Energy Calibration for the Prize of a Fig

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Abstract

Present schemes to determine the beam energy rely on accurate energy measurements in the range from 40 to 60 GeV using resonant depolarization and on precise extrapolations to high energy. Several methods based on measurements of the energy loss due to synchrotron radiation have been studied. Different approaches such as the study of the damping time of transverse oscillations, the radio frequency sawtooth and the dependence of the synchrotron tune on the total accelerating voltage are presented.

1 MOTIVATION

The main motivation for the studies presented below is the aim for a reliable energy determination at the highest energies. The recognized extrapolation methods using NMR and flux-loop measurements cross-calibrated with resonant depolarization in the range from 40 to 60 GeV show nonlinearities of the order of 20 MeV at highest energies. The spectrometer project [1] presents one possibility for an energy measurement at higher energies. The methods mentioned below are alternatives, based mainly on determinations of the energy loss and using the existing infrastructure of LEP.

2 METHODS

2.1 Damping of Coherent Oscillations

The coherent horizontal oscillation is excited by a single kick and the center-of-charge position of the bunch is observed over 1024 consecutive turns. A fit to the “1000-turn” data by a damped oscillation with amplitude dependent frequency yields the coherent damping time \( \tau \) as described in [2, 3]. The coherent damping at LEP is composed of radiation and head-tail damping:

\[
\frac{1}{\tau_{\text{coh}}} = \frac{1}{\tau_0} + \frac{1}{\tau_{\text{head-tail}}} \quad \text{with} \quad \frac{1}{\tau_{\text{head-tail}}} \sim \frac{Q'}{E_0} I_b
\]

where \( Q' \) is the chromaticity, \( I_b \) the bunch current and \( E_0 \) the beam energy. Therefore, by extrapolating to \( I_b = 0 \), the damping rate due to synchrotron radiation \( \tau_{\text{rad}}^{-1} \) can be extracted and the energy loss or energy can be calculated. Figure 1 shows such a measurement at 60 GeV. The energy loss is calculated from the offset of the straight line fit to the data. Table 1 gives the results for measurements at 60 and 45.625 GeV. Although the measurements are in good agreement with the MAD [4] predictions, the resulting relative energy uncertainty is of the order of \( O(1\%) \).

2.2 The Energy Sawtooth

The horizontal beam position is a function of the actual momentum. Comparing the difference of electron and
Figure 3: Energy loss from the sawtooth fits in MeV as function of fill number. The dashed lines denote BOM calibrations.

Figure 4: Distribution of results of the energy loss fits for physics fills at 94.5 GeV before fill 5250. The curve is a Gaussian fitted to the histogram.

Figure 5: Upper plot: Synchrotron tune as function of total RF voltage measured with the 60/60 optics at 50.005 GeV. The curve is a best fit according to eq. (2). Lower plot: Difference between data and fit for the same range.

2.3 $Q_s$ and total RF Voltage

As the synchrotron tune depends on the beam energy as well as on energy loss and total RF voltage, measurements of these dependencies can be used to determine the beam energy. This method has evolved from many discussions with H. Burkhardt. The upper plot of fig. 5 shows a measurement of the synchrotron tune as function of total RF voltage for the 60/60 optics at 50.005 GeV. The curve is a best fit according to

$$Q_s^2 = \left( \frac{\alpha_c \hbar}{2\pi E} \right) \sqrt{V_{RF}^2 - U_0^2}$$

(2)

with the energy loss $U_0 = C_\gamma / \rho E^4$. The bottom plot of fig. 5 shows the difference between data and fit. Residuals and $\chi^2$ show clearly that the simple model is not sufficient to describe the measurements. However, the situation can be improved by some refinements like the correction of the dashed lines, “BOM calibrations” have been made. The fit results seem to scatter around a central value but there are clear “jumps” some of which correspond to BOM calibrations. The other jumps could not yet be accounted for.

To estimate the intrinsic precision of this method a projection onto the ordinate is shown in fig. 4 for all fills before fill 5250. The RMS of the energy loss distribution is relatively small and corresponds to a relative uncertainty of the energy of order $O(5 \cdot 10^{-4})$. This method however is strongly limited by systematic effects. Obvious are the unexplained “jumps”, but there are also other problems: The fit results differ between the octants and the results strongly depend on the selection of rejected pickups. The good intrinsic accuracy and the parasitic measurement favor this method but the systematics are not yet under control.

1During the discussion following the presentation, A. Verdier proposed to use the algorithm presented in [5] for the rejection of bad pickups.
energy for central frequency and tides

\[ E_c = E \left( 1 - \frac{1}{\alpha_c} \frac{f_{RF} - f_{RF}^c}{f_{RF}} \right) \]  

(3)

and the introduction of a “voltage correction factor” \( V_{RF} \to g V_{RF} \) which makes the effective voltage smaller than the nominal one. The first plot of fig. 6 shows the residuals of the fit after applying these corrections. Systematic offsets in the residuals and the still rather large \( \chi^2 \) indicate that the fit model still is not complete. In addition to the energy loss in dipoles due to synchrotron radiation there are other energy losses which have to be taken into account: energy loss from quadrupoles due to sawtooth and closed orbit distortions, energy loss from correctors, parasitic mode losses, corrections due to finite beam size and to the momentum offset due to central frequency and tides.

The energy loss due to finite beam size equals the loss at a position offset of 1 \( \sigma \). Table 2 shows a compilation of all corrections. The overall uncertainty assigned to the sum of these losses \( K \) is \( \Delta K = \pm 500 \) keV. The total energy loss \( U_0 \) used in eq.(2) is finally

\[ \tilde{U}_0 = \frac{C}{\rho} E^4 + K \]  

(4)

In order to test the fit model, it was applied to data generated with the EMIT command of MAD. Several RF configurations have been used: a realistic case where all superconducting units were “switched on”, a case with only one unit with same total voltage and the limit of a homogenous distribution where the voltage is distributed over the whole ring. Figure 7 shows the three datasets generated for a beam energy of 50.005 GeV. The simple model of eq.(2) is only valid in the limit of a homogenous distribution. To improve the model, a term proportional to \( V_{RF}^4 \) has to be added. The weight factor \( M \) is taken from the fit to the MAD dataset. The line drawn in fig. 7 is a fit of the model of eq.(5) to the realistic case using appropriate input parameters. The energy extracted from the fit is in good agreement with the input energy. A systematic uncertainty of \( \pm 10 \) MeV is assigned to the fit results. Figure 8 shows the measurements with the 60/60 optics at 50.005 and 60.589 GeV. The curves are best fits using the following “final” model:

\[ Q_{s}^{2} = \left( \frac{\alpha_c h}{2 \pi} \right)^{2} \left\{ \frac{g^2 V_{R}^2}{E_{c}} + M V_{RF}^4 - \frac{1}{E_{c}} \tilde{U}_0^2 \right\} \]  

(5)

with the relations from equations (3,4). The momentum compaction factor \( \alpha_c \) and the voltage nonlinearity factor \( M \) are taken from MAD. All other parameters were allowed to vary in the fit. The lower plot of figure 6 shows the residuals of the fit to the 50.005 GeV data. It is clearly visible that

<table>
<thead>
<tr>
<th>energy loss mechanism</th>
<th>energy loss [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>quad. / sawtooth</td>
<td>5</td>
</tr>
<tr>
<td>quad. / closed orbit</td>
<td>47</td>
</tr>
<tr>
<td>correctors</td>
<td>33</td>
</tr>
<tr>
<td>mom. offset (( f_{RF}^c / \text{tide} ))</td>
<td>17</td>
</tr>
<tr>
<td>beam size</td>
<td>213</td>
</tr>
<tr>
<td>parasitic mode loss</td>
<td>2250</td>
</tr>
<tr>
<td>( \sum = K )</td>
<td>2565</td>
</tr>
</tbody>
</table>

Table 2: Estimates of the additional energy losses for beam energies of 50.005 and 60.589 GeV. The sum \( K \) is added to the fit model. The overall uncertainty assigned to \( K \) is \( \Delta K = \pm 500 \) keV.

Figure 6: Upper plot: Difference between data and fit after introduction of a “voltage correction factor” and correction for central frequency and tides. Lower plot: Difference between data and fit for the final fit model.

Figure 7: Synchrotron tune as function of total RF voltage as calculated with the EMIT command of MAD for different RF configurations. The curve is a fit to the “realistic” RF distribution using the fit model eq.(5) with appropriate input parameters.

Figure 8: Measurements with the 60/60 optics at 50.005 and 60.589 GeV. The curves are best fits using the following “final” model:

\[ Q_{s}^{2} = \left( \frac{\alpha_c h}{2 \pi} \right)^{2} \left\{ \frac{g^2 V_{R}^2}{E_{c}} + M V_{RF}^4 - \frac{1}{E_{c}} \tilde{U}_0^2 \right\} \]  

(5)

with the relations from equations (3,4). The momentum compaction factor \( \alpha_c \) and the voltage nonlinearity factor \( M \) are taken from MAD. All other parameters were allowed to vary in the fit. The lower plot of figure 6 shows the residuals of the fit to the 50.005 GeV data. It is clearly visible that
the model is able to reproduce the measurements quite accurately. External knowledge was incorporated in the fit by introducing constraints of the type \((a - a_{\text{nom}})^2/\sigma_a^2\) where \(a\) stands for a fit parameter and \(\sigma_a\) for its estimated uncertainty to the \(\chi^2\)-function. \(a_{\text{nom}}\) is the value the parameter is constrained to. The beam energy was constrained to the nominal voltage. The results of the fits imply that the effective voltage is about 5\%.

The voltage correction factor \(K\) was set to the estimates given in tab. 2 with an error of \(\pm 0.5\) MeV. The voltage correction factor \(g\) was constrained to the average value obtained from the measurements with the 60/60 optics. \(\sigma_g\) was estimated from the spread of the results. The value of

\[
g = 0.95415 \pm 0.0005
\]

implies that the effective voltage is about 5\% less than the nominal voltage. The results of the fits for the measurements with the 60/60 optics as well as the nominal energies and the energies measured with resonant depolarization in the following fill are given in table 3. For both measurements the fitted energies are lower than the polarization energies but still agree within their errors. The relative error is of the required magnitude. To estimate the relative precision at high energies, data of a measurement with the 102/90 optics at 90.419 GeV were used. At this preliminary stage it is not possible to give final results, but the absolute error on the beam energy seems to be mostly energy independent.

The systematic uncertainty assigned to the results from studies with MAD is \(\pm 10\) MeV.

At 90.419 GeV the relative uncertainty is

\[
\Delta E/E = 2.8 \cdot 10^{-4}
\]

This can still be improved if tighter bounds can be set on \(K\) and \(g\). If the energy is known from other measurements (polarization, spectrometer) the same fits can be used to extract the momentum compaction factor. The relative uncertainty resulting from this method is

\[
\Delta \alpha/e/\alpha_e \approx 1 \cdot 10^{-3}
\]

whereas conventional measurements have relative uncertainties of 1 - 2\%. All measurements of the momentum compaction factor are in good agreement with the MAD value.

### 3 SUMMARY

Several methods to measure the energy from the energy loss using existing infrastructure have been studied. To be a useful calibration, the relative uncertainty should not exceed a few times \(10^{-4}\). The determination of the energy loss from the damping of coherent oscillations gives a relative error of \(O(1\%)\). The measurement of the energy loss using the energy sawtooth has a good intrinsic accuracy \((O(5 \cdot 10^{-4}))\) and does not require dedicated beam time. However the systematics are not yet under control. The most promising method is the measurement of the synchrotron tune as function of total RF voltage. The relative energy error of this reliable measurement is of the order of \(O(3 \cdot 10^{-4})\) and further improvements are possible.

### 4 ACKNOWLEDGEMENTS

Sincere thanks to all who have contributed to the results presented above in helpful and instructive discussions. Many thanks as well to A. Butterworth and E. Peschardt for their help with the RF in MD and analysis. Especially I would like to thank H. Burkhardt, A. Hofmann, A. Verdier and J.M. Jowett and in particular J. Wenninger for his constant help and support.

### 5 REFERENCES

[1] B. Dehning. Is the spectrometer a fata morgana or a real oasis? These proceedings.


