<td>$0.162 \pm 0.023$</td>
</tr>
<tr>
<td>$\mu_B = 266 \pm 5$ [45,46]</td>
<td>$\mu_B = 389 \pm 4$</td>
<td>proton</td>
<td>$0.017 \pm 0.001$</td>
<td>$0.0096 \pm 0.0043$</td>
</tr>
</tbody>
</table>