Asymmetric Supernovae, Pulsars, Magnetars, and Gamma-Ray Bursts

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ABSTRACT

We outline the possible physical processes, associated timescales, and energetics that could lead to the production of pulsars, jets, asymmetric supernovae, and weak γ-ray bursts in routine circumstances and to a $10^{16}$ G magnetar and perhaps stronger γ-ray burst in more extreme circumstances in the collapse of the bare core of a massive star. The production of a LeBlanc-Wilson MHD jet could provide an asymmetric supernova and result in a weak γ-ray burst when the jet accelerates down the stellar density gradient of a hydrogen-poor photosphere. The matter-dominated jet would be formed promptly, but requires 5 to 10 s to reach the surface of the progenitor of a Type Ib/c supernova. During this time, the newly-born neutron star could contract, spin up, and wind up field lines or turn on an $\alpha - \Omega$ dynamo. In addition, the light cylinder will contract from a radius large compared to the Alfvén radius to a size comparable to that of the neutron star. This will disrupt the structure of any organized dipole field and promote the generation of Large Amplitude Electromagnetic Waves (LAEW). The generation of the LAEW would be delayed by the cooling time of the neutron star $\sim$ 5 to 10 seconds, but the propagation time is short so the LAEW could arrive at the surface at about the same time as the matter jet. In the density gradient of the star and the matter jet, the intense flux of LAEW could drive shocks, generate pions by proton-proton collision, or create electron/positron pairs depending on the circumstances. The LAEW could influence the dynamics of the explosion and might also tend to flow out the rotation axis to produce a collimated γ-ray burst.

Subject headings: supernovae: general – gamma rays: bursts – pulsars: general – ISM: jets and outflows
1. Introduction

Recent evidence has given support for the idea that the core-collapse process is intrinsically strongly asymmetric. The spectra of Type II and Type Ib/c supernovae are significantly polarized indicating asymmetric envelopes (Méndez et al. 1988; Jeffrey 1991; Höflich 1991; Trammel et al. 1993; Wang et al. 1996; Tran et al. 1997; Leonard et al. 1999). The degree of polarization tends to vary inversely with the mass of the hydrogen envelope (Wang, Wheeler & Höflich 1997; Wang & Wheeler 1998; Höflich, Wheeler & Wang 1999; Wheeler, Höflich & Wang 1999; Wheeler 1999). Pulsars are observed with high velocities, up to 1000 km s$^{-1}$ (Strom et al. 1995). Observations of SN 1987A showed that radioactive material rapidly mixed out to the hydrogen-rich layers (Lucy 1988; Sunyaev et al. 1987; Tueller et al. 1991). Cas A shows rapidly moving oxygen-rich matter outside the nominal boundary of the remnant (Fesen & Gunderson, 1996) and evidence for two oppositely directed jets of high-velocity material (Reed, Hester, & Winkler 1999). High velocity “bullets” of matter have been observed in the Vela supernova remnant (Taylor et al. 1993). Other evidence shows that soft gamma-ray repeaters arise in very strongly magnetized neutron stars, “magnetars,” with dipole fields in the range $10^{15}$ G (Kouveliotou et al. 1998; Thompson et al. 1999). Theoretical models have shown that matter-dominated jets can cause supernova explosions (Khokhlov et al. 1999) and that a strongly asymmetric explosion can account for the outward mixing of $^{56}$Ni in SN 1987A (Nagataki 1999). In addition, much attention has recently been paid to the issue of collimation of $\gamma$-ray bursts and the associated affect on energetics and observable properties in the context of high-redshift events (Kulkarni et al. 1999; Rhode, 1999; Sari, Piran & Halpern 1999; Lamb 1999; Chevalier & Li 1999a,b) and the proposed correspondence of SN 1998bw with a much lower energy event (Galama et al. 1998). In this paper, we explore the physics that might unite all these areas.

The discovery of optical afterglows of $\gamma$-ray bursts has raised the estimates of the maximum isotropic $\gamma$-ray emission to unprecedented values, $\sim 3 \times 10^{54}$ ergs for GRB 990123 (Kulkarni, et al. 1999). This has brought a new focus on the likelihood that the prompt $\gamma$-ray bursts and the afterglows are collimated to various extents as well as Doppler boosted and “beamed” (Rhoads 1997, 1999; Sari, Piran & Halpern 1999; Kulkarni et al. 1999; Harrison et al. 1999; Dar 1999). Although the evidence is still preliminary, collimation factors of $\Delta \Omega/4\pi \lesssim 0.01$ have been derived from the decline in some afterglow light curves. Wang & Wheeler (1998), Nakamura (1998) and Cen (1998) have pointed out that if the collimation were strong enough, the energetics might be provided by supernova-like energies. SN 1998bw and GRB 980425 provided a different perspective by suggesting that some $\gamma$-ray bursts are directly associated with some supernovae (Galama et al. 1998; see also Bloom et al. 1999; Reichart 1999; Germany et al. 1999; Galama et al. 1999). If this association is correct, the “isotropic” $\gamma$-ray energy of GRB 980425 is only $\approx 10^{48}$ ergs. Some models of SN 1998bw invoked especially large kinetic energies, in excess of $10^{52}$ ergs in spherically-symmetric models, to account for the bright light curve and high velocities (Iwamoto et al. 1998; Woosely, Eastman & Schmidt 1998), and others took note of the measured polarization and chemical structure to suggest that strongly asymmetric models could account for
the observations with more "normal" energies (Höflich, Wheeler & Wang 1999; Danziger et al. 1999). The former models have been referred to as "hypernova" models, although that term was originally introduced (Paczynski 1998) to mean the generic high energy events associated with the afterglows of the classical \( \gamma \)-ray bursts at large distance.

Many of the popular models for creating \( \gamma \)-ray bursts are based on binary neutron stars (Paczynski 1986) or accretion onto black holes (Paczynski 1991; Woosley 1993). The latter, in particular, are popular because there is no limit, in principle, to the mass of the black hole and hence, again in principle, the energy that can be extracted. All the models struggle with the mechanism for turning the large energies into \( \gamma \)-rays. There is no reason why the energy flux from a black hole source cannot be collimated, as witness the jets from active galactic nuclei and some binary black hole sources. This could reduce the energy requirements even in black hole models. Some models explicitly include this collimation (MacFadyen & Woosely 1999). We note that even the superluminal jets from AGN and blazars have a maximum bulk Lorentz factor of about 10, while the \( \gamma \)-ray bursts require \( \Gamma \gtrsim 100 \) (Baring & Harding 1997, Piran 1999, and references therein). This may suggest that a qualitatively different mechanism is needed to generate the cosmic \( \gamma \)-ray bursts despite the attractive possibilities for black hole models.

Here we will attempt to see how far a more conservative model can go both to produce asymmetric supernovae and perhaps to generate \( \gamma \)-ray bursts, by considering the effects of a newly-born pulsar. This is not a new idea (Ostriker & Gunn 1971; Bisnovatyi-Kogan 1971), but it is worth reconsidering in the context of the polarization of core-collapse supernovae, the growing evidence for "magnetars," the growing understanding of \( \gamma \)-ray bursts and their afterglows, the likely association of SN 1998bw with GRB 980425, and the interesting possibility that supernovae can be induced by energetic jets arising from their cores. All these processes demand or suggest strong asymmetries. Questions arise as to whether these issues are related, whether \( \gamma \)-ray bursts of observed properties could be produced in this context, whether there are more than one type of \( \gamma \)-ray burst mechanism, and whether supernovae may, in some circumstances, produce the \( \gamma \)-ray bursts seen at high redshift. It is very plausible that the formation of a neutron star will engender collimated flow, including Large Amplitude Electromagnetic Waves (LAEW: Usov, 1992, 1994; Blackman & Yi 1998). Here we will show that it is plausible that the properties of SN 1998bw, including a weak \( \gamma \)-ray burst, may be generated by the acceleration of a collimated shock down a density gradient and possible that a \( \gamma \)-ray burst visible at cosmic distances could be produced under some circumstances. Certain aspects of our model are similar to those of Usov (1992, 1994), but he considered accretion-induced collapse and we explicitly investigate the context of a core collapse in the core of a massive star. Some of the ideas we explore here are also presented by Nakamura (1998).

The purpose of this paper is to outline the basic time scales, energetics, and relevant physical processes in order to define areas that need more quantitative work. Key length and time scales and a summary of the core collapse ambiance are given in §2 and §3, respectively. The physics of the proto-neutron star phase including the generation of the magnetic field and associated
torquing of the surrounding plasma is presented in §4. The effect of an axial jet associated with the collapse process is outlined in §5. The important phase when the neutron star cools, contracts, and spins more rapidly is discussed in §6, and the manner in which the energy associated with matter and radiation jets could be propagated outward is given in §7. Discussion and conclusions are given in §8.

2. Basic Length and Time Scales

In the discussion to follow there are a number of key length and time scales. Among these are the radius of the star, $R_{\text{star}} \simeq 10^8$ km for a red supergiant and $R_{\text{He}} \simeq 2 \times 10^5$ km for a helium core. The inner iron core that collapses to form a neutron star has a typical radius, $R_{\text{Fe}} \simeq 4 \times 10^3$ km. The dynamical or sound crossing time is

$$\tau_{\text{dyn}} \simeq 20 s \frac{R_5^{3/2}}{(M/M_\odot)^{1/2}},$$

where $R_5$ is the radius in units of $10^5$ km, and the light travel time is

$$\tau_{\text{light}} = \frac{R}{c} \simeq 0.3 s \, R_5.$$

The dynamical time to emerge from a red giant or even a light crossing time, from $10^3$ to $10^5$ s, is not likely to be associated with observed $\gamma$-ray bursts. For a stripped helium core, the progenitor of a Type Ib or Type Ic supernova, a range of timescales from 1 to 10 s is possible, in the absence of Doppler boosting. Note that the breakout of a collimated jet might involve an area considerably less than the surface area of the helium core and an appropriately smaller time scale. Other relevant length scales are the radius of the light cylinder,

$$R_{\text{LC}} = \frac{c}{\Omega} = \frac{cP}{2\pi} \simeq 50 \text{ km} \left( \frac{P_{\text{ms}}}{P} \right),$$

where $\Omega$ is the rotational frequency and $P$ the rotational period of the neutron star, and the Alfvén radius at which magnetic pressure is balanced by the ram pressure, e.g.,

$$\frac{1}{2} \rho v^2 \simeq \frac{1}{8\pi} B^2,$$

which, with $B \simeq B_{NS} \left( \frac{R}{R_{NS}} \right)^{-3}$, is

$$R_A \simeq 3.0 R_{NS} \left( \frac{B}{10^{11} \text{ G}} \right)^{1/3} \left( \frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{-1/6} \left( \frac{v}{10^8 \text{ cm s}^{-1}} \right)^{-1/3}.$$

During the thermal contraction of the proto-neutron star, $P$, $R_{\text{LC}}$, and $R_A$ will all change significantly. For illustration, we assume that the rotation period decreases from that of the
proto-neutron star, $P_{PNS} \simeq 25$ ms to that of the neutron star, $P_{NS} \simeq 1$ ms. As a result, the light cylinder will contract from $R_{LC} \simeq 10^3$ km, beyond the radius of the stalled shock and comparable in size to the original iron core, to $R_{LC} \simeq 50$ km, comparable to the radius of the neutron star and well within the stalled shock. The light cylinder will contract from beyond $R_A$ to significantly less than $R_A$. This has critical implications for angular momentum transport and the generation of radiation.

3. Context of Core Collapse

We know that pulsars arise from core-collapse. We know that some core collapse events are directly associated with supernovae, e.g., the Crab Nebula, SN 1987A. Circumstantial evidence suggests that supernovae of Type II and Ib/c result from core collapse based on correlations with spiral arms and H II regions in spiral galaxies and on the nebular spectra that are consistent with the cores of massive stars. Polarization data suggests that all these events are significantly asymmetric and that the asymmetry is stronger for explosions with smaller hydrogen envelopes (Wang, Wheeler & Höflich 1997; Wang & Wheeler 1998; Wheeler, Höflich & Wang 1999; Höflich, Wheeler & Wang 1999; Wheeler 1999; Leonard et al. 1999). Explosions induced by jets from within the inner core can plausibly account for this asymmetry (Khokhlov et al. 1999). The imprint of such jets and the possibility to make $\gamma$-ray bursts is largest in massive stars from which the hydrogen envelope has been stripped by winds or binary mass transfer. In the following we will thus concentrate on core collapse explosions and neutron star formation in hydrogen and helium-deficient Type Ib and Type Ic supernovae. Such supernova progenitors must be surrounded by substantial mass from the previous mass loss stages, and this will affect the production and evolution of jets and $\gamma$-ray bursts.

Here we will adopt a generic picture of core collapse in an intermediate mass star of main sequence mass $\simeq 15 - 25 \, M_\odot$ with length scales as given in §2. The iron core undergoes instability and collapses in a time of $\simeq 1$ s. The collapse involves a homologous collapse of the iron core in which the density increases monotonically inward. The outer parts of the stellar core composed of silicon, oxygen, carbon, etc, have lower densities and longer free-fall times. Over the times of interest here, $\simeq 10$ s, these outer layers will “hover” as the collapsing iron core succeeds or fails to generate an explosion.

After core bounce, a proto-neutron star (PNS) forms with a radius of $R_{PNS} \simeq 50$ km and a shock is formed that stalls at a radius of $R_{sh} \simeq 200$ km. Over a cooling time $t_{cool} \simeq 5$ to 10 s, the proto-neutron star de-leptonizes by neutrino emission, cools, and contracts to form the final neutron star structure (Burrows & Lattimer 1986). If the neutron star is rotating and magnetized, this cooling phase will be associated with contraction, spin-up, and amplification of the magnetic field. These changes can significantly alter the physics associated with the neutron star and its interaction with its surroundings and hence the explosion process itself.
4. The Proto-Neutron Star Phase

Right after collapse, the hot proto-neutron star undergoes neutrino-driven convection with the characteristic convective overturn time scale $\tau_{\text{conv}} \simeq 1 \text{ ms} \ F_{39}^{-1/3}$ where $F_{39} \equiv F/10^{39} \text{ erg s}^{-1}$ is the neutrino flux at the base of the convection (Duncan & Thompson 1992). The hot proto-neutron star has a radius $\simeq 50 \text{ km}$ that gradually shrinks on the cooling time scale $\simeq 5 \text{ to } 10 \text{ s}$. This time scale is coincidently, but significantly, similar to the sound crossing time of the helium core, and hence about the time required for a supersonic (but not relativistic) jet to penetrate the outer mantle. There are thus two potential mechanisms to create a $\gamma$-ray burst about 10 s after collapse. A $\gamma$-ray burst could be generated as the jet accelerates down the density gradient at the boundary of the core. Alternatively, a strong flow of LAEW might be generated on the neutron-star cooling time as the neutron star spins up. This might lead to an alternate mechanism of $\gamma$-ray burst formation on a similar timescale (Nakamura 1998). These mechanisms for generating $\gamma$-ray bursts will be discussed in §7.

It is hard to estimate the spin period of the proto-neutron star when it first forms. We assume that any $\gamma$-ray burst stimulated by LAEW from the neutron star will occur near the end of the cooling, contraction stage when the neutron star spins with a period $P_{\text{NS}} \simeq 1 \text{ ms}$. As we will see below, a spin period approaching $P_{\text{NS}} \simeq 1 \text{ ms}$ is necessary (if not sufficient) to power a classical $\gamma$-ray burst. A slower spin or shock breakout associated with a jet might generate the type of $\gamma$-ray burst observed in SN 1998bw/GRB 980425 as we will also show below. For purposes of illustration, we then adopt a spin period of the proto-neutron star to be that which will lead to a spin period of about 1 ms after cooling and contraction. Assuming angular momentum conservation during contraction of the original proto-neutron star, that $I_{\text{PNS}} \equiv kMR_{\text{NS}}^2$ with $k = \text{const} \simeq \frac{2}{5}$, and that $R_{\text{NS}}$ shrinks from 50 km to 10 km during the deleptonization phase, we take:

$$P_{\text{PNS}} \simeq 1\text{ms} \left( \frac{50 \text{ km}}{10 \text{ km}} \right)^2 \simeq 25 \text{ ms}.$$  

This is a substantial energy, but since only a fraction of it could be tapped it is not clear that this energy could power a supernova never mind a classical $\gamma$-ray burst. If the rotational period of the proto-neutron star is longer, this is even more true. The fraction of this rotational energy that can be tapped depends on the behavior of the magnetic field, and we turn to that subject.
4.1. Magnetic Field Amplification

The magnetic field of the proto-neutron star is uncertain. If the pre-collapse core has a field strength comparable to that of a magnetized white dwarf, \(B_0 \simeq 10^6\) G, then a field of \(\simeq 10^{12}\) G could arise from flux freezing. For \(P_{\text{PNS}} \simeq 25\) ms (equation 6) and \(\tau_{\text{conv}} \simeq 1\) ms, the Rossby Number \(R_0 \simeq P_{\text{PNS}}/\tau_{\text{conv}} \simeq 25\) \(\gg 1\) is too large to allow an \(\alpha - \Omega\) type dynamo to operate. An alternative magnetic field amplification mechanism is linear amplification (Meier et al. 1976, Kluźniak & Ruderman 1998). In this process, the differentially rotating neutron star could wrap the poloidal seed field into strong toroidal fields which then emerge from the star through buoyancy. After \(n_\phi\) revolutions of the neutron star, the initial seed (poloidal) field is wrapped and amplified to produce a toroidal field

\[
B_\phi \simeq 2\pi n_\phi B_p, \tag{8}
\]

where \(B_p\) is the initial seed poloidal field. Buoyancy will operate to expel the field if the amplified final field \(B_f\) satisfies

\[
\frac{B_f^2}{8\pi} \simeq f_B \rho c_s^2, \tag{9}
\]

where \(f_B \simeq 0.01\) is the fractional difference in density between the rising flux tube elements and the stellar material. Assuming sound speed \(c_s \simeq c/3\) and density \(\rho \simeq 10^{13}\) g cm\(^{-3}\), one finds

\[
B_f \simeq 2 \times 10^{16} \text{ G } f_B^{1/2} \rho_{13}^{1/2}, \tag{10}
\]

where \(f_{B-2} = f_B/0.01\) and \(\rho_{13} = \rho /10^{13}\) g cm\(^{-3}\). Assuming that the magnetic flux tube (a torus) occupies a volume of \(V_B/V_{\text{PNS}} \simeq 0.1\) where \(V_{\text{PNS}} \simeq \frac{4\pi}{3} R_{\text{PNS}}^3\), the energy contained in the magnetic flux tubes at the buoyancy limit is estimated to be:

\[
E_B \simeq 0.1 \times V_{\text{PNS}} \times \frac{B_f^2}{4\pi} \tag{11}
\]

\[
\simeq 1.6 \times 10^{51} \text{ erg.} \tag{12}
\]

This magnetic energy is ejected from the neutron star in the rising magnetic flux tubes. This energy is comparable to the proto-neutron star rotation energy (equation 7). For the adopted parameters, the proto-neutron star would have to lose substantial rotational energy to the magnetic field before the field would float on dynamical time scales. This magnetic energy is still likely to escape from the proto-neutron star, but the details may be complex and involve the subsequent contraction of the neutron star.

The number of revolutions to reach \(B_f \simeq 2 \times 10^{16}\) G is:

\[
n_f = \frac{B_f}{B_0} \frac{1}{2\pi} \simeq 3 \times 10^3 \left( \frac{B_0}{10^{12}\text{ G}} \right)^{-1}. \tag{13}
\]

For \(B_0 \simeq 10^{12}\) G, the amplification time scale before the field is expelled by buoyancy is thus

\[
t_f \simeq n_f P_{\text{PNS}} \simeq 25 \text{ ms} \times 3 \times 10^3 \sim 75\text{ s}, \tag{14}
\]
at the beginning of the contraction of the proto-neutron star. As the neutron star contracts, it spins up. The timescale for the linear field amplification, $t_f$, will decrease and the rotational energy of the neutron star will increase (cf §6). By the time the neutron star is spinning with a period of 1 ms, $t_f$ would be substantially less than the cooling, contraction time of $\simeq 10$ s. Thus sometime during the contraction a dipole-type strong field is expected to be produced once the $\simeq 10^{16}$ G field emerges from the surface. A dipole field strength of $\simeq 10^{14}$ G is expected from the random sum of flux tubes of $\simeq 10^{16}$ G (Duncan & Thompson 1992; Thompson & Duncan 1993). It is likely that during the contraction phase a substantial fraction of $E_{rot,PNS} \simeq E_{B,PNS} \simeq 10^{51}$ erg would be extracted from the neutron star.

### 4.2. Energy Transport

We now consider the possible energy transport mechanisms from the neutron star to the outer stellar envelope. For $P \simeq 25$ ms, the radius of the light cylinder is (cf. equation 3) $R_{LC,PNS} \simeq 10^5$ km, comparable to the initial radius of the iron core and much larger than the proto-neutron star. For $R \lessdot R_{LC,PNS}$, a rotating dipole field could exert a magnetic torque on the plasma around the star. Typical collapse calculations (Burrows, Hayes & Fryxell 1996; Mezzacappa, et al. 1997) give for the density in the vicinity of the standing shock $\rho \simeq 10^8$ g cm$^{-3}$ with a pre-shock velocity of $v \simeq 10^8$ cm s$^{-1}$. Using those as fiducial values, the characteristic Alfvén radius for the proto-neutron star is (equation 5):

$$R_A \simeq 150 \text{ km} \left( \frac{R_{PNS}}{50 \text{ km}} \right) \left( \frac{B_{PNS}}{10^{14} \text{ G}} \right)^{1/3} \left( \frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{-1/6} \left( \frac{v}{10^8 \text{ cm s}^{-1}} \right)^{-1/3}. \quad (15)$$

Equation (15) implies that for $B_{PNS}$ substantially less than $10^{14}$ G the Alfvén radius would be less than the radius of the proto-neutron star and hence that the magnetic torque would be ineffective. For $B_{PNS} \gtrsim 10^{14}$ G, the torque could have a significant effect. Equation (15) shows that $R_A \ll R_{LC} \simeq 1000$ km, so that a dipole field could be maintained over a significant volume. For the fiducial values chosen in equation (15), $R_A$ is interestingly close to the stalled shock radius $\simeq 200$ km, thus justifying the choice of density and velocity, to which $R_A$ is not, in any case, very sensitive.

There are two relevant timescales pertinent to the action of the torque. One is the time to deflect infalling matter from radial infall to substantially azimuthal flow. The second is the time for this torque to spin down the proto-neutron star and hence to deposit energy in the infalling matter. The time for the torque to deflect infalling matter is approximately the momentum per unit volume of the infalling matter times an appropriate lever arm, $R_A$, divided by the torque per until volume, $\simeq B_z(R)B_\phi(R) \simeq B^2(R)$ (cf. Shapiro & Teukolsky 1983). This effect of the torque thus operates on a time scale

$$t_{tor,1} \simeq \frac{\rho R_A v}{B^2} \simeq \frac{\rho R_A v}{4\pi \rho v^2} \simeq \frac{1}{4\pi} \frac{R_A}{v}, \quad (16)$$
This time is sufficiently short that the torque could substantially alter the flow of matter, preventing the radial infall interior to the shock that is common to all spherically symmetric models of core collapse. Multi-dimensional models that produce non-radial circulation flows in the matter beneath the standing shock would also be substantially affected for conditions similar to those reflected in equation (16).

The time scale to extract energy from the proto-neutron star and deposit that energy in the surrounding plasma can be estimated by equating the torque, $N$, on the proto-neutron star to that on the gas,

$$N = I\dot\Omega \simeq \int_{R_A}^\infty R^2 B_z(R) B_\phi(R) dR \simeq R_A^3 B(R_A)^2.$$  (17)

Defining the timescale appropriate to this action of the torque to be $t_{\text{tor}} = \Omega/\dot\Omega$ and using equation (17) gives:

$$t_{\text{tor},2} \simeq \frac{1}{5\sqrt{\pi}} \frac{\Omega_{\text{PNS}} M_{\text{NS}}}{R_{\text{PNS}} B_{\text{PNS}}^{1/2} v},$$  (18)

The rate of energy deposition in the plasma by this torque is thus:

$$L_{\text{tor}} = I\dot\Omega \simeq 2\sqrt{\pi} \frac{R_{\text{PNS}}^3 B_{\text{PNS}}^{1/2}}{v},$$  (19)

5. The Effect of an Axial Jet

If, as in LeBlanc & Wilson (1970), an MHD jet is formed during the collapse phase, the maximum jet power is estimated to be $E_{\text{rot}}/\tau_{\text{dyn}} \simeq 10^{51}$ erg/s $\simeq 10^{51}$ erg s$^{-1}$. Although the details of the jet dynamics and energetics are not known, roughly $\lesssim 10^{51}$ erg s$^{-1}$ of power could be directed along the jet axis. The LeBlanc & Wilson calculation was criticized by Meier et al. (1976) as requiring extreme parameters of the progenitor star. These issues need to be re-examined in the current context, but we note several things about the Meier et al. analysis. They argue that
the MHD axial flow found by LeBlanc & Wilson will not propagate to the stellar surface as a jet. The calculation of Khokhlov et al. (1999) shows that this is not necessarily correct, at least for a helium core. Meier et al. based their analysis on stellar evolution calculations of the day, but they adopted a stellar core with central density of about $10^{10}$ g cm$^{-3}$ giving a binding energy of about $10^{52}$ ergs. This exaggerates the binding energy of the initial core by about a factor of 10 compared to modern calculations and gives an incorrectly small value of a key parameter of Meier et al., the ratio of the binding energy of the newly-formed neutron star to that of the initial core. Meier et al. also did not consider the possibility of an $\alpha - \Omega$ dynamo that could lead to exponential field growth (Duncan & Thompson 1992). The whole question of the initiation of MHD jets in association with neutron star formation needs to be considered anew. Here we consider some basic properties of jet propagation.

The jet would be stopped by the envelope when the jet is unable to provide the power to move the envelope material at a speed comparable to the jet velocity. This can be expressed as:

$$L_{\text{jet}} \simeq R^2 \Delta \Omega \times \rho_{\text{env}} v_{\text{jet}}^2 \times v_{\text{jet}},$$

where $\Delta \Omega$ is the solid angle of the jet. The jet would then be stopped in a length

$$R_{\text{st}} \simeq \left[ \frac{L_{\text{jet}}}{\Delta \Omega \rho_{\text{env}} v_{\text{jet}}^3} \right]^{1/2}.$$  

(21)

In order to penetrate the star, the energy injected into the envelope at $R \lesssim R_{\text{st}}$ should be enough to unbind the region of the outer stellar mantle impacted by the jet. The amount of stellar material impacted by the jet is

$$\Delta M_{\text{env}} = M_{\text{env}} \frac{L_{\text{jet}}}{4\pi} \simeq 8 \times 10^{-3} M_{\text{env}} \left( \frac{\Delta \Omega}{0.1} \right).$$

(22)

Unbinding this mass requires

$$L_{\text{jet}} \gtrsim \frac{G M_{\text{env}} M_{\text{NS}} \Delta \Omega}{4\pi R_{\text{env}} \Delta t},$$

(23)

where $\Delta t$ is the duration of injection of the jet. Equation (23) can be expressed as:

$$L_{\text{jet}} \gtrsim 3 \times 10^{47} \text{ erg s}^{-1} \left( \frac{M_{\text{env}}}{1 M_\odot} \right) \left( \frac{M_{\text{NS}}}{1.5 M_\odot} \right) \left( \frac{R_{\text{env}}}{10^5 \text{ km}} \right)^{-1} \left( \frac{\Delta \Omega}{0.1} \right) \left( \frac{\Delta t}{1 \text{ s}} \right)^{-1}.$$  

(24)

A jet of $\approx 10^{51}$ erg s$^{-1}$ thus gives ample power to unbind a portion of the envelope occupying about 0.1 steradian.

The dynamics of the jet and impacted envelope material will depend on whether the mass of the jet is greater than or less than the mass of the displaced stellar envelope. The speed of the jet will be

$$v_{\text{jet}} \simeq 2c \left( \frac{2L_{\text{jet}} \Delta t}{M_{\text{jet}} c^2} \right)^{1/2}.$$  

(25)
This implies a relativistic velocity for $L\Delta t \simeq 10^{51}$ ergs and $M_{\text{jet}} \lesssim 0.01M_\odot$. If a comparable amount of energy is put into the displaced envelope material, the impacted envelope mass, $\Delta M_{\text{env}}$, would expand at a speed

$$v_{ej} \simeq \left(\frac{2L_{\text{jet}}\Delta t}{M_{\text{env}} \Delta \Omega}\right)^{1/2}. \quad (26)$$

If this matter were displaced radially out of the star, it could also acquire relativistic speeds for $L\Delta t \simeq 10^{51}$ ergs and $\Delta \Omega \lesssim 0.1$. In practice, the displaced material will tend to be accelerated sideways by shocks induced by the passage of the jet and the energy will be shared by the whole envelope at roughly the sound speed in the envelope.

The propagation of the jet requires more study, but the calculation of Khokhlov et al. (1999) gives some qualitative insight into the behavior to be expected. As the jet propagates, a bow shock runs ahead of it. The speed of the bow shock is less than that of the matter inflow into the jet, a ratio of about 0.5 to 0.7 for the particular case explored by Khokhlov et al., but that ratio will depend on the speed and density of the jet, its opening angle, and the structure of the star through which the jet propagates.

The bow shock of the jet will both heat material and cause it to expand sideways. The opening half angle of the jet will then be approximately

$$\theta \simeq \frac{v_{\text{env}}}{v_{\text{bow}}} \simeq 0.1 \text{ rad} \frac{v_{\text{env},8}}{v_{\text{bow},9}} \simeq 5^{-\theta/5} \frac{v_{\text{env},8}}{v_{\text{bow},9}}. \quad (27)$$

For $v_{\text{env}} \simeq 2 \times 10^8$ cm s$^{-1}$ and $v_{\text{bow}} \simeq 2 \times 10^9$ cm s$^{-1}$, $\Delta \theta \simeq 5^\circ$ or $\Delta \Omega/4\pi \simeq 0.004$. The actual dynamics of the jet will depend on nested cocoon-like shocks from the bow shock and subsequent expansion, as illustrated by Khokhlov et al. (1999).

### 6. The Neutron Star Phase

Concurrent with the propagation of any MHD jet and magnetic torque exerted by the proto-neutron star, the neutron star will contract and new physics can come into play. We assume that the cooling and contraction leads to a neutron star rotating at a period of about 1 millisecond. This implies a much smaller radius for the light cylinder,

$$R_{LC} = \frac{c}{\Omega_{NS}} \simeq 50 \text{ km} \left(\frac{P_{NS}}{1 \text{ ms}}\right). \quad (28)$$

This is only a little bigger than the neutron star radius. If the field remains about the same as the proto-neutron star, $B \simeq 10^{14}$ G, the Alfvén radius will be comparable to $R_{LC}$, but if the field amplifies during the contraction, as is plausible, the Alfvén radius will expand to become substantially greater than $R_{LC}$. Under this circumstance, a stationary dipole configuration at and beyond the Alfvén radius can no longer be maintained.

During the contraction phase, the Rossby number decreases in proportion to the rotational period. Assuming the neutron star convective time scale remains at about 1 ms, the condition
$R_0 \lesssim 1$ will be reached if the neutron star ends up with a period of about 1 ms. This means that an $\alpha - \Omega$-type dynamo could be initiated that would build a strong magnetic field up to $\sim 10^{15} - 10^{16}$ G for the dipole configuration near the neutron star surface (in the absence of the light cylinder). This phase of field growth occurs exponentially in contrast to the linear amplification associated with the relatively slowly-rotating initial proto-neutron star. If the dipole field grows to $B_{NS} \simeq 10^{16}$ G as the radius shrinks to $R_{NS} \simeq 10^6$ cm, then, all else being equal, the Alfvén radius will be $\simeq 140$ km, substantially larger than $R_{LC}$, as just noted. If the ram pressure has declined, $R_A$ will be even larger.

The rotational energy after the contraction is

$$E_{\text{rot,NS}} \simeq \frac{1}{2} I_{NS} \Omega_{NS}^2 \simeq 6 \times 10^{52} \text{ erg} \left( \frac{M}{1.5 M_\odot} \right) \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^2 \left( \frac{R_{NS}}{10^6 \text{ cm}} \right)^2.$$  

Setting aside the baryon-loading problem for the moment, this energy is comparable to the largest energy associated with any $\gamma$-ray burst, $\sim 3 \times 10^{54} \Delta \Omega/4\pi$ erg for GRB 990123 (Kulkarni, et al. 1999) for a degree of collimation, $\Delta \Omega/4\pi \leq 10^{-2}$. This degree of collimation has been deduced for some afterglows, in particular for GRB 990123 itself, and as noted in the previous section, is about the order expected for a matter-dominated jet (see also Khokhlov et al. 1999). The degree of collimation of the subsequent flow of electromagnetic radiation is unclear. We return to that topic below.

The rotational energy of the contracted neutron star is radiated away in the form of a Poynting flux or LAEW at the frequency $\Omega_{NS}$. The luminosity is estimated to be

$$L_{EM} \simeq 4\pi R_{LC}^2 \times \frac{c}{4\pi} |\vec{E} \times \vec{B}| \simeq \frac{\mu_{NS}^2 c}{R_{LC}^4} \simeq \frac{R_{NS}^6 B_{NS}^2 \Omega_{NS}^4}{c^3},$$

assuming the LAEW to be generated at $R_{LC}$ and the magnetic moment of the neutron star to be $\mu_{NS} = B_{NS} R_{NS}^3$. For the conditions of the contracted neutron star which has initiated an $\alpha - \Omega$ dynamo, we expect

$$L_{EM} \simeq 4 \times 10^{52} \text{ erg s}^{-1} \left( \frac{R_{NS}}{10 \text{ km}} \right)^6 \left( \frac{B_{NS}}{10^{16} \text{ G}} \right)^2 \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^4,$$

which will last for a duration of

$$t_{EM} \simeq \frac{E_{\text{rot,NS}}}{L_{EM}} \simeq 2 \text{ s} \left( \frac{M}{1.5 M_\odot} \right) \left( \frac{R_{NS}}{10 \text{ km}} \right)^{-4} \left( \frac{B_{NS}}{10^{16} \text{ G}} \right)^{-2} \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^{-2}.$$  

These LAEW cannot propagate through a plasma if the plasma frequency

$$w_p = \left( \frac{4\pi n_e e^2}{m_e} \right)^{1/2} \approx 9 \times 10^3 n_e^{1/2} \text{ Hz},$$

exceeds the LAEW frequency $\Omega_{NS}$. The corresponding condition on the electron density is:

$$n_e \gtrsim 1 \text{ cm}^{-3} \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^2.$$
For densities exceeding this value, the LAEW would have a very small skin depth and cannot propagate in the plasma. They would effectively be reflected by the plasma. The density exceeds this critical value under any stellar conditions. The LAEW will then initially be bottled up near the site of their production near \( R_{LC} \). The LAEW will push the plasma aside and seek the easiest way out. An obvious possibility is that they will “burn” a channel along the rotation axis, following the path of the previous MHD jet.

We assume that the most intense production of LAEW occurs after the completion of the contraction of the neutron star, and hence that it is delayed by the cooling time \( \simeq 5 \times 10^2 \) s, about the time necessary for the initial matter jet to propagate through the stellar core. A mass of about \( \Delta M \simeq M_{\text{env}} \frac{\Delta \Omega}{4\pi} \simeq 10^{-2} M_{\text{env}} \), will be pushed aside by the initial matter jet. Whether the region impacted by the matter jet will be rarefied depends on the mass of the jet. If the jet is rather massive, as in the case of the calculation of Khokhlov et al. (1999), then the jet remains denser than the stellar environment as it propagates into the envelope. In this case, the jet acts as a “plug” until it disperses after several dynamical times. The electromagnetic waves generated by the pulsar will tend to propagate through the lowest density regions. If there is a density minimum along the rotation axis, there will be a tendency for energy to flow in that direction. The dynamics are likely to be complicated and we will just sketch the possibilities.

The LAEW should be rapidly isotropized as they reflect off the plasma, so they can be considered as a relativistic gas while they are trapped within the stellar core. It is not so clear that they are thermalized since their characteristic wavelength,

\[
\lambda_{\text{LAEW}} \simeq c P_{\text{NS}} \simeq 300 \text{ km},
\]

is much larger than the scale of the region in which they are generated. This is an important issue since if the LAEW are thermalized then the effective temperature will be,

\[
T_{\text{LAEW}} \simeq \left( \frac{E_{\text{LAEW}}}{\frac{4\pi}{3} a R^3} \right)^{1/4} \simeq 2 \times 10^{10} \text{ K} E_{52}^{1/4} R_3^{-3/4},
\]

where \( R_3 \) is the radius in units of \( 10^3 \) km, so that much of the energy would go into the formation of pairs which might be dissipated by adiabatic expansion and difficult to recover. We will ignore this possibility for the moment, assuming that the LAEW cannot thermalize.

If the LAEW act like a relativistic gas, then they will exert a pressure of

\[
P_{\text{LAEW}} \simeq \frac{1}{3} \left( \frac{E_{\text{LAEW}}}{\frac{4\pi}{3} a R^3} \right) \simeq 8 \times 10^{26} \text{ erg cm}^{-3} E_{52} R_3^{-3}. \tag{37}
\]

This may be contrasted to the pressure required to ensure hydrostatic equilibrium in the gravity of the neutron star which we can represent crudely as,

\[
P_{\text{HSE}} \simeq \frac{G M_{\text{NS}}^2}{R^4} \simeq 1 \times 10^{28} \text{ erg cm}^{-3} \left( \frac{M_{\text{NS}}}{1.5 M_\odot} \right)^2 R_3^{-4}. \tag{38}
\]
By this measure, the pressure of the radiation will be less than required to produce HSE for small radii, and will exceed the pressure corresponding to HSE at a radius of

\[ R_{\text{HSE}} \simeq 1 \times 10^4 \text{ km} \left( \frac{M_{\text{NS}}}{1.5M_\odot} \right)^2 E_{52}^{-1}. \]

The regime where the LAEW begin to dominate will, of course, depend on the density profile in the collapsing matter and the whole environment is, in any case, dynamic so that considerations of HSE give only a partial perspective.

More significantly, perhaps, the production of the LAEW near the light cylinder could lead to strong Rayleigh-Taylor instability as the relativistic fluid pushes on the surrounding gas. The subsequent behavior will depend on the rate at which the LAEW “gas” expands quasi-spherically in Rayleigh-Taylor fingers compared to the rate at which the waves will selectively propagate upward along the rotation axis. It seems plausible that a substantial majority of the energy will flow up the axis, the path of least resistance. Another possibility is that the energy density becomes so high compared to the surrounding matter, near the neutron star or further out in the mantle (equation 39), that the stellar matter becomes an insubstantial barrier to the nearly free expansion of the LAEW bubble. Since the total energy in the LAEW could be as high as $10^{52}$ erg, this is a real possibility. Whether the LAEW remain collimated may thus depend sensitively on the timescale on which they are generated and hence the instantaneous energy density throughout the star. These possibilities clearly need to be investigated with appropriate numerical calculations.

The result of the generation of the LAEW could be rather different if they are thermalized. In this case, the LAEW would be replaced with copious pairs (cf. equation 36). The pair gas would also act like a relativistic gas with the same issues of dynamics just discussed. In addition, there would be the complications of annihilation at boundaries with normal matter and the possibility of strong adiabatic losses after a pair jet broke out of the star and underwent free expansion. Such loss of pair thermal energy to rest mass and kinetic energy might be at least partially recovered if the pairs interacted with a surrounding wind of normal matter.


There are two stages when $\gamma$-rays might be generated. The first is when the bow shock that proceeds the initial mass-dominated jet impacts on the stellar photosphere. The second phase is when a collimated flow of LAEW erupts from the surface at about the same time. We consider those in turn.
7.1. Bow Shock Gamma-Ray Emission

Some $\gamma$-ray emission may be generated by the phase of shock breakout as the shock associated with the initial matter-dominated jet runs down the stellar density gradient and breaks through the photosphere. An associated mechanism is for the accelerated matter to reach relativistic speeds and then collide with some external matter.

As the bow shock that proceeds the jet runs down the exponential stellar density gradient in the photosphere it will accelerate, in turn heating and accelerating the matter, a mechanism to produce $\gamma$-rays first described by Colgate (1974, 1975). The question of how this matter expands and radiates and perhaps collides with an external environment is beyond the scope of this paper, but worth more detailed study. Here we remark on the basic properties at the time of shock passage.

Energy is deposited in the atmosphere by the bow shock. The bow shock, in turn, derives its energy from the jet. The power carried by the jet is $\approx \frac{1}{2} \rho_{jet} v_{jet}^2 \Delta A$, where $\Delta A$ is the fractional area of the star that is impacted by the bow shock. This power will be delivered to the photosphere in a time $\approx l_{phot}/v_{jet}$, where $l_{phot}$ is the depth of the photosphere. The energy deposited in the photosphere during the bow shock break out phase is thus:

$$E \approx \frac{1}{2} \rho_{jet} v_{jet}^2 l_{phot} \Delta A \approx 5 \times 10^{45} \text{ erg} \rho_{jet} v_{jet,10}^2 l_{phot,7} \Delta A_{18},$$

(40)

where $v_{jet,10}$ is the velocity of the bow shock in units of $10^{10}$ cm s$^{-1}$, $l_{phot,7}$ is in units of $10^7$ cm and $\Delta A_{18}$ is in units of $10^{18}$ cm$^2$. For the calculation of Khokhlov et al. (1999) the jet density is about $\rho_{jet} \approx 10^3$ g cm$^{-3}$, and the velocity steepens to about 90,000 km s$^{-1}$ as the jet approaches the outer density gradient of the helium core at which point the resolution of the density profile degrades. For $\rho_{jet} \approx 10^3$ g cm$^{-3}$, $v_{jet,10} \approx 1$, $l_{phot,7} \approx 1$, and $\Delta A_{18} \approx 1$, the energy would be $\approx 5 \times 10^{48}$ ergs, comparable to the $\gamma$-ray energy in the burst of SN 1998bw/GRB 980425.

The corresponding temperature, making the crude, and not necessarily correct, assumption of thermalization to a radiation-dominated gas, is

$$T \approx \left( \frac{\rho_{jet} v_{jet}^2}{2a} \right)^{1/4} \approx 10^8 \text{ K} \rho_{bow}^{1/4} v_{bow,10}^{1/2},$$

(41)

For $\rho_{jet} \approx 10^3$ g cm$^{-3}$, $v_{jet,10} \approx 1$, equation (41) gives $T \approx 10^{10}$ K with the possible emission of $\gamma$-rays.

This hot, accelerated matter would then expand and radiate. Whether it could account for the $\gamma$-ray burst in SN 1998bw/GRB 980425 will require more careful consideration. We note that if this energy accumulates in the density gradient of the photosphere the matter will be optically thin, so not susceptible to adiabatic losses while it undergoes free expansion. The photosphere is already optically thin by definition and the opacity will be decreased by Klein-Nishina corrections. On the other hand, the jet is denser and hence more opaque than the photosphere so it is important
to know where the energy represented by equation (40) resides. Although the conditions could
vary widely depending on the nature and propagation of the jet, for the calculation of Khokhlov
et al. (1999), the bow shock material is matter ablated from the jet and the characteristic density
is about the same as the jet, $\rho_{\text{bow}} \simeq 10^3 \text{ g cm}^{-3}$.

The emission properties of the jet/bow shock/stellar atmosphere region as the jet impacts the
atmosphere thus require more careful study. Nevertheless, considerable hard radiation could be
emitted before the phase of homologous expansion is reached. The time scale for this energy to be
radiated is also of importance. The shock break out time, $\simeq 10^{-3} \ell_{\text{phot},7}/v_{\text{jet},10} \text{ s}$, is far too short
to correspond to an observed $\gamma$-ray burst, specifically that in GRB 980425, but the controlling
time scale will be the radiative timescale.

Another possible means of producing $\gamma$-rays from the matter jet is to accelerate the matter in
the photosphere to relativistic speeds and for it then to subsequently collide with a surrounding
medium, perhaps a stellar wind. The mass fraction that can be accelerated in this way for an
explosion with an impulsive energy input that drives a single shock through the outer stellar layers
has been evaluated for CO and He stellar cores by Woosely, Eastman & Schmidt (1998). For their
models, they establish a relationship

$$Q = \Gamma \beta \left( \rho r^3 \right)^{0.2} \simeq 2.5 \times 10^6 \Gamma M_{\text{ex},32}^{0.15},$$

(42)

where the parameter Q is independent of Lagrangian mass and radius in the ejecta, $M_{\text{ex},32}$ is the
mass external to the layer with Lorentz factor $\Gamma$ in units of $10^{32}$ gm, and we have taken $\beta \simeq 1$.

Woosely et al. find that $Q \simeq 3 \times 10^5$. At the risk of overinterpreting their results, we note that for
two otherwise identical models that differ only in the explosion energy input (models CO6A and
CO6C), Q scales with the explosion energy, $E_{\text{ex}}$, approximately as $E_{\text{ex}}^{1/2}$. In the following we have
adopted $Q \simeq 10^5 E_{\text{ex},51}^{1/2}$, where $E_{\text{ex},51}$ is in units of $10^{51}$ erg.

With this scaling, we find the kinetic energy in the homologously expanding material (e.g.
after the immediate shock heating phase discussed above) to be

$$KE \simeq 1.6 \times 10^{44} \text{ erg} \Gamma^{-5.66} E_{\text{ex},51}^{2.83}.$$

(43)

This expression is for a spherical explosion. It is clear that the energy in matter accelerated to
relativistic speeds is insufficient to account even for the weak $\gamma$-ray burst imputed to GRB 990425,
as concluded by Woosely, Eastman & Schmidt (1998).

In the present context, it is relevant to estimate the difference if the shock did not propagate
spherically, but were collimated. In this case, we are interested in the kinetic energy in a jet with
solid angle $\Delta \Omega/4\pi$ or $KE_{\Delta \Omega} = KE \Delta \Omega/4\pi$. The relevant input energy is the energy in the jet,
$E_{\text{jet}} = (\Delta \Omega/4\pi) E_{\text{ex}}$, where $E_{\text{ex}}$ is now to be thought of as the equivalent isotropic energy of the
collimated jet. Making this substitution in equation (43), we find:

$$KE_{\Delta \Omega} \simeq 7.2 \times 10^{47} \text{ erg} \Gamma^{-5.66} f_{\text{coll}}^{-1.83} E_{\text{jet},51}^{2.83},$$

(44)
where \( f_{\text{coll}}^{-2} = \Delta \Omega/4\pi \) in units of \( 10^{-2} \). Equation (44) shows that even with substantial collimation, the amount of energy put into mildly relativistic matter (\( \Gamma \gtrsim a \) few) by this mechanism is still insufficient to account for the putative \( \gamma \)-ray burst in SN 1998bw for an energy typical of the matter jet we have discussed here, \( E_{\text{jet,51}} \simeq 1 \). There might be sufficient energy for \( E_{\text{jet,51}} \simeq 10 \), but even this energy would be insufficient to boost enough stellar matter to \( \Gamma \simeq 10 \) to account for SN 1998bw.

From this analysis, we conclude that, even if it is strongly collimated, the physical process of running a single impulsively-induced shock down the photospheric density gradient of a compact stellar core, accelerating that matter to relativistic speeds, and slamming it into a surrounding medium is unlikely to generate a substantial \( \gamma \)-ray burst of the kind associated with SN 1998bw.

To produce a cosmic \( \gamma \)-ray burst with an energy of \( 10^{52} \text{ erg} \) (isotropic equivalent of \( 10^{54} \text{ erg} \)) and \( \Gamma \gtrsim 100 \) is out of the question. If jets from stellar collapse to produce either neutron stars or black holes are to generate \( \gamma \)-ray bursts in SN 1998bw, never mind the high redshift events, then the physics of the \( \gamma \)-ray production must be associated with the initial shock breakout phase (as discussed above) or by the prolonged emission phase of the jet passing through and beyond the photosphere.

### 7.2. The Contraction/LAEW Phase

We now consider the time evolution of the contracting neutron star and the associated energy emission mechanisms implied by this scenario.

When \( R_{LC} \gg R_A \), the magnetic torque is the dominant energy/momentum transport mechanism (§4.2). When \( R_{LC} \ll R_A \), Poynting flux/LAEW transports the energy/momentum. In our scenario, the proto-neutron star continues spinning up due to contraction even though it transfers angular momentum through magnetic torque. Initially, when \( R_{LC} \gg R_A \), the rate of loss of energy to electromagnetic Poynting flux would be (cf. equation 31)

\[
L_{\text{EM}} \simeq 2 \times 10^{46} \text{ erg s}^{-1} \left( \frac{R_{\text{PNS}}}{50 \text{ km}} \right)^6 \left( \frac{B_{\text{PNS}}}{10^{14} \text{ G}} \right)^2 \left( \frac{\Omega_{\text{PNS}}}{250 \text{ s}^{-1}} \right)^4.
\]  

This will be small compared to the energy deposited by the torque of the spinning neutron star acting at the Alfvén radius (equation 19),

\[
L_{\text{tor}} \simeq 1 \times 10^{49} \text{ erg s}^{-1} \left( \frac{\Omega_{\text{PNS}}}{250 \text{ s}^{-1}} \right) \left( \frac{R_{\text{PNS}}}{50 \text{ km}} \right)^3 \left( \frac{B_{\text{PNS}}}{10^{14} \text{ G}} \right) \left( \frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{v}{10^8 \text{ cm s}^{-1}} \right).
\]  

After contraction when the condition \( R_{LC} \ll R_A \) is reached, we have:

\[
\frac{R_A}{R_{\text{NS}}} \simeq 14 \left( \frac{B_{\text{NS}}}{10^{16} \text{ G}} \right)^{1/3} \left( \frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{-1/6} \left( \frac{v}{10^8 \text{ cm s}^{-1}} \right)^{1/3},
\]
\[ R_{LC} \simeq 30 \text{ km} \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^{-1}, \]  

and hence

\[ \frac{R_{LC}}{R_A} \simeq 0.2 \left( \frac{R_{NS}}{10 \text{ km}} \right)^{-1} \left( \frac{B_{NS}}{10^{16} \text{ G}} \right)^{-1/3} \left( \frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{1/6} \left( \frac{v}{10^8 \text{ cm s}^{-1}} \right)^{1/3} \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^{-1}, \]  

where \( \rho \) and \( v \) are to be evaluated at the Alfvén radius. Since energy injection during the phase when \( R_{LC} > R_A \) will drive the density down in the vicinity of the magnetopause and the standing shock, and \( B_{NS} \) is likely to increase from \( 10^{14} \text{ G} \) to \( 10^{16} \text{ G} \) continuously (but suddenly) during the contraction by the linear amplification mechanism and dynamo, the epoch during contraction when \( R_{LC} \simeq R_A \) will occur is difficult to estimate; however, since essentially all the energy emitted will be used to expel the envelope (i.e. either by torque or LAEW) a more significant explosion/expansion is expected when \( R_{LC} \ll R_A \). At this epoch we expect that if the torque were still active the rate of loss of rotational energy due to this mechanism would be

\[ L_{\text{tor}} \simeq 3 \times 10^{49} \text{ erg s}^{-1} \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right) \left( \frac{R_{NS}}{10 \text{ km}} \right)^3 \left( \frac{B_{NS}}{10^{16} \text{ G}} \right) \left( \frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^4. \]  

This power would in any case be overwhelmed by the loss of energy to the LAEW with the enhanced rotation and magnetic field,

\[ L_{\text{EM}} \simeq 4 \times 10^{52} \text{ erg s}^{-1} \left( \frac{R_{NS}}{10 \text{ km}} \right)^6 \left( \frac{B_{NS}}{10^{16} \text{ G}} \right)^2 \left( \frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^4. \]  

This power output will last for a few seconds (equation 32). The injected energy is then \( \gtrsim 10^{52} \text{ erg} \), which is enough to drive a significant shock. In this scenario, both a \( \gamma \)-ray burst and an asymmetric supernova could be produced during the “magnetar” phase.

The LAEW will likely flow up the rotation axis starting with a cross-sectional area of \( \simeq R_{LC}^2 \) and rising as an intense photon “bubble” with an opening angle

\[ \theta_{\text{LAEW}} \simeq \frac{c_s}{c} \simeq 3 \times 10^{-3} \text{ rad} \simeq 0.2^\circ. \]  

This corresponds to a solid angle of \( \Delta \Omega/4\pi \simeq 2 \times 10^{-6} \). The LAEW may then propagate up the axis in a radiation-dominated jet with smaller cross section than the original matter-dominated jet. If this is the case, the channel carved by the LAEW will remain substantially smaller than the characteristic wavelength of the LAEW until the radiation breaks out of the surface of the star.

To avoid the baryon loading problem and to generate a \( \gamma \)-ray burst with large Lorentz factor, the hole through which the bulk of the LAEW emerge should contain less than

\[ \Delta M \lesssim \frac{10^{52} \text{ ergs}}{\Gamma^2 c^2}, \]  

(Rees & Mészáros 1992). For \( \Gamma \gtrsim 100 \), the requirement is \( \Delta M \lesssim 10^{27} \text{ g} \simeq 5 \times 10^{-7} \text{ M}_\odot \). It is difficult to estimate whether the LAEW jet will entrain such a small amount of matter. There
are two qualitative possibilities for the propagation of the LAEW jet. There may be a density minimum along the axis of the previous matter jet as suggested by the calculations of Khokhlov et al (1999). If the LAEW propagate through such a low density axial channel, they will naturally emerge somewhat more collimated than the original jet. On the other hand, if the jet is denser than the surrounding stellar matter and the jet acts like a plug on the axis, the LAEW might propagate in a cylindrical blanket around the plug. They would then emerge with an annular cross section. This would result in yet another complication for predicting the observational aspects of such an event.

If the matter jet leaves behind a region of very low baryon density, or the LAEW emerge beyond the end of the more slowly propagating matter jet, then the propagating LAEW could enter a low density region in which the density of the environment is lower than the critical Goldreich-Julian density (Goldreich & Julian 1969; Shapiro & Teukolsky 1983) of

\[ \rho_{\text{GJ}} \simeq 10^{-6} \text{ g cm}^{-3} \left( \frac{B(R)}{10^{16} \text{ G}} \right)^{-1} \left( \frac{P}{1 \text{ ms}} \right)^{-1}, \]

where the flux-freezing and force free conditions are broken. At this point, reconnection of magnetic fields and rapid acceleration of particles in a pair plasma follow (e.g. Asseo, Kennel & Pellat 1978; Usov 1992; Michel 1984; Thompson 1994; Blackman, Yi & Field 1995). The pair plasma could in principle be accelerated to a high bulk Lorentz factor up to \( \simeq 10^5 \), although the exact value of the maximum bulk Lorentz factor could be significantly lower than this due to baryon loading and other complicated factors reducing acceleration efficiency (cf. Usov 1992, 1994).

The rest-frame \( \gamma \)-ray luminosity would be determined by the electromagnetic power, i.e.

\[ L_{\text{obs}} \simeq f_\gamma L_{\text{EM}} \simeq 4 \times 10^{51} \text{ erg s}^{-1} \left( \frac{f_\gamma}{0.1} \right) \left( \frac{\Omega_{\text{NS},i}}{10^4 \text{ s}^{-1}} \right)^4 \left( \frac{R_{\text{NS}}}{10 \text{ km}} \right)^6 \left( \frac{B_{\text{NS}}}{10^{16} \text{ G}} \right)^2, \]

where \( f_\gamma \) is the \( \gamma \)-ray emission efficiency factor. During this phase, the pulsar rotation energy is used to power the \( \gamma \)-ray burst while the spin of the pulsar slows down as

\[ \Omega_{\text{NS}} = \left( \frac{\Omega_{\text{NS},i}}{10^4 \text{ s}^{-1}} \right) \times \left[ 1 + 6 \times 10^{-2} \left( \frac{M_{\text{NS}}}{1.5 M_\odot} \right)^{-1} \left( \frac{\Omega_{\text{NS},i}}{10^4 \text{ s}^{-1}} \right)^2 \left( \frac{R_{\text{NS}}}{10 \text{ km}} \right)^4 \left( \frac{B_{\text{NS}}}{10^{16} \text{ G}} \right)^2 \left( \frac{t - t_{\text{em},i}}{1 \text{ s}} \right) \right]^{-1/2}, \]

where \( \Omega_{\text{NS},i} \) is the initial spin frequency of the neutron star and \( t_{\text{em},i} \) is the initial time of the spin-down phase driven by the electromagnetic dipole radiation emission. The time evolution of the \( \gamma \)-ray luminosity is then given by

\[ L_{\text{obs}} \simeq 4 \times 10^{51} \text{ erg s}^{-1} \left( \frac{f_\gamma}{0.1} \right) \left( \frac{\Omega_{\text{NS},i}}{10^4 \text{ s}^{-1}} \right)^4 \left( \frac{R_{\text{NS}}}{10 \text{ km}} \right)^6 \left( \frac{B_{\text{NS}}}{10^{16} \text{ G}} \right)^2 \times \left[ 1 + 6 \times 10^{-2} \left( \frac{M_{\text{NS}}}{1.5 M_\odot} \right)^{-1} \left( \frac{\Omega_{\text{NS},i}}{10^4 \text{ s}^{-1}} \right)^2 \left( \frac{R_{\text{NS}}}{10 \text{ km}} \right)^4 \left( \frac{B_{\text{NS}}}{10^{16} \text{ G}} \right)^2 \left( \frac{t - t_{\text{em},i}}{1 \text{ s}} \right) \right]^{-2}. \]
The LAEW-powered γ-ray emission stage shows a characteristic initial phase during which the luminosity remains nearly constant. This phase is expected to be followed by a phase in which the luminosity decreases as ∝ t⁻². As shown in Blackman and Yi (1998), the γ-ray emission via the synchrotron-Compton process gives the luminosity of the peak ∝ t⁻¹. We note that if the proton fraction changes after some complicated acceleration processes, then the synchrotron-Compton emission process and the above scalings could be significantly different.

To get γ-rays directly from the LAEW, there must be a density in the environment that is below the Goldreich-Julian density so the currents cannot be supported in the plasma and a pair cloud is spontaneously generated. The Goldreich-Julian density scales with the ambient magnetic field. If that field falls off like $R^{-1}$ in the propagating Poynting flux, then beyond the helium core at $R \gtrsim 10^6$ km, the Goldreich-Julian density will be (equation 54) $\rho \lesssim 10^{-11}$ g cm⁻³. Densities this low might occur immediately beyond the star (and the jet) if the star is embedded only in the interstellar medium, but it is likely that the progenitor of a Type Ib/c supernova is surrounded by a wind or other mass resulting from mass-loss processes. For a constant velocity wind at $10^8 v_8$ cm s⁻¹ carrying mass at a rate $10^{-5} \dot{M}_{-5}$ M⊙ yr⁻¹, the density is

$$\rho = 5 \times 10^{-9} \text{ g cm}^{-3} \dot{M}_{-5} v_8^{-1} R_5^{-2},$$

with radius in units of $10^5$ km. This density will be less than the Goldreich-Julian density for $R \gtrsim 10^6$ km, several times the radius of the helium core.

Although the possibility of gamma-ray emission by pair cascade is not ruled out, it is important to consider how the LAEW escape from the stellar core and associated processes. A sub-relativistic MHD jet leaves behind a baryon-rich environment. The common statement is that if the baryon loading is too high a γ-ray burst of observed properties cannot be produced. We show below that this is not necessarily the case.

Since the plasma density (baryons, leptons, and photons) is high, the perfect MHD condition is always maintained throughout the phase when the LAEW propagates within the stellar core. Due to the high radiation density and the large amount of high temperature stellar material, the optical depth for outgoing radiation is exceedingly high. This makes the γ-ray emission efficiency very low. Under these circumstances, the bulk of the energy supplied by the LAEW may be used to accelerate shocks parallel to the rotation axis and the axis of the matter jet. After the shock breaks out, γ-ray emission could occur as the kinetic energy of the shock is dissipated into particle acceleration and subsequent synchrotron-Compton emission. We note one important difference between this situation and that of the original Colgate mechanism. As noted in §7.1, in the Colgate mechanism there is an impulsive input of energy in the stellar core and a single shock that propagates outward. This leads to a single, short pulse of hard emission. This process may, indeed, work when the shock driven by the LAEW first encounters the steep density gradient at the stellar surface or at the tip of the jet, but in the current situation the pulsar and the LAEW continue to drive shocks for an extended time. The density profile will be altered as the continuing shock energy is deposited and that reaction must be considered self-consistently.
It is difficult to estimate the $\gamma$-ray emission efficiency in this process. Even for a very low efficiency $\lesssim 10^{-4}$, however, a $\gamma$-ray burst event such as SN 1998bw/GRB980425 could be generated provided that the LAEW power is large enough, $\gtrsim 10^{51}$ erg s$^{-1}$ as has been assumed. If the efficiency of production of $\gamma$-rays approaches unity, then, with appropriate collimation, a $\gamma$-ray burst detectable at cosmological distances could be produced. Aside from the complex details of the $\gamma$-ray production, the overall bolometric luminosity evolution is expected to follow the time evolution described by equation (57).

In this context, we take note of circumstances when the deposition of energy into baryons does not necessarily doom the production of a $\gamma$-ray burst. As noted by Protheroe & Bednarek (1999), if protons can be accelerated to energies where pion production is efficient, then $\gamma$-rays can be produced in the subsequent pion decay. To reach this regime, the protons must be given at least mildly relativistic energies, in excess of 1 GeV. This requires that the energy of the LAEW be shared with less than 0.005 $M_\odot E_{52}$ of baryons. The shock driven by the LAEW should boost some protons to even higher bulk velocities and the energy deposited by the LAEW will plausibly go into a mass less than the total mass of the precursor MHD jet if their effects are concentrated in the relatively low density matter along the axis of the jet or the matter immediately surrounding the jet. Deposition of the energy of the pulsar spindown via LAEW selectively into the bulk motion of protons with energies substantially above the pion production threshold is thus a distinct possibility. Thus rather than being a handicap, the “baryon loading” of the jet could be an advantage in the conversion of pulsar energy into $\gamma$-ray energy.

Although a quantitative calculation is required, we envisage a process by which the protons are accelerated to high energy with the associated efficient production of pions. To produce the pions, the protons must collide with a “target.” In the present context, a good candidate for the target is the wind expected to be present around the progenitor. The stopping length for proton-proton interaction to produce pions is about 100 cm$^2$ gm$^{-1}$. For the density given by equation (58), the column depth of the wind is

$$l = 600 \, \text{g cm}^{-2} \dot{M}_{-5} v_8^{-1} R_{5}^{-1},$$

(59)

with radius again in units of $10^5$ km. Thus a reasonable wind can provide the stopping medium for the conversion of high-velocity protons to pions in the vicinity of the surface of a hydrogen-stripped star.

The pions produced when the high-energy protons collide with the wind then decay and produce very energetic $\gamma$-rays. Note that the stopping length for $\gamma$-rays is about 30 cm$^2$ gm$^{-1}$, so a “target” that stops protons will be somewhat optically thick to $\gamma$-rays. These high energy $\gamma$-rays do not immediately correspond to the observed $\gamma$-rays in a $\gamma$-ray burst. Rather, they induce a pair cascade through photon-photon collision. This pair fireball could then produce the observed $\gamma$-ray burst. The strength of a $\gamma$-ray burst produced in this way will depend on the relative efficiency of production of pions versus neutrinos. The latter would be an energy sink for this process.
8. Discussion and Conclusions

The point of this paper is not to establish that core collapse and pulsar formation will lead to \(\gamma\)-ray bursts, but to establish that this environment gives a framework in which to quantitatively address questions of physics that are germane to the nature of the core collapse process and to potential \(\gamma\)-ray production. It seems very clear that rotation and magnetic fields have a strong potential to create axial matter-dominated jets that will drive strongly asymmetric explosions for which there is already ample observational evidence in Type II and Type Ib/c supernovae, their remnants, and in the pulsar velocity distribution. The potential to also create strong flows of LAEW serves to reinforce the possibility to generate asymmetric explosions. These asymmetries will affect nucleosynthesis and issues such as fall-back that determine the final outcome to leave behind neutron stars or black holes. In addition, the presence of matter-dominated and radiation-dominated jets might lead to bursts of \(\gamma\)-rays of various strengths. The issue of the nature of the birth of a “magnetar” in a supernova explosion is of great interest independent of any connection to \(\gamma\)-ray bursts. Highly magnetized neutron stars might represent one out of ten pulsar births. Production of a strong \(\gamma\)-ray burst might be even more rare.

We have shown that the contraction phase of a proto-neutron star could result in a substantial change in the physical properties of the environment. The following parameters are relevant:

- \(R_{\text{PNS}} \approx 50\) km
- \(P_{\text{PNS}} \approx 25\) ms
- \(R_{\text{LC}} \approx 10^3\) km
- \(I_{\text{PNS}} = kR_{\text{PNS}}^2M_{\text{NS}} \approx 3 \times 10^{46}\) c.g.s.
- \(\tau_{\text{conv}} \approx 1\) ms
- \(R_0 \sim P_{\text{PNS}}/\tau_{\text{conv}} \gg 1\)
  \(\rightarrow\) no dynamo
- magnetic field amplified within \(\sim 10s\) by linear amplification or flux-freezing
- MHD jet & torque action
- hole punched & envelope torqued
- \(R_{\text{NS}} \approx 10\) km
- \(P_{\text{NS}} \approx 1\) ms
- \(R_{\text{LC}} \approx 50\) km
- \(I_{\text{NS}} = kR_{\text{NS}}^2M_{\text{NS}} \approx 10^{45}\) c.g.s.
- \(\tau_{\text{conv}} \approx 1\) ms
- \(R_0 \gtrsim 1\)
  \(\rightarrow\) \(\alpha - \Omega\) dynamo
- exponential growth of field
- LAEW generated: \(\vec{S} = \frac{\dot{\Omega}}{2}(\vec{E} \times \vec{B})\)
- envelope accelerated and heated by waves

When the rotating magnetized neutron star first forms there is likely to be linear amplification of the magnetic field and the creation of a matter-dominated jet, perhaps catalyzed by MHD effects, up the rotation axis. The energy of the proto-neutron star is sufficient to power a significant matter jet, but unlikely to generate a strong \(\gamma\)-ray burst. The matter jet could generate a smaller \(\gamma\)-ray burst as seems to be associated with SN 1998bw and GRB 980425 by the Colgate mechanism as it emerges and drives a shock down the stellar density gradient in the absence of
a hydrogen envelope, e.g., in a Type Ib/c supernova. As the neutron star cools, contracts, and speeds up, two significant things happen. One is that the rotational energy increases. The energy becomes significantly larger than required to produce a supernova and sufficient, in principle, to drive a cosmic \( \gamma \)-ray burst if the collimation is tight enough and losses are small enough. In addition, the light cylinder contracts significantly, so that a stationary dipole field cannot form and the emission of strong LAEW occurs. Tight collimation of the original matter jet and of the subsequent flow of LAEW in a radiation-dominated jet is expected.

The LAEW will propagate as intense low frequency, long wavelength radiation. They may be isotropized to act as a relativistic fluid, but not thermalized since they have a frequency much less than the plasma frequency. The LAEW “bubble” could be strongly Rayleigh-Taylor unstable, but still may propagate selectively with small opening angle up the rotation axis as an LAEW jet. Alternatively, the impulsive production of LAEW could render the stellar matter nearly irrelevant as a confining medium. If a LAEW jet forms, it can drive shocks which may selectively propagate down the axis of the initial matter jet or around the perimeter of the matter jet. The shocks associated with the LAEW jet could generate \( \gamma \)-rays by the Colgate mechanism as they propagate down the density gradient at the tip of the jet or there could be bulk acceleration of protons to above the pion production threshold. The protons could produce copious pions upon collision with the surrounding wind, thus triggering a cascade of high energy \( \gamma \)-rays, pairs, and lower-energy \( \gamma \)-rays in an observable \( \gamma \)-ray burst. Yet another alternative is that the LAEW could eventually propagate into such a low density environment that they directly induce pair cascade (see also Thompson & Madau 1999).

There are a number of reasons why the processes we have outlined may not be as effective as we have assumed. One is that the rotation of the neutron star may prevent contraction to the high densities and rotation rates on the timescales we have assumed (Fryer & Heger 1999). This could affect the convection in the neutron star and hence the generation of the magnetic field by the \( \alpha - \Omega \) dynamo mechanism. This is a complex issue, of course, since the presence of the magnetic field will lead to energy loss and conditions of greater contraction, as we have invoked here. Another complication we have ignored is that the mean dipole field that forms may have its axis tilted with respect to the spin axis. This may not be a critical factor, since the subsequent dynamics may be dominated by the density distribution surrounding the neutron star that is predominantly set by the angular momentum, not the magnetic field. The question of what fraction of the pulsar energy goes to drive quasi-spherical expansion and what fraction propagates as co-linear LAEW clearly requires greater study. We also noted that if the LAEW are thermalized in the stellar core the result will be the copious production of electron/positron pairs. At first, such a pair cloud will behave like a relativistic fluid so there may be little difference in the dynamics. As Rayleigh-Taylor instabilities ensue, the pairs might get mixed with ordinary matter and the positrons annihilate. Even if the pair bubble escapes up the axis, as we envisage for non-thermalized LAEW, it will expand as it propagates, especially after breaking through the stellar surface. Much of the thermal energy could then be lost to kinetic energy by adiabatic
expansion. If there is no “working surface” with which this pair cloud could collide, then much of the energy could be lost. On the other hand, the sort of wind expected (equation 58) would give a stopping length (equation 59) that will easily stop the pairs. The issue would then be the efficiency of conversion of their kinetic and rest mass energy into $\gamma$-rays.

There is an interesting question of the opposite sort concerning the possibility that the production of large energy in LAEW could be too effective. For instance, if an LAEW jet could propagate through the naked core of a Type Ic, but would get stopped and dissipated in a red supergiant, then we might find that the explosion energy of Type II is systematically higher than that of Type Ic. The fact that there is no clear evidence for this may suggest that the production of $10^{52}$ ergs in LAEW is not a common occurrence in core collapse, but this question requires further study.

The question of whether or not pulsar spin-down will produce a $\gamma$-ray burst depends on such factors as the initial rotation rate, the strength of the dipole field that evolves, the tilt of such a field compared to the spin axis, and the density of the progenitor wind. Issues of uncertain physics aside, it is clear that this mechanism might not be robust in the production of $\gamma$-ray bursts, but might produce $\gamma$-ray bursts of varying strength depending on natural variation in the circumstances of a given collapse event.

Any $\gamma$-rays emitted by any of these processes are likely to be strongly collimated. The luminosity of the emitted radiation will depend on the geometry of that emission. We have noted here that the energy produced by the spin-down of the pulsar could emerge from the stellar surface along the axis of a low-density matter jet, or in an annulus surrounding a high density jet. Either of these cases will give a Lorentz factor that depends strongly on the aspect angle of the observer. Computation of the resulting luminosity is thus distinctly non-trivial.

Simple $\gamma$-ray burst models invoking collimation assume that the energy in a collimated burst scales simply as $\Delta\Omega/4\pi$ compared to that deduced in an isotropic geometry. This assumes that the collimated jet nevertheless expands as the section of a sphere with a cross section scaling as $R^2$. We note that if the relativistic flow is truly collimated, this may yet be an overestimate of the energy required to power a given observed $\gamma$-ray burst. The calculation of a jet emerging from a stellar core by Khokhlov et al. (1999) shows that the dynamical jet that precedes any possible $\gamma$-ray burst is nearly linearly collimated and does not expand in cross section as $R^2$. In the absence of an understanding of the dynamics of the subsequent flow of LAEW, we do not know the geometry of the relativistic flow, but if the cross sectional area grows less strongly than $R^2$, then the solid angle $\Delta\Omega/4\pi$ will be a decreasing function of distance. The energetics (and hence the derived rates of occurrence) of $\gamma$-ray bursts will depend not only on the fact, but the geometry of the collimation. If the cross section expands less rapidly than $R^2$, then substantially less energy may be required to produce a given observed $\gamma$-ray burst.

The observed nature of any $\gamma$-ray burst will depend on whether or not one is directly witnessing a strongly relativistic bulk flow. There is general agreement that the observed
afterglows of the cosmic $\gamma$-ray bursts represent relativistic blast waves. The question of whether
that is true or not for the primary $\gamma$-ray burst is still in contention with models based on internal
shocks in a “central machine,” in which the burst duration is the fundamental physical time scale
of the energy production process, vying against models invoking relativistic flows and external
shocks for which there is strong Lorentz contraction of the timescales between the production
mechanism and the observer. Fenimore & Ruiz (1999) have recently argued that a central machine
is favored (see also Heinz & Begelman 1999).

One of the implications of this uncertainty is the location of the $\gamma$-ray burst. If an external
blast wave is involved, then a $\gamma$-ray burst with a time scale in the observer frame of $t_{\text{obs}}$ has a
propagation distance of $\Gamma^2 c t_{\text{obs}}$. For a $\gamma$-ray burst of 10 s this distance is about $3 \times 10^{15}$ cm for
a Lorentz factor of $\Gamma \approx 100$. This is much larger than the radius of the hydrogen-deficient stellar
core we are considering here, $R_{\text{core}} \approx 10^{10}$ cm. It is not clear why the energy produced by shocks
and LAEW would be dumped at a radius as large as $10^{15}$ cm. Shocks should deposit their energy
as they emerge from the star, and the density falls below the Goldreich-Julian density at only
$10^{11}$ cm even for a relatively dense wind. If relativistic protons are generated by shocks or other
mechanisms, they could also plausibly be stopped near the stellar surface by a modest wind. Thus
for all the mechanisms we sketch here, the most logical site of the production of any $\gamma$-ray burst is
near the stellar surface. This implies that if this general process of pulsar spin-down from massive
star core collapse has any role in producing $\gamma$-ray bursts it will most plausibly serve as a “central
engine.” We note that in this case, the natural time scale for any $\gamma$-ray burst is about 5 to 10 sec,
the cooling, spin down time for the neutron star. On the other hand, the eruption of shocks and
LAEW from the stellar surface may occur in a region small compared to the stellar surface, so
considerably shorter timescales might be manifest for central burst peaks. If the emission from
the pulsar is prolonged, then one might also witness time scales associated with the processes of
production of the LAEW, for instance various instabilities, as well. The shortest time scales of
any substructure would be $\approx 10^{11}$ cm/$\Gamma^2 c$ or about 0.3 ms for $\Gamma = 100$.

The deposition of a large amount of energy at the stellar surface could, of course, result in
a subsequent relativistic blast wave and associated afterglow. The processes we are discussing
would produce a maximum “isotropic equivalent” energy of $4\pi E/\Delta\Omega \approx 10^{54}$ erg for $E_{\text{52}} \approx 1$
and $\Delta\Omega/4\pi \approx 0.01$. The external mass required to decelerate this energy, $E\Gamma^{-2} c^{-2}$, requires a
spherically-distributed mass of $\approx 5 \times 10^{-5} E_{\text{52}} M_\odot$ for $\Gamma = 100$. The mass in the wind out to a
radius $R$ is

$$M_{\text{wind}} = 3 \times 10^{-6} M_\odot \dot{M}^{-1} v_8^{-1} R_{15},$$

(60)

with $R$ in units of $10^{15}$ cm. Thus energy emitted by the processes we have outlined will not be
decelerated until a radius of about $10^{16}$ cm. Even ignoring issues of entrainment and asymmetries
in the wind density profile, this radius is sensitive to the energy of the burst, the degree of
collimation, the value of the Lorentz factor and the parameters of the wind.

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