Abstract

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We discuss the calculation of the production of forward jets at HERA with and without resolved virtual photons.

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Forward jet production at HERA

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Abstract. We discuss forward jet production data recently published by the HERA and ZEUS collaborations at HERA. We review how several Monte-Carlo models compare to the data. QCD calculations based on the BFKL formalism and on fixed NLO perturbation theory with and without resolved virtual photons are described.

1 Introduction

It is by now firmly established that the proton structure function $F_2(x, Q^2)$ shows a steep rise in the small $x$-Bjorken region, i.e., below $x = 10^{-3}$ [1,2]. This rise is compatible with DGLAP evolution [3], where $F_2$ is fitted at a fixed input scale $Q_0^2$ and then evolved up to $Q^2$ by summing up $\ln Q^2$ terms. In the small $x$ region an alternative way to describe the evolution might be to sum up $\ln(1/x)$ terms, as it is done in the BFKL approach [4]. The BFKL equation actually directly predicts the behaviour of $F_2$ as a function of $x$. An attempt to unify these two approaches is the CCFM evolution equation [5], which reproduces both, the DGLAP and the BFKL behaviour, in their respective regimes of validity.

The existing data on $F_2$ do not allow to unambiguously determine whether the BFKL mechanism is needed in the $x$-range covered by HERA. Therefore, alternatively the cross section for forward jet production in deep inelastic scattering (DIS) has been proposed as a particularly sensitive means to investigate the parton dynamics at small $x$ [6]. The proposal is based on the observation that the DGLAP and BFKL equations predict different ordering of the transverse momenta $k_{T,j}$ along the parton cascade developing in DIS jet production, see Fig. 1. While the DGLAP equation predicts a strong $k_{T,j}$ ordering and only a weak ordering in the longitudinal momentum fractions $x_i$, i.e.,

$$Q^2 = k_{T,n}^2 \gg \ldots \gg k_{T,1}^2 \quad \text{and} \quad x = x_n < \ldots < x_1 \quad (1)$$

the BFKL approach predicts a strong ordering in the $x_i$, but no ordering in $k_{T,j}$. The idea is now to observe jets at very small $x$, with $x \ll x_1$ in the forward region with $Q^2 \approx k_{T,1}^2$. In this way the DGLAP evolution [4] is suppressed, whereas the BFKL evolution is left active.

The forward jet cross section has been measured recently at HERA [7,8] and in the following I will review the various attempts that have been made to describe this data.
2 Data and Monte-Carlo models

The H1 and ZEUS collaborations have measured forward jet cross sections at small $\frac{Q^2}{s}$ for rather similar kinematical conditions [7,8]. The jet selection criteria and kinematical cuts are summarized in Tab. 1.

In Fig. 2 and 3 the collected data from ZEUS and H1 in the forward region is shown together with predictions from various Monte-Carlo models. The data from both groups is on the hadron level. For the ZEUS data, the Monte-Carlo models ARIADNE [9], LEPTO [10], HERWIG [11] and LTD [12] have been used for predictions. ARIADNE includes one of the main features present in the BFKL approach, which is the absence of the strong $k_T$ ordering. LTD is based on the CCFM approach and finally LEPTO and HERWIG are

<table>
<thead>
<tr>
<th>H1 cuts</th>
<th>ZEUS cuts</th>
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<tr>
<td>$E_T &gt; 11$ GeV</td>
<td>$E_T &gt; 10$ GeV</td>
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<tr>
<td>$y_e &gt; 0.1$</td>
<td>$y_e &gt; 0.1$</td>
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<tr>
<td>$E_{T,jet} &gt; 3.5$ (5) GeV</td>
<td>$E_{T,jet} &gt; 5$ GeV</td>
</tr>
<tr>
<td>$1.7 &lt; \eta_{jet} &lt; 2.8$</td>
<td>$\eta_{jet} &lt; 2.6$</td>
</tr>
<tr>
<td>$0.5 &lt; E_{T,jet}^2/Q^2 &lt; 2$</td>
<td>$0.5 &lt; E_{T,jet}^2/Q^2 &lt; 2$</td>
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<tr>
<td>$x_{jet} &gt; 0.035$</td>
<td>$x_{jet} &gt; 0.036$</td>
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Fig. 2. Forward jet data of ZEUS [7] as a function of Bjorken-$x$ for $E_{T,jet} > 5$ GeV. (a) linear scale, (b) logarithmic scale. The shaded band gives the error due to the uncertainty of the jet energy scale. The data is compared to various Monte-Carlo models.

Fig. 3. Forward jet data of H1 [8] as a function of Bjorken-$x$ for two $E_{T,jet}$ cuts of 3.5 GeV and 5.0 GeV. (a) and (c) contain Monte-Carlo model predictions, whereas (b) and (d) show the results of a LO BFKL (full) and a fixed NLO (dashed) calculation.
based on conventional leading log DGLAP evolution. Except for HERWIG, the same Monte-Carlo models are shown in the H1 plots. Instead of HERWIG, the RAPGAP model [15] is shown, which is also based on DGLAP evolution but contains an additional resolved virtual photon component.

As is clear from Fig. 2, ARIADNE describes the forward jet cross section reasonably well, apart from the smallest \( x \)-bin where it gives slightly too small cross sections. The other three used models predict cross sections which lie significantly below the data. Similar results can be extracted from Fig. 3 (a) and (c). LDC and LEPTO lie below the data by a factor of 2, whereas ARIADNE gives a reasonable good description. As an interesting result, also RAPGAP gives a good description of the H1 data.

3 BFKL approach

Of course, attempts have been made to calculate the forward jet cross section directly within the BFKL formalism. BFKL calculations in LO by Kwieciński et al., Bartels et al. and Tang [14] based on the BFKL approach overshoot the older forward jet data [13]. This can also be seen in Fig. 3 (b) and (d), where the LO BFKL calculation on the parton level [16] (full line) is compared to the recent H1 data [8]. These older calculations suffer, however, from several deficiencies. They are asymptotic and do not contain the correct kinematic constraints of the produced jets. Furthermore, they do not allow the implementation of a jet algorithm as used in the experimental analysis. Also NLO ln(1/\( x \)) terms in the BFKL kernel [17] predict large negative corrections which are expected to reduce the forward cross section as well.

Recently the BFKL calculations have been improved by taking into account higher order consistency conditions as a way of including sub-leading corrections to the BFKL equation [18]. The consistency constraint (CC) requires that the virtuality of the emitted gluons along the chain should arise predominantly from the transverse components of momentum. By including this CC the authors in [18] claim to recover a dominant part of higher order effects. Furthermore, the CC is said to subsume energy-momentum conservation over a wide range of the allowed phase space, which is another source of sub-leading contributions.

Including the CC conditions, good agreement between the predictions and the forward jet data is found, for both the H1 and the ZEUS data, as shown in Fig. 4. The predictions depend on the choice of scale and on an additional infrared cut-off parameter \( k_0 \). However, the \( k_0 \) dependence is much less than the uncertainty due to the choice of scales. Further details about the BFKL calculations from [18] can be found in these proceedings [19], also describing the calculation for forward \( \pi^0 \) production.
Fig. 4. Forward jet data of H1 [8] and ZEUS [7] compared to prediction based on the BFKL formalism including sub-leading corrections [18]. The three curves correspond to three choices of scales and infrared cut-off.

4 Fixed NLO QCD calculations

Calculations in fixed order perturbation theory have already been performed for the older H1 forward jet measurement [15] by Mirkes and Zeppenfeld [20] using their fixed order program MEPJET [21]. The calculations were done in next-to-leading order (NLO) accuracy, i.e., taking the matrix elements up to $O(\alpha_s^2)$ into account. It was found that the NLO calculations are a factor of 2 to 4 below the data. Similar predictions have been made with help of the
DISENT program [22] for the more recent H1 measurements [8], shown as the dashed lines in Fig. 3 (b) and (d), which confirm the earlier findings. Since the forward jet data can be successfully described by the RAPGAP model which includes resolved virtual photons, it is fair to ask whether also fixed order calculations including a resolved virtual photon component will be able to describe the data.

4.1 Low $Q^2$ jet production in NLO

NLO calculations for jet production with slightly off-shell direct and resolved virtual photons have become available recently [23], extending calculations done in the photoproduction regime [24]. The NLO calculations [23] are performed with the phase space slicing method. As is well known the higher order (in $a_s$) contributions to the direct and resolved cross sections have infrared and collinear singularities. For the real corrections singular and nonsingular regions of phase space are separated by a technical cut-off parameter $y_t$. Both, real and virtual corrections, are regularized by going to $d$ dimensions. The NLO corrections to the direct process become singular in the limit $Q^2 \to 0$ in the initial state on the real photon side. For $Q^2 = 0$ these photon initial state singularities are usually also evaluated with the dimensional regularization method. Then the singular contributions appear as poles in $\epsilon = (4 - d)/2$ multiplied with the splitting function $P_{q\gamma}$ and have the form $-\frac{1}{\epsilon} P_{q\gamma}$ multiplied with the LO matrix elements for quark-parton scattering.

These singular contributions are absorbed into PDF’s $f_{q\gamma}(x)$ of the real photon. For $Q^2 \neq 0$ the corresponding contributions are replaced by

$$-\frac{1}{\epsilon} P_{q\gamma} \to -\ln(s/Q^2) P_{q\gamma}$$

(2)

where $\sqrt{s}$ is the c.m. energy of the photon-parton subprocess. These terms are finite as long as $Q^2 \neq 0$ and can be evaluated with $d = 4$ dimensions, but become large for small $Q^2$, which suggests to absorb them as terms proportional to $\ln(M^2_{\gamma}/Q^2)$ in the PDF of the virtual photon. Parametrizations of the virtual photon have been provided by several groups [25]. By this absorption the PDF of the virtual photon becomes dependent on $M_{\gamma}$, which is the factorization scale of the virtual photon, in analogy to the real photon case. Of course, this absorption of large terms is necessary only for $Q^2 \ll M^2_{\gamma}$. In all other cases the direct cross section can be calculated without the subtraction and the additional resolved contribution. $M^2_{\gamma}$ will be of the order of $E^2_T$. But also when $Q^2 \approx M^2_{\gamma}$, one can perform this subtraction. Then the subtracted term will be added again in the resolved contribution, so that the sum of the two cross sections remains unchanged. In this way also the dependence of the cross section on $M^2_{\gamma}$ must cancel, as long as the resolved contribution is calculated in LO only.

In the general formula for the deep-inelastic scattering cross section, one has two contributions, the transverse ($d\sigma^U_{q\gamma b}$) and the longitudinal part ($d\sigma^L_{q\gamma b}$).
Since only the transverse part has the initial-state collinear singularity the subtraction in [23] has been performed only in the matrix element which contributes to $d\sigma_{\gamma b}^{T}$. Therefore the longitudinal PDF’s $f_{L/\gamma}^{b}$ are not needed. It is also well known that $d\sigma_{\gamma b}^{L}$ vanishes for $Q^2 \rightarrow 0$. The calculation of the resolved cross section including NLO corrections proceeds as for real photoproduction at $Q^2 \rightarrow 0$, except that the cross section is calculated also for final state variables in the virtual photon-proton center-of-mass system.

The NLO calculations in the low $Q^2$ region are implemented in the fixed order program JETVIP [26]. Various measurements at HERA in which the jet analysis has been done with JETVIP point to the presence of a resolved virtual photon component up to moderate virtualities of $Q^2 \simeq 5 \, \text{GeV}^2$ [27].

4.2 Comparison to forward jet data

Recently, we have performed a NLO calculation including the virtual resolved photon for the forward jet region [28] with the help of JETVIP. The results for the ZEUS kinematical conditions are shown in Fig. 5 a,b. In Fig. 5 a we plotted the full $\mathcal{O}(\alpha_s^2)$ inclusive two-jet cross section (DIS) as a function of $x$ for three different scales $\mu^2 = \mu_R^2 = 3 M^2 + Q^2$, $M^2 + Q^2$, and $M^2/3 + Q^2$, with a fixed $M^2 = 50 \, \text{GeV}^2$ related to the mean $E_T^2$ of the forward jet and compared them with the measured points from ZEUS [7]. The choice $\mu^2 > Q^2$ is mandatory if we want to include a resolved contribution. Similar to the results obtained with MEPJET and DISENT, the NLO direct cross section is by a factor 2 to 4 too small compared to the data. The variation inside the assumed range of scales is small, so that also with a reasonable change of scales we can not get agreement with the data. In Fig. 5 b we show the corresponding forward jet cross sections with the NLO resolved contribution included, labeled DIR$_S + \text{RES}$, again for the three different scales $\mu$ as in Fig. 5 a. Now we find good agreement with the ZEUS data. The scale dependence is not so large that we must fear our results not to be trustworthy.

In Fig. 5 c,d we show the results compared to the H1 data [8] obtained with $E_T > 3.5 \, \text{GeV}$ in the HERA system. In the plot on the left the data are compared with the pure NLO direct prediction, which turns out to be too small by a similar factor as observed in the comparison with the ZEUS data. In Fig. 5 d the forward jet cross section is plotted with the NLO resolved contribution included in the way described above. We find good agreement with the H1 data inside the scale variation window $M^2/3 + Q^2 < \mu_R^2 < 3 M^2 + Q^2$. We have also compared the predictions with the data from the larger $E_T$ cut, namely $E_T > 5.0 \, \text{GeV}$, and found similar good agreement [28].

As described in [28] the NLO resolved contribution supplies higher order terms in two ways, first through the NLO corrections in the hard scattering cross section and second in the leading logarithmic approximation by evolving the PDF’s of the virtual photon to the chosen factorization scale. This way the logarithms in $E_T^2/Q^2$ are summed, which, however, in the considered kinematical region is not an important effect numerically. Therefore, the
enhancement of the NLO direct cross section through inclusion of resolved processes in NLO is mainly due to the convolution of the point-like term in the photon PDF with the NLO resolved matrix elements, which gives an approximation to the NNLO direct cross section without resolved contributions. One can therefore speculate that the forward jet cross section could be described within a fixed NNLO calculation, using only direct photons.

5 Conclusions

We conclude that two alternative models exist for describing the forward jet data. The BFKL calculation including CC describes the data well. This is supported by the ARIADNE model, where strong $k_T$ ordering is absent and which also describes the data. It is however not clear, why the LDC model does not describe the data, although BFKL dynamics should be included in the respective regime of validity.
Similarly, the NLO theory with a resolved virtual photon contribution as an approximation of the NNLO DIS cross section, which is presently not available, gives a good description of the forward jet data. Assumeably, the RAPGAP model produces these higher order effects through parton shower contributions in the resolved cross section.

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