INJECTION AND EXTRACTION FOR CYCLOTRONS

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Abstract

External ion sources for cyclotrons are needed for polarised and heavy ions. This calls for injection systems, either radial or axial. Radial injection is also needed when a cyclotron works as a booster after another cyclotron or a linear accelerator (usually tandem). Requirements for injection differ from separated sector cyclotrons where there is plenty of room to house inflectors and/or strippers, to superconducting cyclotrons where the space is limited by a small magnet gap, and high magnetic field puts other limitations to the inflectors.

Several extraction schemes are used in cyclotrons. Stripping injection is used for $H^-$ and also for heavy ions where the $q/m$ ratio is usually doubled. For other cases, electric and magnetic deflection has to be used. To increase the turn separation before the first deflector, both resonant and non-resonant schemes are used.

In this lecture, external injection systems will be surveyed and some rules of thumb for injection parameters will be given. Extraction schemes will also be reviewed. Most of the text has been adapted and modified from the reports by David Clark from Berkeley [1] and by Werner Joho from PSI, Zürich [2].

1 INTRODUCTION TO INJECTION SYSTEMS

The first external injection to a cyclotron was designed for polarised beams. Keller and his group at CERN tested their method on a model cyclotron but they never used it with the CERN 600 MeV synchrocyclotron [3]. The first injection into a cyclotron was reported by Powell’s group at the University of Birmingham in 1992 [4]. They got a transmission of 3% from the source to accelerated beam, without bunching. The beam was injected axially. The first radial injection was reported by Thirion in 1963 [5]. Polarised atomic deuteron beam was injected into the centre of the cyclotron where the beam was ionised.

After these first external injections, other methods of injection were proposed and tested. Today, most heavy-ion cyclotrons utilize ECR ion sources together with axial injection where a spiral inflector is used. Radial injection is used both in separated sector cyclotrons and in superconducting cyclotrons where the beam is stripped to higher charge state in the centre of the cyclotron. External injection is the only way to get polarised ions to acceleration due to the large size of the ion source.
2 NEUTRAL BEAM INJECTION

2.1 Thermal Velocity Beams

Keller’s group at CERN proposed to inject neutral beam from a polarised source in the cyclotron median plane. The atoms were ionised in the centre by electron bombardment. The group tested this method with a 4.5 MeV model cyclotron, but they never installed it on the 600 MeV machine. Since the injected beam was of protons, it was difficult to reduce the hydrogen background below the atomic beam pressure.

Thirion’s group injected polarised deuterons with the same method in the 22 MeV classical cyclotron. For deuterons the hydrogen background was not a problem, and a polarisation of 55% was obtained. The deuteron current on target was 0.03 nA. Maillard and Papineau used an arc-type ioniser in the centre of the Saclay cyclotron, and they reported a beam intensity of 5 nA, with the same polarisation [6]. An external ioniser could give a further factor of 10 higher beam current.

2.2 Fast Neutral Beams

A beam of polarised protons accelerated to 40 keV, neutralised, injected into the cyclotron centre, and stripped there with a foil was used on a U-120 Cyclotron at the Nuclear Research Institute in Rez, Czechoslovakia. This method has the advantage that it does not ionise residual gas. Another advantage is that the beam can be focused after the ion source, before the neutralisation.

3 AXIAL INJECTION

The first external injection system was installed on the Birmingham cyclotron by Powell in 1962. The beam focusing in the injection line was done with einzel lenses, and a 45° gridded electrostatic mirror was used to bend the beam into the horizontal plane. Such a mirror had a 65% transparency after two traversals. Today, mirrors have been replaced by spirals which have the possibility of 100% transmission. Also hyperboloid inflectors are used.

3.1 The Electrostatic Mirror

The simplest inflector is an electrostatic mirror. It consists essentially of a pair of planar electrodes which are at an angle of 45° to the incoming beam. The other electrode has to be a grid to allow the beam to enter the bending electric field. The grid reduces the transmission. A mirror is the smallest inflector type, but on the other hand, the required voltage must then be high. A mirror also increases the effective emittance of the beam. The structure of the Birmingham mirror inflector is shown in Fig. 1.

3.2 Spiral Inflector

A spiral inflector, also known as a helical channel, was first used at Grenoble [7]. A spiral inflector consists of a cylindrical capacitor which has been twisted to take into account the spiralling of the ion trajectory which results from the bending in the magnetic field.

The basic assumption with the spiral inflector geometry is that the electric field $\vec{E}$
is always perpendicular to the velocity \( \vec{v} \) of the ion. Thus, the central trajectory lies on an equipotential surface. This allows for lower voltages than with a mirror. The spiral used in Grenoble is shown in Fig. 2.

3.2.1 The beam trajectory in a spiral inflector

To study the beam trajectory in a spiral inflector we define two coordinate systems. The magnetic field is fixed in \((x, y, z)\) frame with unit vectors \(\hat{i}, \hat{j}, \hat{k}\) so that \(\vec{B} \parallel \hat{k}\). The coordinate system following the particle trajectory is \((\hat{h}, u, v)\), where \(\hat{v}\) is parallel to the beam velocity, \(\hat{h}\) is perpendicular to \(\hat{v}\), and is parallel to the cyclotron median plane. The unit vector \(\hat{u}\) can be defined with \(\hat{u} = \hat{h} \times \hat{v}\). The positions of the two frames are shown in Fig. 3.
The unit vectors of the moving frame can also be defined with the velocity vector \( \vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \)

\[
\begin{align*}
\hat{h} &= \frac{v_y \hat{i} - v_x \hat{j}}{\sqrt{v_x^2 + v_y^2}}, \\
\hat{v} &= \frac{v_z \hat{i} + v_y \hat{j} + v_z \hat{k}}{\sqrt{v_x^2 + v_y^2 + v_z^2}}, \\
\hat{u} &= \hat{h} \times \hat{v}.
\end{align*}
\]

The assumption used to solve analytically for the central trajectory are:

1. The magnetic field \( B \) is constant, and parallel to \( \hat{k} \).
2. \( E_h \) is proportional to the \( xy \) plane component of the particle velocity.
3. \( E_u \) is constant along the central trajectory.
4. \( E_v = 0 \).

Using the above mentioned assumptions, J.L. Belmont and J.L. Pabot solved analytically the central trajectory for a spiral inflector [7]. The solution in parametric form is

\[
x = \frac{A}{2} \left( \frac{2}{1 - 4K^2} + \frac{\cos(2K - 1)b}{2K - 1} - \frac{\cos(2K + 1)b}{2K + 1} \right),
\]
where $0 \leq b \leq \frac{\pi}{2}$. The ion mass $m$, velocity $v_0$, the magnetic field $B$ and the electric field $E$ fix the parameters $A$, $b$ and $K$:

\[
\begin{align*}
b &= \frac{v_0 t}{A}, \\
A &= \frac{mv_0^2}{qE_u}, \\
K &= \frac{A}{2\rho} + \frac{k'}{2},
\end{align*}
\]

where $\rho$ is the bending radius in the magnetic field $\rho = \frac{mv_0}{qB}$. $A$ is the corresponding bending radius for an ion in a radial electric field $E_u$ (see Fig. 4), and it is inversely proportional to the potential difference of the electrodes.

The parameter $k'$ fixes the direction of the electric field. If $\theta$ is the angle between $\hat{u}$ and $\vec{E}$, then

\[
\begin{align*}
\vec{E} &= E \left\{ \sin \theta \hat{h} + \cos \theta \hat{u} \right\}, \\
k' &= \frac{\tan \theta}{\sin \theta}, \\
E &= E_u \sqrt{1 + (k' \sin b)^2}.
\end{align*}
\]

The coordinates of the centre of curvature at the exit of the inflector are

\[
\begin{align*}
x_c &= A \left\{ \frac{1 - 2K \sin(K\pi)}{1 - 4K^2} - \frac{\sin(K\pi)}{2K - k'} \right\}, \\
y_c &= -A \left\{ \frac{2K}{1 - 4K^2} + \frac{1}{2K - k'} \right\} \cos(K\pi).
\end{align*}
\]
Figure 5: a) Cross-section of the spiral electrodes in $hu$-plane, and b) projection of the central ray on $xy$-plane.

In a spiral inflector, there are two free parameters, $A$ and $k'$. The spiral size in $z$-direction is determined by $A$, and in the $xy$-plane by both parameters. This allows for flexible central region design. The beam can be injected along the symmetry axis, and yet the beam can still be easily centered in the central region.

3.3 The Hyperboloid Inflector

The geometry for the spiral inflector electrodes is complicated, and the construction is relatively difficult. Electrodes which are surfaces of revolution are easier to construct since such surfaces can be constructed on a lathe.

The simplest potential which satisfies Laplace's equation and at the same time possesses radial symmetry is a hyperbolic potential of the form

$$V_p = \frac{1}{2} K z^2 - \frac{1}{4} K r^2 + C,$$

where $K$ and $C$ are constants, and $z$ and $r$ are cylindrical spatial coordinates. The electrodes are thus hyperboloids of revolution about the $z$ axis. Assuming a constant magnetic field (in $z$-direction) the parametric representation of the central trajectory is [8]:

$$x = \frac{r_0}{2} \{-b \cos(akt) + a \cos(bkt)\},$$

$$y = \frac{r_0}{2} \{-b \sin(akt) + a \sin(bkt)\},$$

$$z = \frac{r_0}{2} \sin(kt),$$

where $a = \frac{\sqrt{6}}{2} + 1$ and $b = \frac{\sqrt{6}}{2} - 1$. The ion mass $m$, charge $q$, and the injection radius $\rho = \frac{m v_0}{q B}$ fix the parameters $k$ and $r_0$:

$$k^2 = \frac{q K}{m},$$

$$r_0 = (2\sqrt{6})\rho.$$
Figure 6: Electrodes for a hyperboloid inflector, central trajectory and the projection of the central trajectory in the median plane.

The central trajectory follows a contour \( r^2 - 2z^2 = r_0^2 \). There are no free parameters. When the injection radius has been fixed we get also the electrode geometry. The electrode geometry is obtained from equations

\[
\begin{align*}
    r^2 - 2z^2 &= r_1^2, \\
    r^2 - 2z^2 &= r_2^2,
\end{align*}
\]

where \( r_1 < r_0 < r_2 \), and \( r_2 - r_1 \) is the electrode gap. Hyperboloid geometry is shown in Fig. 6. The required potential difference comes from

\[ U = \frac{1}{4} K (r_2^2 - r_1^2) \approx 4 U_{inj} (r_2 - r_1) / r_0, \]

where \( U_{inj} \) is the injection voltage.

A hyperboloid inflector is usually bigger than a spiral. The advantage of a hyperboloid is that the two transverse sub phase spaces do not correlate as in a spiral. If we neglect the end effects a hyperboloid is a linear element, and we can derive the transfer matrices without any approximations [8], [9]. Due to non-correlated transfer the matching into the cyclotron acceptance is easy compared to that with a spiral inflector.

### 3.4 The Parabolic Inflector

Another alternative to the spiral inflector is the parabolic inflector. It consists of two bent sheet-metal plates. One of the electric field components is zero at all points within the inflector. As for the hyperboloid inflector, there are no free parameters, and the size becomes bigger than for a spiral using the same injected beam. Parabolic inflectors are not used in cyclotrons, today. More information on this kind of inflector can be found in reference [8].

### 3.5 Axial Hole

In axial injection, the beam enters the central region through a hole in the symmetry axis of the magnet. For conventional magnets, the magnetic field is almost zero in the hole, increasing to some 1.7 T in the gap. Such a field acts as a magnetic mirror, i.e. particles that enter the edge of the field at an angle larger than a certain angle (mirror angle) will be bent back. Some focusing is needed in the axial hole. One can get a
good idea about the focusing needed (position and strength) by tracking the cyclotron acceptance backwards from the inflector entry with no extra focusing in the hole. Usually, solenoids, glaser lenses (short solenoids) or electrostatic einzel lenses are used.

In superconducting cyclotrons the axial magnetic field on the axis is relatively big, and can focus the beam. The field inside of the yoke depends on the main field level, and it must be taken into account when determining the focusing strengths for the other focusing elements in the hole. Axial injection in superconducting cyclotrons was studied by Bellomo and co-workers in the course of the MSU K1200 (K800 at that time) design [10]. Especially for a mirror inflector an off axis injection is needed to ensure proper centering of the beam.

The two transverse acceptances at the inflector entry do not usually coincide. Hence, the optimum injected beam is not symmetrical. The axial magnetic field rotates the beam, and the sub phase spaces get correlated. For efficient injection, a beam rotator or skew quadrupoles should be used to match the beam into the machine acceptance. Although the hyperboloid does not introduce correlation between the two transverse phase spaces the axial field before the inflector entry rotates the beam, and that correlation must be corrected in the injection line.

4 RADIAL INJECTION

4.1 Trochoidal Injection

Radial injection was used for the first time at the Lebedev Institute in Moscow [11]. They used the magnetic field difference between hill and valley to send the beam on a trochoidal trajectory to the central region as shown in Fig. 7. At the centre an electrostatic channel is used to inflect the beam into a centered orbit. The extraction energy of the Lebedev Institute cyclotron was only 300 keV while the injection energy was 30 keV.

Henry Blosser’s group studied the same method for a 42 MeV proton cyclotron with an injection energy of 250 keV. As can be seen in Fig. 9 the loops overlap and the injection is more difficult.

Today, trochoidal injection is not used.

4.2 Electric Field Cancelling Magnetic Field

In 1965, another method of radial injection was suggested by Beurtey and Thirion at Saclay [12]. The method uses a system of electrodes, shown schematically in Fig. 10, shaped to provide horizontal electric field to cancel the force from the magnetic field of a 5 keV proton beam, and to focus it on its path to the cyclotron centre. This method gave a 4 % transmission to acceleration and 1.4 % to extracted beam. The external beam intensity was 70 nA.

4.3 Injection From Another Accelerator

Today, the most usual radial injection is from a tandem to a sector focused cyclotron, with a stripper in the centre of the cyclotron. The Oak Ridge Isochronous Cyclotron (ORIC) operates together with the 25 MV tandem. The first coupled operation took place on January 27, 1981. A Beam of 38 MeV $^{16}$O$^{2+}$ was injected into ORIC, stripped
Figure 7: Radial trochoidal injection in median plane at Lebedev Institute, Moscow.

Figure 8: Lebedev radial injection centre region, showing channel inflector.
Figure 9: Michigan State calculated trochoidal injection at 250 keV

Figure 10: Schematic layout of a radial injection where electric field cancels magnetic force
Figure 11: Median plane section of the Chalk River superconducting cyclotron
to $8^+$ and accelerated to 324 MeV [13]. Another example is the Chalk River superconducting
cyclotron which accelerates beams injected from the tandem [14]. The Chalk River cyclotron
employs charge-exchange injection using a 20 $\mu$g/cm$^2$ C foil stripper at the inner equilibrium orbit. Figure 11 shows the median plane section where the injected
beam can be seen, too.

In a radial injection from another accelerator the magnetic rigidity of the injected
beam must match to the magnetic field and the pole size of the cyclotron. The injected
beam should be bent less than 180° in the field. On the other hand, the velocity of the
injected beam should be such as to give the desired charge state after the stripper. This
limits somewhat the beams available.

4.4 Injection Into Separated Sector Cyclotron

Radial injection into a separated sector cyclotron is similar to that into a sector
focused cyclotron. The main difference, however, being that there is much more space
available for focusing elements, and the magnetic field in the valley is practically zero
compared to that in the sectors. Usually, bending elements are used in the centre of the
machine to inflect the beam into the centered orbit. A good example of such an injection
is the two GANIL separated sector cyclotrons [15] shown in Fig. 12.
5 LONGITUDINAL MATCHING

A cyclotron can accelerate only a portion of an RF cycle. Usually the RF-acceptance is some ±20 RF-degrees (out of 360). Ion sources deliver a DC-beam (Electron Beam Ion Source, EBIS, is a pulsed source but the repetition rate is far too slow for isochronous cyclotrons with a frequency of the order of 10 MHz). To increase the total efficiency of the accelerator (and hence the accelerated beam current) the beam from the ion source must be bunched. A buncher, which is located upstream in the injection line, accelerates particles that would come too late to the first accelerating gap, and decelerates particles that would come too early to the first gap. The buncher is thus an acceleration cavity. It usually works at the principal RF-frequency of the cyclotron. Sometimes the wave form is modulated with the second harmonic frequency which increases somewhat the bunching efficiency. The beam intensity increases normally with a factor of 3–4.

6 INTRODUCTION TO EXTRACTION SCHEMES

Today’s cyclotrons have improved the extraction efficiency towards 100%. A beam power of over 50 kW (TRIUMF, PSI) means that already a loss of 10% of the beam at extraction would pose serious problems with cooling and activation. Practically 100% can be achieved with a stripping extraction. Other extraction schemes use static electromagnetic fields to “peel” the beam out of the cyclotron. Both resonant and non-resonant extraction can be used. For a deflector, the turn separation must be increased both with acceleration and with magnetic bumps which excite resonances. Both integer and half integer resonances are used.
7 EXTRACTION SYSTEMS FOR CYCLOTRONS

7.1 Stripping Extraction

H⁻ cyclotrons use stripping extraction. The beam hits a foil (usually carbon) at the extraction radius, the electrons are stripped and we are left with a naked proton which will be bent outwards from the machine (the charge changes its sign). The extraction energy is easily changed by changing the radial position of the stripper foil.

Positive ions can be extracted with stripping if they have enough electrons to be stripped. For example in the JINR U-200 cyclotron in Dubna, when the accelerated beam is 
\(^4\text{He}\)\(^{1+}\) it becomes naked in the stripper, and the \(q/m\) ratio is doubled. The stripper is located in the hill. After stripping the bending radius is reduced to one half, and on the way back outwards the particles pass into the valley where the field is smaller. There the bending radius is larger, and the particles are extracted from the cyclotron. A drawback in this kind of extraction is that efficient ECR ion sources cannot be used for light and medium heavy ions since the ion source already can make the ions fully stripped. Hence, high energy (\(E \propto q^2\)) cannot be reached.

7.2 Extraction By Acceleration

In a cyclotron the particles move outwards due to their energy increase. This makes it possible to peel the beam off with static electromagnetic channels. A radial increase of the orbit can be achieved not only by acceleration, but also by magnetic bumps:

\[
\frac{dR}{dn} = \frac{dR}{dn}(\text{accel}) + \frac{dR}{dn}(\text{magn}).
\]

The acceleration part can be written as

\[
\frac{dR}{dn}(\text{accel}) = R \times \frac{E_\text{G}}{E} \times \frac{\gamma}{\gamma + 1} \times \frac{1}{\nu_r^2},
\]

where \(R\) is the average radius of the orbit, \(E_\text{G}\) energy gain/turn, \(E\) kinetic energy, \(\gamma = 1 + E/E_0\) and \(\nu_r\) the radial betatron frequency. From Eq. (13) we see three ways to get a high extraction rate:

1. build cyclotrons with a large average radius,
2. make the energy gain per turn as high as possible,
3. accelerate the beam into the fringe field, where \(\nu_r\) drops.

Note, that the last requirement calls for a large energy gain, too, since the phase slip in the fringe field must be kept small. Note also that the first item holds only when the maximum energy is not increased with radius. Having the same maximum magnetic field, a bigger cyclotron with the same energy gain per turn has a smaller turn separation because the energy is proportional to the square of the radius, and then \(\frac{dR}{dn}(\text{accel}) \propto \frac{1}{\nu_r^2}\).

The third item is especially important in high energy cyclotrons, such as the PSI 580 MeV cyclotron where in the isochronous extraction region \(\nu_r = 1.6\). If the extraction takes place in the fringe field where \(\nu_r = 1.1\) we get a factor of 2 larger turn separation. For medium energy cyclotrons where precessional extraction is used the radial betatron frequency is usually near 0.8 (instead of, say, 1.1 in the isochronous part of the field).
7.3 Resonant Extraction

In most cyclotrons the radial gain per turn is not enough for a good extraction efficiency, and therefore magnetic perturbations are used to enhance the turn separation in the extraction. Particles (actually, a bunch of particles coherently) are forced to oscillate around their equilibrium orbit with a bump in the magnetic field where there is a resonance. Here we distinguish two different classes: integer and half integer resonances.

7.3.1 The integer resonance \( \nu_r = N \)

There are two ways of using the integer resonance, brute force and precessional extraction.

**BRUTE FORCE** We introduce a magnetic bump in the axial field of the form

\[
\Delta B(r, \theta) = b_N \cos N(\theta - \theta_N).
\]

(14)

If the radial betatron frequency \( \nu_r \) is close to the integer value \( N \), the beam is driven off centre. The maximum additional radial gain per turn is then

\[
\frac{dR}{dn} (\text{brute force}) = \pi \times R \times \frac{b_N}{N \times B_0}.
\]

(15)

\( B_0 \) is the average field at radius \( R \). Since, for a given energy, the magnetic rigidity \( BR \) is constant, the radial gain per turn increases with a factor of \( R^2 \) favouring larger machines. For a typical conventional cyclotron \( (B_0 \approx 1.7 \ T) \) a bump of 0.1 mT \( (1 \ G) \) introduces a radial gain of about 0.2 mm. To get a desired turn separation bigger bumps are needed (brute force). This method has been used for example in the AEG compact cyclotron [16].

**PRECESSIONAL EXTRACTION** Brute force extraction calls for relatively large first harmonic bumps in the magnetic field. However, there is a more elegant way of increasing the turn separation. When the beam goes through \( \nu_r = 1 \) resonance with a first-order perturbation in the field, it starts to oscillate around its equilibrium orbit with a frequency of \( |\nu_r - 1| \) (after the resonance). Since the betatron frequency decreases two consecutive turns oscillate with a slightly different frequency, and the phase difference between the turns increases. This precession introduces a relatively big turn separation with a modest first-order perturbation in the field.

Crossing of \( \nu_r = 1 \) resonance produces a coherent amplitude

\[
x_c = \pi \times R \times \frac{b_1}{B_0} \times n_{eff}, \quad n_{eff} = \frac{1}{\sqrt{|\frac{dn}{dn}|}},
\]

(16)

where \( \frac{dn}{dn} \) is the rate of change of \( \nu_r \) at the crossing point, \( n_{eff} \) is the effective duration of the resonance (typically around ten revolutions). The precession introduces a maximum turn separation

\[
\frac{dR}{dn} (\text{precession}) = 2 \times x_c \times \sin \pi (1 - \nu_r)
\]

(17)
Figure 13: Precessional extraction in the Jyväskylä K130 cyclotron

Remember now that the field at the resonance, and especially after it, is no longer isochronous which means phase slip. In order to be able to accelerate beyond the resonance the energy gain per turn must be large enough. Typically the extraction takes place near $\nu_r = 0.8$.

An example of precessional extraction in radial phase space $(r, p_r)$ is shown in Fig. 13. Note that the radial momentum $p_r$ has been scaled with $qB_0 \ (p_r \rightarrow \frac{1}{qB_0}p_r)$ so that the unit becomes unit of length.

7.3.2 The half-integer resonance $\nu_r = N/2$ (regenerative extraction)

Instead of a dipole bump as in the previous section, consider a gradient bump

$$\Delta B(r, \theta) = g_N(r - r_N) \cos N(\theta - \theta_N)$$

(18)

Since this perturbation has a field gradient, it changes the focusing properties. Clearly, it then must change the radial betatron frequency. Vogt-Nilsen [17] derived a formula for the frequency shift $\nu_r \rightarrow \tilde{\nu}_r$:

$$\cos \tilde{\sigma} = \cos \sigma - \pi R^2 \times g_N^2 \times \frac{2N \times \nu_r \times B_0^2(N^2 - 4\nu_r^2)}{\sin \sigma}$$

(19)

$$\sigma = \frac{2\pi}{N \nu_r}, \quad \tilde{\sigma} = \frac{2\pi}{N \tilde{\nu}_r}$$

The frequency shift is thus biggest for $\nu_r \approx N/2$. From equation (19) one can determine the so called stop band where the new betatron frequency $\tilde{\nu}_r$ is complex. Then the beam comes unstable which property is used in regenerative extraction. The equation (19) can
Figure 14: Stop band $\nu_r = N/2$. Inside the stop band, the betatron frequency is imaginary which is responsible for the exponential growth of the amplitude.

be expanded around $\nu_r = N/2$, and then we get a useful formula:

$$\left( \tilde{\nu}_r - \frac{N}{2} \right)^2 = \left( \nu_r - \frac{N}{2} \right)^2 - \Delta \nu_r^2$$  \hspace{1cm} (20)

$$\Delta \nu_r = \frac{R \times g_N}{2NB_0} = \text{stop-band half width.} \hspace{1cm} (21)$$

Equation (20) is shown graphically in Fig. 14.

For a particle with positive rotation around the cyclotron centre the betatron oscillation is described by:

$$x(\theta) = c_1 \cos Re(\tilde{\nu}_r) \times (\theta - \theta_N + 45^\circ) e^{i \mu \theta}$$
$$+ c_2 \cos Re(\tilde{\nu}_r) \times (\theta - \theta_N - 45^\circ) e^{i \mu \theta}$$

$$\mu = Im(\tilde{\nu}_r), \quad \mu_{\text{max}} = \Delta \nu_r.$$  \hspace{1cm} (22)

The maximum amplitude increase per turn is given by

$$\frac{dx}{dn} = 2\pi \times x \times \mu_{\text{max}} = \pi \times R \times \frac{g_N \times x}{N \times B_0}$$  \hspace{1cm} (23)
Equations (22) and (23) together with figure 14 show three facts:

1. Inside the stop band the amplitude initially grows or decreases exponentially, according to the starting position, but finally ends up growing.

2. Even if \( \nu_r \) is slightly different from \( N/2 \), the resonance locks the phase of the oscillation to a fixed value.

3. Particles near the central particle hardly move.

For medium energy cyclotrons \( \nu_r \) is around 1, and therefore the half integer resonance \( \nu_r = 2/2 \) can be used for extraction. The \( \cos 2\theta \)-dependence of the field perturbation is normally achieved with a negative gradient bump (the peeler) and a positive gradient bump (the regenerator) about 90° displaced. Sometimes the falling fringe field is used as the peeler.

Regenerative extraction is mainly used in synchrocyclotrons since it favours big betatron amplitudes. The radial gain by acceleration in a synchrocyclotron is much lower than that for an isochronous cyclotron. An example of regenerative extraction is the reconstructed Uppsala synchrocyclotron (which can be run also in isochronous mode). The speciality of the Uppsala cyclotron is that it can utilize also precessional extraction [18]. Table 1 shows a comparison between integer and half integer resonance extraction [2].

<table>
<thead>
<tr>
<th>( \nu_r = 1 )</th>
<th>( \nu_r = 2/2 )</th>
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</thead>
<tbody>
<tr>
<td>Flat bump, shape not critical</td>
<td>Gradient bump, shape critical</td>
</tr>
<tr>
<td>No shift of betatron frequency</td>
<td>Shift of betatron frequency towards half-integer value</td>
</tr>
<tr>
<td>Energy selective</td>
<td>Radius selective</td>
</tr>
<tr>
<td>Constant radial gain per turn</td>
<td>Exponential growth of radial gain (asymptotically)</td>
</tr>
<tr>
<td>Post-resonance acceleration to ( \nu_r \approx 0.8 ) gives additional turn separation through precession</td>
<td>Remedy against low energy gain per turn and bad beam quality (synchrocyclotrons)</td>
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Table 1: Characteristics of integer and half-integer resonance

7.3.3 Resonance extraction at high energies

In most cyclotrons the radial betatron frequency \( \nu_r \) is near unity at extraction. For an isochronous field \( \nu_r \approx \gamma \), and for the high proton energies (such as 580 MeV at PSI) \( \nu_r \) is too far from unity to utilize 1/1 or 2/2 resonances. Possibilities then are to use the \( \nu_r = 3/2 \) resonance [19] or the \( \nu_r = 4/2 \) resonance [20].

7.3.4 Non-linear resonances

The first non-linear resonance, the third-order resonance, can also be used for cyclotrons. For a three sector synchrocyclotron the \( \nu_r = 3/3 \) resonance can be succesffully used in a slow extraction (such as from synchrotrons).
8 STABILITY OF VERTICAL MOTION IN EXTRACTION

Especially for precessional extraction where the radial betatron frequency goes through 1, the vertical betatron frequency \( \nu_z \) also varies relatively greatly. This can lead to several resonance crossings. For medium energy cyclotrons the resonances which can cause growth of the vertical amplitude are: \( \nu_z = 0.5 \), \( \nu_r = 2 \nu_z \), and in addition for ring cyclotrons \( \nu_z = 2/2 \) and \( \nu_z = 1 \). The resonances are described in Table 2 [2].

In medium energy cyclotrons with precessional extraction the resonances \( \nu_r = 1 \), \( \nu_r = 2 \nu_z \) and \( \nu_z = 0.5 = 1/2 \) may all be crossed shortly before extraction. In order to preserve vertical stability, one has to limit the radial amplitude, induced from the \( \nu_r = 1 \) resonance, to a few mm.

The coupling resonance \( \nu_r = 2 \nu_z \) depends strongly on the beam quality. In cyclotrons this resonance can be crossed easily, while in synchrocyclotrons with poor beam quality some particles are lost.

9 EXTRACTION ELEMENTS

We have already mentioned the electrostatic deflector and electromagnetic channel as extraction elements. An electrostatic deflector is always the first element since its effective thickness at the entrance can be zero, with a V-slit. It is usually made of copper or tungsten. Graphite has also been used. The septum must be water cooled since some particles always hit it. At high energy, however, \( dE/dx \) gets smaller which reduces the heating.

Activation is a problem. Proton bombardment on Cu introduces \(^{65}\text{Zn}\) which has a half life of 244 days, and the \( \gamma \) energy is 1115.52 keV. The copper dee electrodes get
Figure 15: Cross-section of the EMC windings in the K130 cyclotron. The windings inwards from the channel minimize the EMC field inside the cyclotron, especially near the $\nu_r = 1$ resonance.

Figure 16: A simple radially focusing channel

activated the same way, too. Graphite would be a better septum material as far as the activation is concerned.

The limiting factor in deflectors is sparking which sets the maximum value for the deflecting electric field.

The second element is usually an electromagnetic channel EMC which consists of a set of windings. It reduces the axial magnetic field increasing the bending radius. Quite often the current in the windings is very high. For example, in the Jyväskylä K130 cyclotron the maximum current in the EMC is about 1200 A. The cross-section of the EMC in the K130 cyclotron is seen in Fig. 15.

After the beam has been deflected out from the magnetic field it travels usually almost half a turn in the radially decreasing fringe field which is radially defocusing (this does not happen in stripping extraction). To keep the beam radially confined, focusing channels are needed. Usually the channels are passive, and they consist of iron rods which shape the field. Sometimes there are both radially- and axially-focusing channels. Figure 16 shows a typical set of iron rods that is radially focusing. There can be more rods to
compensate for the first harmonic component [21]. An example of such a passive focusing channel is shown in Fig. 17.

In addition to focusing and deflecting elements, there must be coils to produce the desired first-harmonic bump near the $\nu_r = 1$ resonance. For regenerative extraction, one needs both a peeler and a regenerator. They can consist of iron rods just as the focusing channels (both the peeler and the regenerator introduce a gradient to the field).

10 CONCLUSIONS

During the last few years there has been progress in injection to cyclotrons due to the development of ECR ion sources. This has moved the focus to axial injection, whereas radial injection uses basically the same methods as in the 60's. This applies also for the extraction. Of course, extraction from superconducting cyclotrons has needed development on the hardware side due to high requirements on voltages in electrostatic deflectors and current densities in electromagnetic channels.

REFERENCES

