SELF-CONSISTENT EFFECTS OF SPACE CHARGE COMPENSATION ON INTENSE ION BEAMS

J.L. Lemaire¹, X. Fleury², A. Piquemal¹
¹CEA/DRIF/DPTA, BP12, 91680 Bruyères-le-Châtel, France
²CMAP/Ecole Polytechnique, 91128 Palaiseau, France

Abstract

It is usually assumed that beams are partially space charge compensated for the design of high intensity, low energy beam transport. Such continuous beams are confined in space by means of magnetostatic lenses, the transverse matching into the RFQ accelerator being achieved with solenoids. Along this low energy transport, beam neutralization is kept almost constant, but severe problems can appear at the entrance of the RFQ where longitudinal bunching takes place. The electric field pulls out the neutralizing electrons, leading to a redistribution of the charged particles. We analyze theoretical solutions of this phenomenon in a self-consistent approach in view of minimizing emittance growth and halo development that could result.

1. Introduction

High permeance proton sources are needed to produce intense beams for industrial projects like TRISPAL.

In the low energy part of the accelerator machine the transport of such beams is critical up to kinetic energies of a few MeV, because the beams are space charge dominated. It was proposed for a long time to transport such proton beams in a charge compensated regime, where the protons are neutralized by trapped electrons.

This well known effect occurs naturally when the residual gas pressure is relatively high as it is the case after the ion source, even if the gas flowing from the source is pumped out efficiently.

The protons ionize the molecules of the residual gas and produce electrons which are trapped in the collective potential well of the beam.

As this was observed in many experiments [1–3], the beam tends to be partially neutralized, depending on characteristic parameters and vacuum pressure.

This is often a favorable situation since the transported beam current can be enhanced correlatively, and this saves power for the external restoring forces which insure the confinement of the beam; the companion electrons screen the primary beam, diminishing the net defocusing force due to coulombian repulsion and participate to the confinement of the whole beam.

But these time dependent mechanisms of neutralization are not necessarily homogeneous in space: they can produce axi or non-axisymmetric instabilities which contribute by non-linear effects to energy redistribution into the beam. This drives the density to a more or less steady profile [4].

Emittance degradation and particle losses in the low energy part of the machine are a real concern for machine designers, it is thus important to be able to predict the optical qualities of the beam and emittance growth.

This is why transport must be simulated using a refined and a self-consistent description.

In this paper, we first describe the system from relevant parameters and time scales of the model that depend on physics: we then derive a set of self-consistent equations for a 1D1/2 model. After analyzing theoretical solutions, we draw conclusions for future studies.

2. Model and hypotheses

We consider a cylindrical DC beam with parameters:

\[ T_0 = 100 \text{ keV}, \quad I_0 = 100 \text{ mA}, \quad S_0 = 1 \text{ cm}^2 (R_0 \approx 5 \times 10^{-3} \text{ m}). \]

The study is restricted to a region surrounding a waist where external confinement can be absent. Magnetic focusing is assumed ahead and behind this region; mechanical walls are absorbent and grounded.

- The primary beam (p) is assumed cold, hence its phase space distribution function has the following expression:

\[ f_p(r,v,t) = n_p \delta(v - v_p) \]

where \( v \) is reduced to the axial velocity \( v_p \); with our parameters \( n_p = 1.410^{12} \text{ m}^{-3} \).

- The residual gas (g) mainly consists of hydrogen molecules (\( \text{H}_2 \) at about \( 10^{-3} \text{ hPa} \)) which are considered at rest compared to the other moving species. With these parameters \( n_g = 3.510^{19} \text{ m}^{-3} \).

Physical processes. We assume that the only source of secondary charges is the gas ionization:

\[ p + \text{H}_2 \rightarrow p + \text{H}_2^+ + \epsilon \]

and the generated plasma is composed of four species where 1, 2 are the primary species and 3, 4 are the secondary ones.

From the processus (2), we can estimate the electron density variation versus time:

\[ \frac{dn_e}{dt} = \sigma \cdot n_p \cdot n_g \cdot v_p \]

and deduce an approximate neutralization time scale.

\[ \tau_n = (\sigma \cdot n_p \cdot n_g)^{-1} \]

assuming that the density \( n_p \) and \( n_g \) are near constants [5]. With our parameters, \( \tau_n \approx 330 \text{ ns} \).

We consider that elastic scattering cross section at large energy transfer is small compared to the ionization one, it is thus assumed that residual gas depletion is inexistent ahead of the beam at any time scale of our study provided:

\[ n_i \leq n_p \ll n_g \]

The radial potential \( \phi \) of a primary beam with a parabolic density profile \( \rho \) can be deduced from the Poisson equation:
It comes out that:

- the primary protons have negligible deviation from incident trajectory and their velocity is almost unaffected,
- the secondary ions have a recoil energy less than 10 eV,
- the electrons created by ionization have energies picked at 0 eV but 50% of them have energies higher than 18 eV.

Usually, it is admitted that both ions and electrons are created at rest, this corresponds to the double differential cross section:

\[
\frac{d\sigma_i}{dE_e d\Omega_0} = \sigma_i \delta(p_e)
\]

where \( p_e \) represents the momentum of electrons created at energy \( E_e \) and with no energy transfer taken into account.

In our model, we take some more realistic initial condition: ions are still created at rest but electrons have a mean energy of about 10 eV. As mentioned before, the differential cross section is then:

\[
\frac{d\sigma_i}{dE_e} = \frac{\sigma_i \exp(-E_e/T_e)}{T_e}
\]

and their velocities are distributed as a maxwellian distribution function with a temperature \( T_e \). This temperature will be an adjustable physical parameter which can be checked by experiment.

**Relaxation time:** the velocity distribution function of the secondary particles is driven to thermodynamic equilibrium by the binary collisions: e-e collisions drive the distribution to a maxwellian, while e-i and e-g participate essentially to the isotropisation of the velocities. The relaxation time is then expressed by

\[
\tau_{i,e} = \frac{3.510^{11}}{\Lambda n_e T_e^{3/2}}
\]

where \( \Lambda \) is the Coulomb logarithm.

With \( \tau_i' = 3m_e >> \tau_e \) one can conclude that neutralization equilibrium is reached well before electrons are thermalized.

### 3. System of equations

The system of equations for the different species can be resumed as follows:

- for the electrons

\[
\frac{df_e}{dt} = C^{ce}_e (f_e)
\]

where \( C^{ce}_e \) is the collision operator and can be calculated from the continuity equation by:

\[
C^{ce}_e (f_e(r,v_r,v_\theta)) = n_e n_p (r) v_r \frac{d\sigma_i}{dv_r dv_\theta}
\]

This gives the number of electrons of velocity \( (v_r,v_\theta) \) created per unit volume in the phase space and per second.

From the relation (11) it is easy to derive the final form:

\[
\frac{d\sigma_i}{dv_r dv_\theta} = \frac{\sigma_i m_e}{2\pi T_e} \exp\left(-\frac{m_e (v_r^2 + v_\theta^2)}{2T_e}\right)
\]
- for the ions
  \[ \frac{df_i}{dt} = C_i^{\text{c}}(f_i) \]

where \( C_i^{\text{c}}(f_i) \) is the collision operator and can be calculated by the same expression as (9) to give:

\[ \frac{d\sigma_i}{dv_r} = \sigma_i \delta(v_r) \]  

(13)

We obtain finally at stationarity [12]:

\[
\begin{align*}
\nu_r \frac{d\phi}{dr} + \frac{e}{m_e} \frac{\partial \phi}{\partial r} \frac{d\phi}{dr} &= \\
&= n_g \sum_n p_n \sigma_i \left[ \frac{m_e}{2\pi T_e^{\frac{3}{2}}} \exp\left(- \frac{m_e(v_r^2 + v_0^2)}{2T_e^{\frac{3}{2}}} \right) \right] \\
\nu_r \frac{d\sigma_i}{dr} - \frac{e}{2m_p} \frac{\partial \sigma_i}{\partial v_r} &= n_g \sum_n p_n \sigma_i \delta(v_r) \\
\Delta \phi(r) &= - \frac{e}{\varepsilon_0} (n_p(r) + n_i(r) - n_e(r)) \\
\phi(r_c) &= 0
\end{align*}
\]

For the closure of the system, we suppose that \( f_i \) and \( f_r \) are not correlated, but the two kinetic equations are coupled by the Poisson equation. For the boundary conditions, we take absorbent walls. This complete set of equations is to be solved by numerical techniques.

**Conclusion**

A space charge neutralization may be needed to keep small the emittance degradation in a transport system made of magnetic lenses.

But this can be done only in a some dynamical equilibrium between the present charge species, where the degree of neutralization is kept near a constant value.

If this equilibrium cannot be maintained, a proton density redistribution will happen when the beam enters into the RFQ: the electrons participating to the self-confinement will be rapidly released by the electrostatic field.

In this case, the adiabatic matching and bunching into the RFQ might fail.

It is too early, at this stage of the study to draw definitive conclusions related to our concern.

But we saw that the study of the dynamics of the companion electrons is essential to understand the mechanisms of the equilibrium during the transport, and the rapid decompensation at the entrance of the RFQ.

We derived a 1D1/2 model close to reality since it represents a cylindrical beam which is transported in an axisymmetric magnetic system.

The assumed parameters like \( T_e \) and \( \phi(R_0) \) and the density profile will be measured experimentally to refine the initial conditions and hypothesis.

The numerical simulations that we are presently carrying out, will provide the density profile of the protons and electrons at equilibrium.

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**References**


