QCD hadron spectrum with domain wall fermions

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We present the QCD hadron spectrum for the cases of both quenched and two-flavor dynamical domain wall fermions. We compare the results obtained using the Wilson gauge action and a renormalization group improved gauge action. Finite volume effects and the dependence on the finite extent of the fifth dimension are discussed.

1. INTRODUCTION

The domain wall fermions (DWF) formalism[1][2] uses an extra space-time dimension to separate the chiral limit from the continuum limit. In this paper, we report a study of the hadron spectrum obtained from both quenched and dynamical DWF.

Our conventions are: \( m_f \) is the 4-d bare quark mass that explicitly mixes the two chiralities on the domain walls, \( L_s \) is the extent of lattice in the fifth dimension, and \( m_0 \) is the 5-d bare quark mass, which is often called domain wall height. The masses in physical units in this paper are obtained by using the rho mass to set the scale.

2. QUENCHED QCD SPECTRUM

Last year, we reported that with domain wall fermions at \( \beta = 5.7 \) on an \( 8^3 \times 32 \) lattice with \( m_0 = 1.65 \) and \( L_s = 48 \), \( m_N/m_\rho = 1.42(10) \) as \( m_f \to 0 \)[3]. Considering the moderate size of the lattice, this value is favorable compared with Wilson and staggered results. However, an \( L_s \) study for \( 8^3 \times 32 \) lattices at this \( \beta \) shows that \( m^2_\pi(m_f \to 0) \) fits well to the form of \( \chi \exp(-\alpha L_s) + B \) with a non-zero value of \( B \approx 0.048 \), which gives \( m_\pi/a(m_f \to 0) \approx 213 \text{MeV} \) at infinite \( L_s \)[3]. In Figure 1 we show the pion mass data on which the conclusion was based.

In order to investigate how much of this 213MeV pion mass comes from the effects of finite volume, we have studied a \( 16^3 \times 32 \) lattice at \( \beta = 5.7 \), \( L_s = 24 \). We find \( m^2_\pi(m_f \to 0) = 0.077(2) \), only 0.021 smaller than the 0.098(7) value we obtained for \( 8^3 \times 32 \) at \( L_s = 24 \). This finite volume shift is less than half of the 0.048 infinite \( L_s \) limit for \( 8^3 \times 32 \) suggesting this non-zero limit of \( m^2_\pi \) is not a finite volume effect.
To examine this question more carefully, we assume the effects of $L_s$ can be represented by a residual quark mass, $m_{\text{res}}(L_s)$, and express $m_{\pi}^2$ to first order in chiral symmetry breaking as:

$$m_{\pi}^2(V, m_f, L_s) = c_0(V) + c_1(V) m_f + m_{\text{res}}(L_s).$$

We find $c_1(8^3) = 4.54(9)$, $c_1(16^3) = 4.75(3)$. If we require that $c_0$ vanishes as $V \to \infty$ and assume that $16^3$ is sufficiently large that $c_0(16^3) \approx 0$ then either $m_{\text{res}}$ does not vanish as $L_s \to \infty$ or $m_{\text{res}}(L_s)$ has a strong, unphysical dependence on the spatial volume with $m_{\text{res}}(24)$ increasing from 0.011(2) to 0.0162(5) as the volume is increased from $8^3$ to $16^3$. Thus, our results appear to require one of three unexpected possibilities: i) quenched $m_{\pi}$ does not vanish in the chiral limit; ii) $m_{\text{res}}$ does not vanish as $L_s \to \infty$ or iii) $m_{\text{res}}$ has a strong volume dependence for $L_s = 24$.

To support our weak matrix element study[4], we have also measured hadron masses using DWF and the Wilson gauge action at a weaker coupling, $\beta = 6.0$. On a $16^3 \times 32$ lattice with $m_0 = 1.80$, $L_s = 16$ and $m_f$ ranging from 0.01 to 0.04, we have (Table 1) $m_{\pi}/m_{\rho} = 1.37(5)$, $m_{\pi}^2 = 0.014(2)$ as $m_f \to 0$, which gives $m_{\pi}/m_{\rho} \to 0$ as a $230(15)$MeV.

From the above discussion, we can see that to decrease the pion mass while using DWF and the Wilson gauge action, large $L_s$ is needed. However, the simulation difficulty is proportional to $L_s$. It might be helpful to use the renormalization group improved gauge action of Iwasaki which smooths out the gauge fields on the lattice[5].

We have simulated using the Iwasaki gauge at $\beta = 2.2827$ and $c_1 = -0.331$ which is equivalent to quenched $\beta = 5.7$ with the Wilson action. The resulting pion masses are promising. Compared with the large $m_{\pi}^2(m_f \to 0)$ values using the Wilson action, much smaller values are obtained from lattice size $8^3 \times 32$ as shown in Figure 1. Studies with $L_s$ ranging from 16 to 32 and $m_0$ ranging from 1.40 to 1.90 show that $m_{\pi}$ has little dependence on $L_s$ and $m_0$. A simulation at larger volume ($16^3 \times 32$) also shows that the finite volume effect is small(Figure 1). These suggest that using the Iwasaki gauge action may enable us to study chenqued DWF at smaller $L_s$, but we may still have a non-zero pion mass at infinite $L_s$.

### Table 1

Valence extrapolations ($a + b m_f^{(\text{val})}$) for $m_{\pi}^2$, $m_{\rho}$ and $m_{\pi}$. Quenched fits are obtained on a $16^3 \times 32$ lattice with $m_0 = 1.80$. Dynamical fits are obtained on an $8^3 \times 32$ lattice with $m_0 = 1.90$.

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<th>mass</th>
<th>$\beta$</th>
<th>$L_s$</th>
<th>$a$</th>
<th>$b$</th>
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<td>6.55(8)</td>
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</table>

### 3. DYNAMICAL QCD SPECTRUM

To support our thermodynamics studies[6], we have measured the dynamical QCD hadron masses at zero temperature to set the scale. Unless indicated otherwise, all the simulations discussed in this section are done with lattice size $8^3 \times 32$, $m_0 = 1.90$, $L_s = 24$, $m_f^{(\text{dyn})} = 0.02$, and valence quark mass ranging from 0.02 to 0.22 with an increment of 0.04. Valence extrapolations for some of the simulations discussed below are shown in Table 1.

Using DWF and the Wilson action, at $N_f = 4$ the transition occurs at about $\beta = 5.325$. At this $\beta$, we find $m_{\pi} = 1.18(3)$, $m_{\rho} = 0.654(3)$ at $m_f^{(\text{dyn})} = 0.02$, which gives $T_c = 163(4)$MeV and $m_{\pi}/a = 427(11)$MeV. Using a larger $16^3 \times 16$ volume only reduces $m_{\pi}$ to 0.652(3), which suggests that the finite volume effect is very small. A Ward identity evaluation[7] shows that the residual mass caused by the mixing between the two
walls plays an important role in this heavy pion mass. However, that study shows these residual mass effects do vanish in the limit of large $L_s$ for these full QCD simulations.

We have also measured the masses for $m_f^{(dyn)} = 0.06$ at $\beta = 5.325$. Performing real dynamical extrapolations using the two dynamical points obtained from $m_f = 0.02, 0.06$, we get $m_\pi^2 = 0.320(6) + 5.38(11)m_f^{(dyn)}$. Compared with valence extrapolation, the dynamical extrapolation slightly decreases the pion mass as $m_f \to 0$.

As in the quenched studies, we have also investigated the renormalization group improved gauge action. We have simulated at $\beta = 1.90$, $c_1 = -0.331$ which is about the transition point for $N_t = 4$ with DWF and the Iwasaki gauge action\cite{6}. We obtain $m_\rho = 1.16(2)$, $m_\pi = 0.604(3)$ at $m_f^{(dyn)} = 0.02$, which gives $T_c = 166(3)$MeV and $m_\pi/a = 400(7)$MeV. Surprisingly this is about the same value as that obtained at $\beta_c$ using the Wilson action. Therefore, although the Iwasaki action helps to reduce the pion mass in our quenched study, this is not true for the dynamical case.

We have also measured the hadron masses at $\beta = 2.0$ for both $m_f^{(dyn)} = 0.02, 0.06$. As with the Wilson gauge action, we can draw the same conclusion that the dynamical extrapolation $m_\pi^2 = 0.088(10) + 6.9(2)m_f^{(dyn)}$ gives a slightly smaller pion mass as $m_f \to 0$.

To study the finite $L_s$ effect, we are currently doing a simulation at $\beta = 2.0$, $c_1 = -0.331$, and $L_s = 48$. Figure 2 shows the valence extrapolations of the pion, rho, nucleon mass values for $L_s = 24$. Figure 2 shows the valence extrapolations of the pion, rho, nucleon mass values for $L_s = 24$. For $L_s = 24$, we have $m_\pi(m_f^{(val)}) = 0.02$ vs $0.475(7)$. For $L_s = 48$, we have obtained $m_\pi(m_f^{(val)}) = 0.02$ vs $0.420(10)$. This confirms that the mixing between the two walls is at least a major cause of the heavy pion mass at $\beta_c$, and this effect can be reduced by increasing $L_s$.

4. CONCLUSIONS

Although for the quenched QCD spectrum the Iwasaki action lowers the pion mass compared with that obtained using the Wilson action, the non-zero value of $m_\pi(m_f \to 0)$ is still problematic. From our dynamical QCD spectrum study, we have obtained a heavy pion mass at $\beta_c$ for $N_t = 4$ for both the Wilson and Iwasaki gauge actions at $L_s = 24$. This can be improved by increasing the size of $L_s$. These calculations were performed on the QCDSF machines at Columbia and RIKEN/BNL.

REFERENCES
4. T. Blum and A. Soni, these proceedings.
6. P. M. Vranas, these proceedings.
7. G. Fleming, these proceedings.