SU(4) pure-gauge phase structure and string tensions*†

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We present numerical evidence that the SU(4) pure-gauge dynamics has a finite-temperature first-order phase transition. For a $6 \times 20^3$ lattice, this transition occurs at the inverse-square coupling of $8/g^2 \sim 10.79$. Below this and above the known bulk phase transition at $8/g^2 \sim 10.2$ is a confined phase in which we find two different string tensions, one between the fundamental $4$ and $4^*$ representations and the other between the self-dual diquark $6$ representations. The ratio of these two is about 1.5. The correlation in the adjoint representation suggests no string forms between adjoint charges.

There are renewed interests in SU($N_c$) pure Yang-Mills theory with large $N_c$:

1) Finite-temperature phase structure of quantum chromodynamics (QCD) would be easier to understand if the SU($N_c$) pure Yang-Mills system has a second order phase transition for $N_c \geq 4$ \cite{1}. With standard large-$N_c$ analysis where $N_c g^2$ is held fixed, the $Z(N_c)$ deconfinement transition occurs at $T_d \sim O(1)$, separating confining phase with free energy $F \sim O(1)$ and deconfining phase with $F \sim O(N_c^2)$. The deconfining temperature $T_d \sim O(1)$ is not effected if $N_f$ and $g^2 N_c$ are held fixed and $N_c \to \infty$. If the transition is first order, it is not effected either. So large $N_c$ is not a reasonable guide for $T \neq 0$ QCD phase structure with Columbia phase diagram \cite{2}, unless SU($N_c$) pure Yang-Mills dynamics has second order deconfining phase transition for all $N_c \geq 4$.

2) New developments in M/string theory \cite{3} predict such things as glueball spectrum at large $N_c$ and large $g^2$ or ratio between different string tensions for $N_c \geq 4$ \cite{4}.

3) The dimensionless ratio $T_d/\sqrt{\sigma}$ of the deconfining temperature $T_d$ and string tension $\sigma$ is expected to be independent of $N_c$ with a value $\sqrt{3/\pi(D-2)}$ with $D$ being the space-time dimensions \cite{5}.

Here we report the results of our numerical investigation on the order of deconfining phase transition and the ratio of string tensions for $N_c = 4$ \cite{6}. We use single-plaquette action defined in the fundamental $4$-representation of the SU(4) gauge group. Combinations of pseudo-heat bath and Metropolis and over-relaxation algorithms are used in updating $4$, $6$ and $8 \times 8^3$, $12^3$, $16^3$ or $20^3$ lattices. Various workstations are used for the numerical calculations, while migration to the RIKEN BNL QCDSF mother boards is planned. We look at the following observables: plaquette, Polyakov loops, $L(\vec{x}) = (1/N_c) \text{tr} \prod_{t=1}^{T} U(\vec{x}, t)$, in $4$ (fundamental), $6$ (antisymmetric diquark), $10$ (symmetric diquark) and $15$ (adjoint) representations, deconfinement fraction, and Polyakov loop correlation $\langle L(\vec{0}) L(\vec{r}^*) \rangle \sim r^{-1} \exp(-F(r)/T) \sim \exp(-L_t \sigma r - \ln r)$ in $4$, $6$, $10$ and $15$ representations.

This SU(4) pure Yang-Mills system is known to have a bulk phase transition near $\beta = 8/g^2 \sim 10.2$, separating two confining phases \cite{7}; across this transition the plaquette jumps discontinuously but the average Polyakov line in the fundamental $4$ representation remains zero on both sides. However if the lattice extent in temperature direction $L_t$ is too small this bulk transition drives a first-order finite-temperature deconfining

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Figure 1. Time histories of the fundamental Polyakov loop magnitude and argument (in units of $\pi$) from a $6 \times 20^3$ lattice at $\beta = 10.79$ (above) and magnitude histogram (below). Confined and deconfined phases coexist at this temperature suggesting a first-order deconfining phase transition.

As is shown in Figure 1, on a $6 \times 20^3$ lattice we confirmed coexistence of confined and deconfined phases at temperature $\beta = 10.79$. This strongly suggests a first-order deconfining phase transition. Work in progress confirms this phase coexistence as we extend the simulation from the present 3500 evolution steps (1 evolution = 5 heat bath + 1 over relaxation steps) to 20000 steps. We plan further study with finite-size scaling.

String tensions in SU($N_c$) pure Yang-Mills system is classified by its center Z($N_c$) $N_c$-ality. With $N_c = 4$, the fundamental ($4$) charge has 4-ality $k = 1$, the two diquark ($6$ and $10$) charges $k = 2$, and adjoint ($15$) $k = 0$. The string tensions between these charges and their anticharges are predicted to behave as $\sigma_k \propto \min\{k, N_c - k\}$ by a standard strong-coupling analysis, $k(N_c - k)$ by another strong coupling analysis [4], and $\sin(k\pi/N_c)$ by a SUSY strong coupling analysis [4]. Generally the ratio $\sigma_k/\sigma_1$ should fall in the interval $1 \leq \sigma_k/\sigma_1 \leq 2$ [9]. Note also that $N_c = 4$ is the first example with different string tensions: in SU(3) pure Yang-Mills system the fundamental ($3$) and the symmetric diquark ($6$) tensions are the same [10].

In our numerical calculation on a $6 \times 16^3$ lattice at $\beta = 10.70$ (see Figure 2): we find a clear difference between 4- and 6-string tensions extracted from 4- and 6-Polyakov loop correlations. From fitting these data we have $\sigma_4 = 0.068(4)$ and $\sigma_6 = 0.108(17)$. At a stronger coupling of $\beta = 10.65$
Figure 3. Polyakov line correlation in 4 (+), 6 (×) and 15 (∗) representations on a 8 × 123 lattice at β = 10.85. The adjoint (15) signals now suggest there is no string for this representation. From the slopes of the former two lower-dimensional representations we confirm different string tensions for them, and by comparison there is no tension seen in the adjoint representation.

For thermodynamics of SU(4) pure-gauge theory we confirmed that the bulk transition and T ≠ 0 phase change are separated on L t ≥ 6 lattices, the bulk transition is at β t ~ 10.2 and the finite-temperature 1st-order phase transition is at β d ~ 10.79 (L t = 6) and ≥ 10.9 (L t = 8), and easier to establish than weakly first-order SU(3). For string tensions, at L t = 6 we find signals for different string tensions in fundamental 4 and antisymmetric 6 representations, but no signal yet for symmetric diquark 10 and adjoint 15 representations. These strings satisfy a relation, 1 < σ6/σ4 < 2, as they should, and the ratio does not show any strong temperature dependence. Combining these findings for thermodynamics and string tensions at L t = 6, we find an inequality: T d/√σ4(T = 0) < T d/√σ4(T ~ T d) ~ 0.64 < 1/(2(3/π)(D − 2)), just like in SU(2) and SU(3) pure-gauge results. At a weaker coupling of β = 10.85 using a L t = 8 lattice we now have rather good evidence that there exists no string in the adjoint (15) representation. We plan further investigation on larger and finer lattices, probably using smaller partitions of the QCDSP parallel supercomputer at RIKEN BNL Research Center.

REFERENCES