Interaction of Stored Ion Beams with the Residual Gas

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Abstract
Different kinds of interaction between ion beams and the residual gas at energies from 0.1 to 1000 GeV/u are discussed in this contribution. Especially in the case of highly-charged, heavy ions, stored beams are affected predominantly by inelastic atomic processes and by scattering at screened nuclear potentials of the residual gas atoms. Significant results are loss of beam particles by electron capture or stripping, growth of beam emittances, loss of mean beam energy, and production of residual gas ions. The importance of beam-gas interaction to both the design and operation of heavy ion storage rings is demonstrated by a few experimental results.

1. Introduction
This lecture would be superfluous if the residual gas density in ion storage rings could be reduced easily to arbitrarily low values. However, though gas pressures below $10^{-13}$ mbar are routinely attained in small systems, vacuum requirements in large facilities have to be optimized with respect to costs, tolerable technical complication, and required beam life times. Design pressures in planned or recently completed storage rings range from $10^{-9}$ to below $10^{-11}$ mbar. Future progress in ultrahigh vacuum (UHV) technique may make it possible to improve on these figures.

Both the nature and strength of the interaction between ion beams and residual gas molecules in storage rings depend strongly on the atomic number $Z_i$, charge state $q_i$ and specific kinetic energy $T_i$ of stored ions – we will not consider interactions between electron beams and residual gas. Relevant processes for proton beams at highly relativistic energies are elastic and inelastic nuclear scattering and nuclear reactions with total cross sections in the order of 1 barn ($10^{-24}$ cm$^2$). With 30 GeV–proton beams in the Intersecting Storage Rings (ISR) at CERN, where residual gas pressures below $10^{-11}$ mbar [1] had been achieved, typical storage times in the order of days were observed. Since interest in low- and medium–energy storage rings for heavy ions has grown in the past decade, attention has been focused more and more on the atomic interaction between ions and residual gas atoms. Atomic cross sections exceed the nuclear ones by many orders of magnitude, especially those for highly charged heavy ions at energies far below 1 GeV/u.

Relatively short beam lifetimes between 1 s and 100 s have been measured for different ion beams in the 1 MeV/u–range at pressures around $10^{-10}$ mbar in the Test Storage Ring (TSR) in Heidelberg [2]. At energies in the 100 MeV/u range and under comparable UHV conditions in the ESR at GSI Darmstadt [3], storage times of many hours are observed for fully stripped Ne–, Ar– and Kr ions as well as for H-like Bi$^{+62}$– and Au$^{+78}$ ions.
Relevant processes with respect to lifetime times of heavy ion beams at energies between 0.1 MeV/u and 1 GeV/u are:

*Electron capture:*  \( P^{+q} + T \rightarrow \tilde{P}^{+(q-n)} + \tilde{T}^{+(n+m)} + me^- \)

The equation describes the transition of \( n \) electrons from the neutral target atom \( T \) to bound states of the projectile ion \( P^{+q} \) with initial charge state \( +q \). In addition, the target particle may loose \( m \) electrons to unbound states and both the projectile and the target ion may be left in excited states (marked by tilded symbols). In most practical cases, capture of a single electron \((n = 1)\) is by far the most probable capture process. It may take place without emission of radiation (direct capture) or with simultaneous emission of photons (radiative electron capture, REC). The latter dominates at high energies, where the binding energies of the target electrons are negligible compared to their kinetic energy in the fast moving system of the projectile.

*Electron loss:*  \( P^{+q} + T \rightarrow \tilde{P}^{+q+n} + \tilde{T}^{+m} + (n + m)e^- \)

For a projectile with charge state \( q < Z_i \), there is certain ion velocity \( v_i \) close to the orbit velocity of the outermost electron of the ion, where capture and loss cross sections are balancing each other. Electron capture dominates at velocities below this equilibrium point, whereas electron loss becomes more and more dominating at higher \( v_i \).

*Angular scattering:*  \( \tilde{p}_i (\text{init}) \approx \tilde{p}_f (\text{final}) + \tilde{p}_u \)

Angular scattering of the projectile is described here by considering the conservation of the total momentum of the colliding system. For small scattering angles the absolute values of initial and final momenta of the projectile are nearly identical. \( \tilde{p}_f (\text{final}) \) contains a transverse component, which is approximately equal to the total momentum \( \tilde{p}_u \) of the backscattered target particle, but with opposite sign. This means that the majority of target recoils is observed in the laboratory frame at forward angles close to 90 degree. A large number of small-angle scattering processes (*multiple scattering*) in stored beams leads to transverse emittance growth.

*Gas ionization:*  \( P^{+q} + T \rightarrow P^{+q} + \tilde{T}^{+m} + me^- \)

Especially in the case of high \( q \) the cross sections for secondary ion production are much larger than those for electron capture and loss. This means that the major fraction of processes takes place in collisions with relatively large impact parameters, i.e. in distant collisions. The energy for ionization and excitation of the target atoms is taken from the kinetic energy of the projectile \( T_i \). Therefore, after a large number of ionizing collisions, a slight reduction of \( T_i \) is observed.

Information about important experimental and theoretical work on atomic ion–gas interaction is given in the following – with emphasis on heavy ions due to the strong dependence of interaction rates on nuclear or atomic charge states.

2. Electron capture and loss

Theoretical and experimental investigations of charge–changing processes are reported in a large number of publications. The early authors Bohr [4], Bell [5] and Bohr & Lindhard [6] published theoretical cross sections for electron capture and electron loss (stripping) of fast ions in gaseous media. Their models were based on Fermi–Thomas distributions for the electronic states of both the target atom and the projectile ion. More and more results
from experiments with accelerated beams [7] – [20]. are reported in later publications. A rigid quantum-mechanical treatment of charge exchange phenomena in fast collisions between two multi-electron systems turned out to be very difficult if not impossible. Therefore, many attempts were made to describe and extrapolate measured cross section data by means of empirical formulae, often making use of the theoretical work of N. Bohr.

Experimental cross sections for electron capture \( \sigma_c \) and electron loss \( \sigma_l \) depend rather strongly on the ion parameters \( (Z_i, q \text{ and } \beta_i = v_i/c \text{ or } T_i) \) and the atomic number \( Z_i \) of the target. The major fraction of data for highly charged heavy ions was measured at low specific energies \( 10 \text{ keV/u} \leq T_i \leq 10 \text{ MeV/u} \). For application in storage ring design, cross section estimates over wide ranges of \( T_i \) and for a large number of different \( Z_i \) and \( q \) are necessary. A few facts deduced from experimental data may be helpful to extrapolate cross sections far from investigated regions.

The dependence of \( \sigma_c(q) \) and \( \sigma_l(q) \) on \( q \) and \( \beta_i \) is very different for the two charge state regimes \( q < \bar{q} \) and \( q \geq \bar{q} \), indicated in the following by \( q_\prec \) and \( q_\succ \), respectively. \( \bar{q} \) is the equilibrium charge state, which would be measured after a large number of collisions in the given target gas (see below). For \( q_\succ \), we find that \( \sigma_c(q) \) is approximately \( \propto q^2 \), nearly independent of \( Z_i \). For \( q_\prec \) the q-dependence of \( \sigma_c(q) \) is stronger and there is also a strong dependence on \( Z_i \). At the equilibrium \( q = \bar{q} \) we expect that \( \sigma_c \) and \( \sigma_l \) are balancing each other. Hence, for a given energy \( T_i \), we would have to estimate first \( \bar{q} \) and secondly the most confident of the cross sections \( \sigma_c(\bar{q}) \) or \( \sigma_l(\bar{q}) \). Capture and loss estimates for arbitrary \( q \) - - not too far from \( \bar{q} \) - - require then only the dependence on \( q \) and not that on the full set of relevant parameters \( Z_i, q, \beta_i \) and \( T_i \). Supposing that we have a good estimate for the capture cross section at \( q = \bar{q} \), we may scale capture and loss cross section for other charge states by

\[
\sigma_c(q) \approx \sigma_c(\bar{q}) \left( \frac{q}{\bar{q}} \right)^a \quad \text{and} \quad \sigma_l(q) \approx \sigma_c(\bar{q}) \left( \frac{q}{\bar{q}} \right)^b
\]

\( a \approx 4, b \approx -2.3 \) for \( q_\prec \) and \( a \approx 2, b \approx -4 \) for \( q_\succ \)

where \( a \) and \( b \) are fitted to experimental cross sections for heavy ions \( (Z_i \geq 18) \) in heavy gases at different energies up to \( 10 \text{ MeV/u} \).

Several formulae (see [8] and references therein) for the equilibrium charge state \( \bar{q} \) in gaseous media have been published. For gaseous media the formula

\[
\bar{q} \approx Z_i \left( 1 - \exp \left( -\frac{\beta_i}{\alpha_i Z_i^{0.57}} \right) \right)
\]

is well suited, which, for \( \beta_i \to 0 \), is compatible with the well known Bohr criterion [4]. \( \alpha_i = 1/137 \) the fine structure constant. The criterion expresses that \( \bar{q} \) is approximately the charge state for which the orbit velocity of the outermost electron is equal to the ion velocity. The criterion may be applied also indirectly by \( I(Z_i, \bar{q}) \approx (E_{0,e}/E_0)T_i \). \( I(Z_i, \bar{q}) \) in eV is the binding energy of the outermost electron of the ion in the (fictive) charge state \( \bar{q} < Z_i \), \( E_{0,e} = 5.11 \times 10^5 \text{ eV} \) is the electron rest energy and \( E_0 = 9.315 \times 10^8 \text{ eV/u} \) (if \( T_i \) in eV/u) the rest energy of the atomic mass unit. \( \bar{q} \) is then easily determined by interpolating quantum-mechanically calculated binding energies published in data tables [21] for all \( q \) of all \( Z_i \).
None of the following selected cross section formulae includes multiple capture or loss. Multiple loss is found mainly with electron-rich ions \((Z_i - q \gg 1)\) colliding with heavy target atoms at energies above the equilibrium point \((q_<)\), and the cross section may reach the same order of magnitude as that for single loss. Multiple capture mainly takes place with highly-charged ions \((q - q \gg 1)\) in collisions with heavy target atoms at lower energies. Its maximum contribution to the total capture cross section is in the order of 10%.

Electron capture

Bohr and Lindhard (1954) [6]:

\[
\sigma_c(q) \approx \pi a_0^2 Z_i^{1/3} q^2 \left( \frac{\beta_i}{\alpha_s} \right)^{-3}
\]  

(3)

The formula is a modification [8] of the author’s original formula, in which an effective charge state \(Z_{i,\text{eff}}\) of the projectile ion had been used instead of the charge state \(q\). It should deliver reasonable results in the velocity range \(\alpha_s \leq \beta_i \leq \alpha_s Z_i^{2/3}\) and for heavy target gases \((Z_t \geq 7)\). A cursory estimate for the capture of very loosely bound electrons from light target atoms (e.g. hydrogen) to highly charged, fast ions is

\[
\sigma_c(q) \propto q^3 \left( \frac{\beta_i}{\alpha_s} \right)^{-7}
\]  

(4)

In comparison with (3) the dependence on \(q\) as well as on \(\beta_i\) has changed strongly. Experimental capture cross sections for \(q_<\) and high \(\beta_i\) seem to follow better (4), whereas (3) is applicable for \(q_>\) and low \(\beta_i\).

Nikolaev et al. (1965) [9]:

\(\sigma_c\) for arbitrary ions is estimated by means of experimental \(\sigma_c(p)\) for protons at the same
Figure 2: Comparison of two scaling rules for $\sigma_e$
Solid curves are calculated with (6), dashed curves are according to (5). Though the agreement in the MeV/u-range is surprisingly good, the curves diverge strongly towards higher energies, where (5) seems to be in better agreement with a few experimental points [12].

Figure 3: Effective total charge-changing cross sections for uranium ions $\sigma_e(q)$ according to (5) and $\sigma_t(q)$ for $q < 92$ extrapolated by (1). The assumed gas composition is typical for a pressure of $10^{-10}$ mbar. Nitrogen stands for all other heavy fractions (e.g. CH$_4$, H$_2$O and CO).
energy $T_i$ and for the same target medium $Z_i$:

$$\sigma_e(q) \approx 3.21 \times 10^5 q^2 \beta_i^2 \sigma_e(p)$$

(5)

For $\sigma_e(p)$, experimental data up to 20 MeV may be used [10], and extrapolation to higher energies may be done by $\sigma_e(p) \approx \sigma_0 T_i^{-4.25}$. With $T_i$ in MeV/u, $\sigma_0 \approx 5 \times 10^{-20}$ cm$^2$/atom for nitrogen and $\approx 7 \times 10^{-21}$ cm$^2$/atom for hydrogen.

Schlachter et al. (1983) [11]:

$$\sigma_e(q) \approx \frac{1.1 \times 10^{-8} q^{0.5}}{Z_i^{1.8} E_i^{4.8}} \left(1 - \exp(-0.037 E_i^{2.2})\right) \left(1 - \exp(-2.44 \times 10^{-5} E_i^{2.6})\right)$$

(6)

In this purely empirical scaling rule the "reduced" ion energy is $E_i = T_i Z_i^{-1.25} q^{-0.7}$ with $T_i$ in keV/u. The strong dependence on both $q$ and $T_i$ is typical for $q < i$, i.e. the formula should not be used to estimate capture cross sections for high charge states at low energies.

H. Gould et al. (1984) [12]:

The cross section for radiative capture of electrons by highly-charged ions at high energy is correlated to the reverse process of photoionization. The authors found reasonable agreement of experimental capture cross section for U$^{+91}$ and U$^{+91}$ at 440 MeV/u and 960 MeV/u in copper with

$$\sigma_{REC}(q) \approx Z_i \sum_{n=n_0}^{\infty} \sigma_{\phi,n} F_n \frac{\gamma_i - 1 + \frac{I_n}{E_{0,e}}}{(\gamma_i + 2 \frac{I_n}{E_{0,e}})^2 - 1}$$

(7)

where $n_0$ is the main quantum number for the electronic state in the ion, $\sigma_{\phi,n}$ the cross section for photoionization of the $n$-th shell, $F_n$ the unpopulated fraction of the $n$-th shell, and $I_n$ the average binding energy of electrons in the $n$-th shell in eV. Numerical values for $I_n$ and $\sigma_{\phi,n}$ are published in data tables.

Electron loss

N. Bohr (1948) [4]:

For the electron loss cross section of heavy ions in light targets ($Z_i > Z_t$) Bohr found

$$\sigma_l(q) \approx 4 \pi \alpha_0^2 \left( \frac{Z_i^2 + Z_t}{q^2} \right) \left( \frac{\beta_i}{\alpha_i} \right)^{-2} \approx 4 \pi \alpha_0^2 \left( Z_i^2 + Z_t \right) \left( \frac{\beta_i}{\alpha_i} \right)^{-2} \sum_{n=q}^{\infty} \frac{1}{B_n}$$

(8)

where $B_n$ are binding energies of the ion electrons in units of 13.6 eV. The charge state dependence in Bohr's original formula was modified by Dmitriev et al. [13] by introducing binding energies for the contributing electrons. Another modification of (8) was used by Gould et al. [12] to interprete their results with U$^{+90}$ and U$^{+91}$ at 440 and 960 MeV/u in Cu-targets:

$$\sigma_l(q) \approx 4 \pi \alpha_0^2 \left( Z_i^2 + Z_t \right) \left( \frac{\beta_i}{\alpha_i} \right)^{-2} \frac{f_k}{B_k} \left( \ln \frac{\gamma_i^2 - 1}{\alpha_i^2} - \ln(0.0048 B_k) \right)$$

(9)
Figure 4: Loss of uranium beams by charge-changing processes during acceleration
Effective total cross sections according to figure 3.

where \( f_k \) is proportional to the oscillator strength for the transition from K-shell to continuum. The authors applied \( f_k = 0.58 \) for \( U^{79} \) and \( f_k = 0.29 \) for \( U^{81} \).

For light ions in heavy targets \((Z_i \leq Z_t^{1/3})\) Bohr finds:

\[
\sigma_i(q) \approx \pi \alpha_o^2 Z_i^{1/2} \frac{\beta_i}{\alpha_s} \sum_{n=q}^{Z_i-1} \sqrt{B_n^{-1}}
\]

(Equations (8) and (10) are obviously applicable for charge states \( q \leq \bar{q} \) or for high energy. In these cases the loss cross sections expectedly decrease with increasing \( \beta_i \). The contribution of tightly bound electrons \((n \geq \bar{q})\) is negligible.

Beam loss during storage and acceleration

The actual loss rate \( R_{cc} \) in a stored beam due to charge-changing collisions with residual gas atoms is determined by the product \( \sigma_{tot} \rho_i \beta_c \), where \( \sigma_{tot} \) is the sum of capture and loss cross sections and \( \rho_i \) the mean spatial density of residual gas atoms (not molecules!) in the beam pipe. The inverse of \( R_{cc} \) is the beam life time with respect to charge-changing. The effective cross section \( \sigma_{eff} = \sum_{k=1}^{n} \sigma_{tot,k} F_k \) has to be applied for \( n \) different constituents of the residual gas, where the \( k \)-th gas component with \( \sigma_{tot,k} \) has the fraction \( F_k \) (see figure 3).

During acceleration or deceleration in synchrotron mode, one has to apply the energy dependence of cross sections. Though not following a clean power function, it can be approximated within small energy intervals by a power of the specific energy \( T_i \). Doing so one gets the transmission \( D_{cc}(n) \) for the \( n \)-th interval of acceleration by the integral

\[
\ln D_{cc}(n) = - \frac{\rho A \sigma_{tot,n-1}}{qRB_Ti_{n-1}^c} \int_{T_i,n-1}^{T_i,n} T^c dT
\]

\[
= - \frac{\rho A \sigma_{tot,n-1}}{qRB_Ti_{n-1}^c} \left\{ \begin{array}{ll}
(ln T_i,n - \ln T_i,n-1) & \text{if } c = -1 \\
(c+1)^{-1}(T_i,n-T_i,n-1) & \text{otherwise}
\end{array} \right.
\]
The slope of the bending field $\dot{B}$ is assumed to be constant within the integration interval. $A_i$ is the mass number of the beam ion, $q$ its charge state, and $R$ the bending radius in the main dipole magnets. The total transmission $D_{ee}$ is the product of all $D_{ee}(\pi)$ in the accelerating cycle and the relative beam loss is $1 - D_{ee}$ (see figure 4).

Results from ion storage rings

Beam lifetimes have been investigated systematically at the TSR with partially- and fully-stripped ions with $1 \leq Z_i \leq 8$ at energies up to 12 MeV/u, for protons at 21 MeV. Stripping processes were found to limit predominantly the lifetimes of partially stripped ions. In the case of fully stripped ions the beam life is determined by multiple scattering (see next section) rather than by electron capture [2].

Rates of charge-changing processes in a stored heavy ion beam are easily observed by means of longitudinal Schottky scans, if the secondary charge states are produced in sections of the orbit with vanishing or small momentum dispersion and, if the momentum acceptance of the ring is sufficiently large to store these charge states. This so-called multicharge operation has been realized in the ESR, where two charge states of Kr--ions and three charge states of Au-- and Bi--ions have been stored and cooled simultaneously. Figure 5 shows a spectrum containing three bands from three different charge states +82, +81, and +80 of Bi--ions stored and cooled simultaneously at 230 MeV/u. The cooler aligns all beam components to exactly the same mean velocity. Therefore the frequency difference is caused by different orbit lengths: $\Delta f/f = \gamma^{-2} \Delta q/q$, where $\gamma$ is the transition point of the lattice. The lower charge states are populated by successive REC in the electron cooler. At pressures below $10^{-10}$ mbar and at energies in the 100 MeV/u-range the REC rates are orders of magnitude higher than charge-changing rates caused by interaction with residual gas atoms. However, in the spectrum of figure 6, we see also a
weak Schottky band from the fully stripped Bi\(^{83}\) produced by stripping collisions with residual gas molecules at a pressure slightly below \(1 \times 10^{-16}\) mbar. Approximately 30\% of the stripped ions are produced in orbit sectors with small dispersion function.

Electron capture and loss cross sections will be investigated systematically at the ESR by means of the internal gas jet target, where different gases with variable target density can be studied. Instead of being stored the secondary charge states will be counted event by event by means of position sensitive detectors. These multi-wire type detectors inside a stainless steel pocket with a 100\(\mu\)m thick window are installed at orbit positions, where the dispersion function has amplitudes of about 6 m, and can be moved to desired radial positions inside the beam pipe aperture.

3. Multiple Scattering

Reviewing some basic theoretical aspects of the Coulomb scattering theory, we will follow mainly the classical treatment by Jackson [22], though there are other publications [23] treating the problem of multiple scattering in more detail.

The momentum transfer to a projectile with atomic number \(Z_t\), mass number \(A_t\) and velocity \(\beta_t c\) after the collision with an atom with corresponding \(Z_t, A_t,\) and \(\beta_t = 0\) is determined by the time integral

\[
\Delta p_\perp = Z_t c \int_{-\infty}^{\infty} E_\perp(t) dt
\]  

(12)

where the time dependent \(E_\perp(t)\) is the electric field component perpendicular to the direction of \(\beta_0\) as seen by a target particle at rest. If the impact parameter \(b\) of the collision is not too small, and the mass of the target particle sufficiently high, the corresponding integral of the longitudinal component \(E_{0\parallel}(t)\) vanishes. In this case, the momenta transferred to both the projectile and the target are directed transversely to the motion of the
Figure 7: Differential scattering cross section vs. scattering angle $\theta$

The explanation of $\theta_{\text{min}}$ and $\theta_{\text{max}}$ is given in the text.

projectile. After transformation of the co-moving Coulomb field to the laboratory system we get

$$E_\perp(t) = \frac{eZ_i \gamma_i b}{(b^2 + \gamma_i^2 \beta_i^2 c^2 t^2)^{3/2}}$$

(13)

Now (12) is easily integrated and we get for a small deflection of the incident particle

$$\theta \approx 2 \tan \frac{\theta}{2} \approx \frac{2Z_i Z_u r_u}{A_i \beta_i^2 \gamma_i b} = \frac{b_R}{b}$$

(14)

where $r_u = e^2 / E_0 = 1.546 \times 10^{-18}$ m$^2$ is the classical radius of a unit mass particle and $b_R$ may be interpreted as the impact parameter leading to the deflection $\theta \approx 1$. We get then the differential small angle Rutherford cross section (see figure 7):

$$\frac{d\sigma}{d\Omega} \approx \frac{b}{\sin \theta} \left| \frac{db}{d\theta} \right| = \frac{b_R^2}{\theta^4}$$

(15)

Deviations from the $\theta^{-4}$-dependence given by (15) are due to the screening of the nuclear Coulomb fields by bound electrons at large $b$ (small $\theta$) and caused by the finite sizes of colliding nuclei at small $b$ (large $\theta$):

$$\theta_{\text{min}} \approx 2.86 \times 10^{-6} \frac{Z_t^{1/3}}{A_i \beta_i \gamma_i} \quad \text{and} \quad \theta_{\text{max}} \approx 0.119 \frac{1}{Z_t^{1/3} A_i \beta_i \gamma_i}$$

(16)

Interesting quantities for application are mean square scattering angles, $\langle \theta^2 \rangle$, after a single scattering process, and $\langle \Theta^2 \rangle$ after a large number of scattering processes. The first is given by

$$\langle \theta^2 \rangle = \frac{\int \theta^2 \frac{d\sigma}{d\Omega} d\Omega}{\int \frac{d\sigma}{d\Omega} d\Omega} = 2 \theta_{\text{min}}^2 \ln(\frac{\theta_{\text{max}}}{\theta_{\text{min}}}) = 4 \theta_{\text{min}}^2 \ln(\frac{204}{Z_t^{1/3}})$$

(17)
Successive scattering processes are independent events and will produce approximately Gaussian angular distributions. After a large number $n$ of scattering processes, the mean square width of an initial Gaussian is increased by $\Delta(\Theta^2) = n(\Theta^2)$. Introducing the spatial density of target atoms $\rho_t$, the integral single-scattering cross section $\sigma \approx \pi b_R^2/\theta_{min}^2$ and the time of flight through the scattering medium $\Delta t$ we get $n = \rho_t \sigma \beta_t c \Delta t$ and

$$\frac{d\langle \Theta^2 \rangle}{dt} \approx 4\pi \rho_t \beta_t c b_R^2 \ln(204 Z_t^{-1/3})$$

(18)

The projection of the mean square scattering angle on a convenient plane, which is useful for our application, is given by the relation $\langle \vartheta^2 \rangle = \langle \Theta^2 \rangle/2$. After replacing $b_R$ according to (14) and setting numbers for all constants we get

$$\frac{d\langle \vartheta^2 \rangle}{dt} \approx k P_t m_t Z_t^2 \ln(204 Z_t^{-1/3}) \left( \frac{Z_t}{A_t} \right)^2 \frac{1}{\beta_t^3 \gamma_t^2}$$

(19)

where $k \approx 4.8 \times 10^{-4}$ rad mbar$^{-1}$ s$^{-1}$, $m_t$ is the number of atoms per molecule, and $P_t$ in mbar the pressure of the gaseous target medium (at room temperature). The “target factors” for a few residual gas molecules typically found in UHV-systems are given in the following table, demonstrating that the fraction of heavy atoms should be as small as possible (see also [24]).

<table>
<thead>
<tr>
<th>Molecule</th>
<th>$m_t Z_t^2 \ln(204 Z_t^{-1/3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$</td>
<td>10.6</td>
</tr>
<tr>
<td>He</td>
<td>20.3</td>
</tr>
<tr>
<td>Ne</td>
<td>455</td>
</tr>
<tr>
<td>N$_2$</td>
<td>485</td>
</tr>
<tr>
<td>CO</td>
<td>466</td>
</tr>
<tr>
<td>O$_2$</td>
<td>592</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>762</td>
</tr>
<tr>
<td>Ar</td>
<td>1411</td>
</tr>
</tbody>
</table>

At given $T_t$ and residual gas conditions (pressure and composition), ions are scattered the less strongly the heavier the ion mass. The growth rate of the mean square angle in a beam of fully-stripped uranium ions is only 15 % of that for a proton beam. At the low energy end ($\gamma_t \approx 1$) the scattering rate is proportional to $T_t^{-3/2}$, whereas at relativistic energies ($\beta_t \approx 1$) the rate decreases proportional to $T_t^{-2}$.

Formula (19) makes it possible to calculate the influence of multiple scattering on beam parameters in one of the transverse phase planes of a storage ring (see e.g. [25]). If we start with a Gaussian for the transverse beam particle distribution — this is the easiest way — the r.m.s. emittance $\varepsilon$ in $\pi$ rad $m$ is given by

$$\varepsilon = \frac{4\beta_l(s) \langle \vartheta^2 \rangle(s)}{1 + \alpha_l^2(s)}$$

(20)

where $\beta_l(s)$ is the betatron amplitude function, $\alpha_l(s)$ the Twiss parameter and $\langle \vartheta^2 \rangle(s) = \sigma_\vartheta^2$ the square angular standard deviation of beam particles at the given orbit position $s$. 
The factor 4 is due to the relation $\hat{\vartheta} = 2\sigma_\vartheta$ commonly applied in the case of a Gaussian to define the half angular spread in the beam. The increase of the emittance due to multiple scattering, taking place at all positions of the orbit circumference, is

$$\frac{dc}{dt} \approx 4\langle \beta_\perp \rangle (d\langle \vartheta^2 \rangle/dt)$$

where $\langle \beta_\perp \rangle$ is the mean value of $\beta_\perp$ for given ring optics, $d\langle \vartheta^2 \rangle/dt$ the quantity according to (19), and $(1 + \sigma_\perp^2) \approx 1$. Integration of (21) delivers time intervals, which are of interest with respect to useful beam storage times:

Arbitrary limit for emittance growth:

$$\Delta t \approx \frac{\varepsilon_{\text{final}} - \varepsilon_{\text{init}}}{4\langle \beta_\perp \rangle (d\langle \vartheta^2 \rangle/dt)}$$

Doubling of the beam size:

$$(\Delta t)_2 \approx \frac{3\varepsilon_{\text{init}}}{4\langle \beta_\perp \rangle (d\langle \vartheta^2 \rangle/dt)}$$

Beginning of beam loss:

$$(\Delta t)_{\text{loss}} \approx \frac{A_\perp - \varepsilon_{\text{init}}}{4\langle \beta_\perp \rangle (d\langle \vartheta^2 \rangle/dt)}$$

where $A_\perp$ is the ring acceptance in the considered transverse plane.

Finally, we should look briefly at the angular distribution of beam particles if single (wide angle) scattering is included. The distribution consists of a central Gaussian generated by multiple scattering and single scattering tails decreasing proportionally to $\vartheta^{-3}$. 
Introducing the relative projected angle $\alpha = \theta / \sqrt{\langle \theta^2 \rangle}$, the complete scattering distribution is described by the central part for multiple scattering

$$P_{MS}(\alpha)d\alpha = \frac{1}{\sqrt{\pi}} \exp(-\alpha^2)d\alpha$$  \hspace{1cm} (25)

and single scattering tails

$$P_{SS}(\alpha)d\alpha = \frac{1}{8 \ln(204Z_t^{-1/3}) \alpha^3}d\alpha$$  \hspace{1cm} (26)

In the region where $P_{MS} \approx P_{SS}$ (for nitrogen $\alpha \approx 2.4$), there is a smooth transition from small angle multiple scattering to wide angle single scattering, called plural scattering (see figure 8). The relative intensity in this region is in the order of $10^{-3}$, and the intensity fraction covered by plural and single scattering tails is below of 1%.

**Emittance measurements at storage rings**

Transverse emittances of stored beams can be determined – in principle – from measurements of the transverse beam transfer function at low harmonic numbers of the revolution frequency. However, this very sophisticated method requires precise knowledge about betatron amplitude functions, $\beta_\perp(s)$, chromaticities, $\xi_\perp$, the transition point, $\gamma_t$, and the momentum distribution in the beam. More direct information is available from devices measuring the transverse distribution of beam particles via residual gas ions produced by the circulating beam. These ions are accelerated to a channel plate detector with a position sensitive anode. Detectors of this type also deliver valuable information about
ion production rates as a function of ion species and energy (see next section). They are already installed in many storage rings and are in preparation also for the ESR in Darmstadt.

Non-destructive measurements of transverse particle distributions are also feasible by means of secondary charge states of stored, highly charged ions. For this purpose position sensitive detectors are located at orbit positions with large amplitude of the dispersion function \(D \approx 6 \text{ m}\), where the secondary charge state is well separated from the primary beam and can be detected easily without disturbing the main beam. The plot of figure 9 shows horizontal and vertical beam profiles of Bi\(^{80}\) produced from Bi\(^{81}\) by REC in the electron cooler. If we want to investigate emittance growth caused by multiple scattering, we have to switch off the electron cooler. In this case we have to deal with much smaller charge-changing rates caused by the residual gas atoms.

4. Momentum Loss and Gas Ionization

Data tables for the stopping of heavy ions were published by Northcliffe & Schilling (N&S). They scaled experimental results with lighter ions for all ion species, for different target materials, and for energies up to 10 MeV/u [27]. Though later experiments with very heavy ions [28] showed rather large deviations from the N&S data, we prefer — for simplicity — to apply the well known Bethe–Bloch formula for the electronic stopping power [26]:

\[
\frac{dE_i}{dx} = K_{BB} P_i m_t Z_i Z_i^2 \beta_i^{-2} \left\{ 11.1 - 0.9 \ln Z_i + \ln(\gamma_i^2 - 1) \right\}
\]

We deduce that the energy transferred to the electrons of a target atom increases with the square of the projectile charge \(Z_i\). For partially-stripped ions \(Z_i\) may be substituted by the atomic charge state \(q < Z_i\). With the constant \(K_{BB} = 0.0137 \text{ eV mbar}^{-1} \text{ m}^{-1}\), the residual gas pressure \(P_i\) in mbar at room temperature, the number of atoms per gas molecule \(m_t\), and \(E_i = A_i T_i\) in eV we obtain \(dE_i/dx\) in eV/m. The formula is valid if \(11.1 - 0.9 \ln Z_i > - \ln(\gamma_i^2 - 1)\). This is, for instance, the case at energies above 40 keV/u for nitrogen and above 100 keV/u for argon. It should be noted that, quite similar to charge changing and angular scattering, (27) has to be applied separately to different gas components by using corresponding partial pressures.

**Momentum loss rate**

For design and operation of ion storage rings it might be useful to calculate the rate of loss of mean momentum \(p_i\) of beam particles:

\[
\frac{d\ln p_i}{dt} = \frac{1}{p_i} \frac{dp_i}{dt} = \frac{1}{p_i} \frac{dE_i}{dx} = \frac{c}{A_i E_0 \beta_i \gamma_i} \frac{dE_i}{dx}
\]

where \(E_0 = m_e c^2 = 9.3148 \times 10^8 \text{ eV/u}\). Curves for \(d\ln p_i/dt\) for protons, Ne\(^{10}\) and U\(^{92}\) at \(10^{-10}\) mbar (90% \(H_2\), 10% \(N_2\)) are plotted vs. \(T_i\) in figure 10. The decrease of mean momentum should be observed especially during storage of coating heavy ion beams. It can be demonstrated easily by measuring either the frequency shift in longitudinal
Figure 10: Momentum loss rate of different ions vs. kinetic energy

The curves are calculated for a residual gas at $10^{-10}$ mbar containing 90% $H_2$ and 10% $N_2$. The slope change at energies above 1 GeV/u ($\gamma_i \approx 2$) is due to the increase of the stopping power $\propto 2 \ln \gamma_i$.

Schottky-spectra or the displacement of the beam position in the bending plane of the storage ring.

Ionization of the residual gas

For a qualitative comparison of the ionizing power of protons with that of highly charged heavy ions, we may express the stopping of beam particles in units of eV/s instead of eV/m:

$$\frac{dE_i}{dt} = \beta_i c \frac{dE_i}{dz} = p_i \beta_i c \frac{d\ln p_i}{dt} = A_i E_0 \beta_i^2 \gamma_i \frac{d\ln p_i}{dt}$$  \hspace{1cm} (29)

Numerical values for $dE_i/dt$ are deduced easily from momentum loss rates shown in figure 10, assuming identical vacuum conditions. The values listed in the following table are normalized to 13.6 eV, the ionization energy of the hydrogen atom in ground state. Thus, we get some rough feeling for the secondary ion production of different ion species. The numbers would be equal to the number of ions produced by a single beam ion per second, if excitation of residual gas atoms could be neglected. In fact, some fraction of $dE_i/dt$ has to be accounted for gas excitation.

<table>
<thead>
<tr>
<th>Ion energy in MeV/u</th>
<th>$dE_i/dt$ in Ry/s</th>
<th>$13.6$ eV/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>protons</td>
<td>Ne$^{+19}$</td>
</tr>
<tr>
<td>1</td>
<td>0.0029</td>
<td>0.29</td>
</tr>
<tr>
<td>10</td>
<td>0.0014</td>
<td>0.14</td>
</tr>
<tr>
<td>100</td>
<td>0.0006</td>
<td>0.06</td>
</tr>
<tr>
<td>1000</td>
<td>0.0004</td>
<td>0.04</td>
</tr>
</tbody>
</table>
More precise information about secondary ion production rates can be found in recent publications on experimental ionization cross sections of heavy ions (see [29, 30] and references therein). Measurements with ions up to uranium at energies from 1 MeV/u to a few hundred MeV/u have been reported. Ionization cross sections \( U^{+92} \) in \( N_2 \) are in the order of \( 1 \times 10^{-16} \) cm\(^2\)/molecule in the range from 100 MeV/u to 1 GeV/u. Some cross sections for single and double ionisation, and also for free electron production of fully stripped neon and uranium ions in \( H_2 \) are given in the following table. They have been calculated by means of a theory, which is in good agreement to some experimental data [31]. Assuming a clean \( H_2 \)-vacuum at \( 10^{-10} \) mbar we calculate a total \( H^+ \) production rate for \( U^{+92} \) at 1000 MeV of approximately 80 per second. Our rough estimate according to (29) gave much lower values.

### \( \text{Ne}^{+10} \rightarrow H_2 \)

<table>
<thead>
<tr>
<th>Ion energy in MeV/u</th>
<th>Ioniz. cross sections in ( 10^{-16} ) cm(^2)/molec.</th>
<th>( H^+ )</th>
<th>( 2H^+ )</th>
<th>free e(^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.2</td>
<td>5.4</td>
<td>22.0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.09</td>
<td>0.41</td>
<td>2.91</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.283</td>
<td>0.007</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.072</td>
</tr>
</tbody>
</table>

### \( \text{U}^{+92} \rightarrow H_2 \)

<table>
<thead>
<tr>
<th>Ion energy in MeV/u</th>
<th>Ioniz. cross sections in ( 10^{-16} ) cm(^2)/molec.</th>
<th>( H^+ )</th>
<th>( 2H^+ )</th>
<th>free e(^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>155</td>
<td>211</td>
<td>583</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>78.2</td>
<td>42.3</td>
<td>163</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>14.4</td>
<td>4.56</td>
<td>23.5</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>11.5</td>
</tr>
</tbody>
</table>

High rates of residual gas ions in heavy ion storage rings may enhance beam and vacuum instabilities. The so-called pressure bump effect had been observed with high proton currents [32, 33]. Whether or not critical situations are to be expected with heavy ion beams depends on the vacuum pressure, on the cleanliness of inner surfaces of beam pipes, and on the beam-wall potential difference. Estimates are easily done by scaling experimental data for proton beams in the ISR [24] to the heavy ion in question.

### Results from ion storage rings

A positive aspect of high secondary ion production rates is the application to non-destructive diagnosis. As mentioned above, the channel plate detector delivers – besides ion production rates – transverse distributions and mean position of beam particles. It should be noted that this type of detector is independent of a time structure in the beam and that event rates do not depend on ion beam temperatures. This is important especially in the case of extremely cooled beams, for which the amplitudes of Schottky signals might be reduced to far below the electronic noise level.
Figure 11: Momentum loss of stored Ar\textsuperscript{18} and Bi\textsuperscript{82} in the residual gas
Nevertheless, the influence of energy loss and energy loss straggling on stored heavy ion beams are easily observed by means of longitudinal Schottky spectra as demonstrated by two examples from the ESR. The first effect observed in the longitudinal Schottky spectrum after switching off the electron cooling is a fast broadening of the band due to intra-beam scattering (IBS) as shown in the upper diagram of figure 11. After a few minutes we see a significant frequency shift due to energy loss and a slight increase in the frequency spread. The latter effect is probably caused by IBS rather than by energy loss straggling. After 30 min storage time at $5 \times 10^{-11}$ mbar residual gas pressure the circulation frequency of $\text{Ar}^{+18}$ ions at 250 MeV/u is reduced by $4.3 \times 10^{-4}$. This shift is according to a relative loss of momentum $\Delta p/p = \eta^{-1}\Delta f/f = 9 \times 1^{-4}$ or a stopping rate $dE_i/dt \approx 48.8$ MeV/h. The stopping of $\text{Bi}^{+82}$ ions at 238 MeV/u under similar UHV conditions is illustrated in the lower plot of figure 11. In this case the relative frequency shift after 300 s is found to be $1.7 \times 10^{-4}$ and $dE_i/dt \approx 392$ MeV/h. The relation between both values measured at approximately the same ion velocity and same residual gas density is in rather good accordance to the $Z_i^2$-dependence given in (27).

5. Summary

The interaction of stored heavy ion (HI) beams with residual gas molecules compares with that of proton (p) beam as follows:

- **Electron capture rate**: $R_c(\text{HI}) \approx R_c(p) \times Z_i^a$, where $Z_i$ may be exchanged by the atomic charge state $q$ in the case of partially stripped ions. The value of parameter $a$ may increase from 2 at low energies to 5 at very high energies, and the decrease with increasing energy is generally somewhat weaker for heavy ions.

- **Electron loss rate**: Electron loss by partially stripped ions is, compared to electron capture, the dominant process at high energies. Rates increase with the number of projectile electrons and the atomic number of residual gas atoms.

- **Multiple scattering rate**: $d\theta/dt(\text{HI}) \approx d\theta/dt(p) \times Z_i^2/A_i^2$ is evidently smaller in the case of heavy ions, typically only 25% of the proton scattering, if $Z_i/A_i = 0.5$ is assumed.

- **Momentum loss rate**: $dlnp/dt(\text{HI}) \approx dlnp/dt(p) \times Z_i^2/A_i$ increases approximately with $Z_i$ ($Z_i \propto A_i$ assumed).

- **Gas ionization rate**: $dn/dt(\text{HI}) \approx dn/dt(p) \times Z_i^2$ increases so strong that one might expect an additional limitation — besides different space charge effects — for stored beam currents in the case of very heavy, high-$Z_i$ ions.
The results from heavy ion storage rings in the MeV/u range at the TSR (Heidelberg) and in the 100 MeV/u range at the ESR (Darmstadt) proved that resonable beam life times are attainable with residual gas pressures between $10^{-10}$ mbar and $10^{-12}$ mbar. The lower pressures are necessary mainly in the case of partially stripped ions at low energies. Conversely, from the heavy ion synchrotron SIS (Darmstadt) we know, that an average residual gas pressure of about $5 \times 10^{-9}$ mbar is sufficient to accelerate Bi$^{+67}$ or Au$^{+65}$ from 11.5 MeV/u to 800 MeV/u within two seconds without major beam loss due to stripping of ions.

At present, no experimental data are available on electron capture and loss cross sections for heavy ions at energies in the GeV/u range, which is the aim of the "Lead-Project" at CERN. Partially stripped Pb–ions are planned to be accelerated to several GeV/u in the proton synchrotron (PS) and, after stripping of remaining electrons, to about 90 GeV/u in the SPS. Even further acceleration and storage of Pb–ions in the proposed Large Hadron Collider (LHC) is envisaged. Some valuable data up to 2 GeV/u may be expected in the near future from SIS and ESR.

References


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[26] CERN Data Booklet 1988, p. 64
[33] O. Gröbner; The Dynamic Behaviour of Pressure Bumps in the ISR; CERN ISR–VA/76–25 (1976)