Testing Reciprocity and CPT-Invariance for Neutral Kaons

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Abstract

We point out that the time - reversal - invariant explanation of the recently observed CPLEAR asymmetry is valid at the level of only about two standard deviations; this asymmetry has been claimed to be the first direct observation of time - reversal - noninvariance in the neutral kaon system. We suggest how, by avoiding the assumptions made at present, future data can provide model - independent tests of structural T- and CPT-invariances of the neutral kaon complex.

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By assuming CPT- invariance for the K_{l3} amplitudes (viz. amplitudes of β -decay of the neutral kaon), the CPLEAR collaboration recently obtained [1] their “first direct observation of time - reversal - noninvariance in the neutral kaon system”. The need for assuming this CPT- invariance was confirmed by an independent analysis [2] of the data of ref. [1]; it was pointed out [2] that the relevant CPT- noninvariant amplitudes must be numerically well-restricted for establishing T- noninvariance from the CPLEAR asymmetry [1]. Contrary to the claim communicated to the author ref. [2], as mentioned in the footnote-cum-reference (26) of ref. [2], we now show that the required limitation of the relevant CPT-noninvariant amplitudes has not been achieved, and that the T-invariant (and CPT-noninvariant) explanation of the CPLEAR asymmetry is valid at the level of only about two standard deviations. In that sense, a direct and model- independent observation of failure of reciprocity in the neutral kaon system has not yet been achieved.

Now the details. The observed CPLEAR asymmetry A_T^{exp} , for long times, is given [2] as

$$A_T^{exp} = 2Re(\epsilon_L + \epsilon_S) - 2z \quad (1)$$

where $Re(\epsilon_L + \epsilon_S)$ represents structural T-noninvariance, and z arises from CPT- noninvariance in K_{l3} amplitudes. Using CPT- invariance, the first term is given numerically [2] as

$$2Re(\epsilon_L + \epsilon_S) = (6.4 \pm 1.2) \times 10^{-3} \quad (2)$$

The second term in Eq. (1) is given numerically (see Eq. (7) of ref. [3]) as

$$-2z = (0.8 \pm 1.2) \times 10^{-3} \quad (3)$$

The experimental number [1] is

$$A_T^{exp} = (6.6 \pm 1.3 \pm 1.0) \times 10^{-3} \quad (4)$$

where the first error is statistical, and the second error is systematic. Adding the two errors in quadrature, one may write Eq. (4) as

$$A_T^{exp} = (6.6 \pm 1.6_+) \times 10^{-3} \quad (5)$$

If, therefore, A_T^{exp} arises from the CPT-noninvariant and T- invariant contribution (3), Eq. (1) becomes

$$(6.6 \pm 1.6_+) \times 10^{-3} = (0.8 \pm 1.2) \times 10^{-3} \quad (6)$$

On the other hand, the T- noninvariant (and CPT-invariant) choice gives

$$(6.6 \pm 1.6_+) \times 10^{-3} = (6.4 \pm 1.2) \times 10^{-3} \quad (7)$$

which, of course, is satisfied very well. Present data thus prefer the T- noninvariant choice to the T- invariant choice, but the latter cannot be discarded; Eq. (6) can hold numerically at the level of only about two standard deviations. Thus one cannot regard T-noninvariance for the neutral kaon complex as directly established.

The numerical analyses (see, e.g., refs. [1-3]) in question all assume the Weisskopf - Wigner approximation (henceforth called WWA) which is characterised by the introduction of the short-lived and the long-lived kaon states $|K_{S,L}\rangle$ which propagate, in time, independently of each other. The WWA is inadequate as a good theory; see, e.g., refs. [4-6]. The CPLEAR asymmetry is predicted by the WWA to be constant if CPT- invariance and the $\Delta S = \Delta Q$ rule are assumed; see, e.g., refs. [1,2]. Because of these assumptions, and the experimental errors on data-points used to extract A_T^{exp} , this “constancy” prediction of the WWA can be allowed, but not required, by present data. The detailed WWA prediction for the asymmetry (called A_3 below) which tests CPT- invariance is also not well-tested at present.

In view of the fundamental nature of reciprocity, it would be nice to avoid the assumption of CPT- invariance of the K_{l3} amplitudes in order to achieve a direct test of reciprocity. Could one avoid also the WWA? The present note takes a step in these two directions.

Given below is our complete set of six asymmetries which can provide general tests of reciprocity (T-invariance) and CPT-invariance of the structure of the neutral kaon complex; the lack of these two invariances provides the only two “indirect” CP-violations. Our asymmetries $A_{3,4}$ have been previously considered for evaluation using the WWA; see, e.g., refs. [7,8]. The remaining asymmetries $A_{1,2,5,6}$ provide good help to get $A_{3,4}$ which respectively test CPT-invariance and T-invariance; the complete set gives a useful overall picture. We shall assume strong interactions to obey CP-invariance and to conserve strangeness. We use strong interactions to provide strangeness-tagging; some such taggings have been previously considered [9,10].

We consider the two low-energy antiproton-proton annihilations

$$\bar{p}p \rightarrow \pi^+ K^- K^0 \quad , \quad (8)$$

$$\bar{p}p \rightarrow \pi^- K^+ K^0 \quad , \quad (9)$$

studied by the CPLEAR collaboration, as providing the strangeness of the neutral kaons (produced at proper time $t=0$) through the pion and kaon charges. Let us write the four time-dependent “structural” amplitudes $a(t)$, $b(t)$, $\bar{b}(t)$, $\bar{a}(t)$ to denote, respectively, the transitions [initial $K^0 \rightarrow K^0$ (at time t)], [initial $K^0 \rightarrow \bar{K}^0$ (at time t)], [initial $\bar{K}^0 \rightarrow K^0$ (at time t)], [initial $\bar{K}^0 \rightarrow \bar{K}^0$ (at time t)]. We shall not use any model for these amplitudes; the WWA provides such a model; see, e.g., refs. [6,11]. Then, the reaction (8) leading to the final states $(\pi^+ K^- K^0)$ and $(\pi^+ K^- \bar{K}^0)$ at time t determines the amplitudes a and b respectively. Similarly, the reaction (9) leading to the final states $(\pi^- K^+ \bar{K}^0)$ and $(\pi^- K^+ K^0)$ provides the amplitudes $\bar{a}$ and $\bar{b}$ respectively. We use [9] the strong interaction

$$\bar{K}^0 p \rightarrow \Lambda \pi^+ \quad (10)$$

to tag the $\bar{K}^0$ created at time t , and the strong interaction

$$K^0 p \rightarrow K^+ n \quad (11)$$

to tag the K^0 created at time t . In summary, the amplitudes $a, b, \bar{a}$ and $\bar{b}$ arise from final states of the type $\pi^+ K^- K^+$, $\pi^+ K^- \Lambda$, $\pi^- K^+ \Lambda$, $\pi^- K^+ K^+$ respectively. Thus the K_{l3} amplitudes are circumvented [9]. Of course, there are new problems associated with the reactions (10,11) - namely, the lack [9] of large and precisely known cross-sections. Hopefully, these problems would be sorted out soon, or some other strong-interaction-tagging utilized.

We introduce the t -dependent rates R_{++} , R_{+-} , R_{-+} , R_{--} where the first and second subscript respectively denotes the strangeness of the initial (time =0) and final (time = t) neutral kaon. Apart from uninteresting overall factors, one has

$$R_{++} = |a|^2, \quad R_{+-} = |b|^2, \quad R_{-+} = |\bar{b}|^2, \quad R_{--} = |\bar{a}|^2, \quad (12)$$

which gives the six t -dependent asymmetries as

$$\begin{aligned} A_1 &= \frac{R_{++} - R_{+-}}{R_{++} + R_{+-}} = \frac{|a|^2 - |b|^2}{|a|^2 + |b|^2} \quad , \quad A_2 = \frac{R_{++} - R_{-+}}{R_{++} + R_{-+}} = \frac{|a|^2 - |\bar{b}|^2}{|a|^2 + |\bar{b}|^2} \quad , \\ A_3 &= \frac{R_{++} - R_{--}}{R_{++} + R_{--}} = \frac{|a|^2 - |\bar{a}|^2}{|a|^2 + |\bar{a}|^2} \quad , \quad A_4 = \frac{R_{+-} - R_{-+}}{R_{+-} + R_{-+}} = \frac{|b|^2 - |\bar{b}|^2}{|b|^2 + |\bar{b}|^2} \quad , \\ A_5 &= \frac{R_{--} - R_{+-}}{R_{--} + R_{+-}} = \frac{|\bar{a}|^2 - |b|^2}{|\bar{a}|^2 + |b|^2} \quad , \quad A_6 = \frac{R_{--} - R_{-+}}{R_{--} + R_{-+}} = \frac{|\bar{a}|^2 - |\bar{b}|^2}{|\bar{a}|^2 + |\bar{b}|^2} \quad . \end{aligned} \quad (13)$$

Here, the asymmetries A_3 and A_4 vanish, for all t , if respectively CPT- invariance and T- invariance hold, see, e.g., refs. [4,6-9]; both these asymmetries vanish if CP- invariance holds. One may, therefore, expect $A_{3,4}$ to be small, in contrast to $A_{1,2,5,6}$. While A_1 utilizes reaction (8), A_6 utilizes reaction (9). While A_2 uses reaction (11), A_5 utilizes reaction (10). Thus, $A_{2,5}$ are likely to be comparatively free from uncertainties associated with strong-interaction-tagging. Similarly, $A_{1,6}$ would avoid possible normalisation problems of comparing reactions (8) and (9).

Only three of the six asymmetries are independent because there are only three independent amplitude-ratios which may be taken as the set $[(b/a), (\bar{a}/a) \text{ and } (\bar{b}/b)]$; their modulus squares uniquely fix $A_{1,3,4}$ respectively, through formulas of the type

$$A_1 = \frac{1 - |b/a|^2}{1 + |b/a|^2} . \quad (14)$$

Since the ratios $(\bar{b}/a), (b/\bar{a}), (\bar{b}/\bar{a})$ can be expressed in terms of the independent set, the asymmetries $A_{2,5,6}$ are uniquely fixed by $A_{1,3,4}$:

$$\begin{aligned} \frac{1 - A_2}{1 + A_2} &= \frac{1 - A_4}{1 + A_4} \frac{1 - A_1}{1 + A_1} , \\ \frac{1 - A_5}{1 + A_5} &= \frac{1 + A_3}{1 - A_3} \frac{1 - A_1}{1 + A_1} , \\ \frac{1 - A_6}{1 + A_6} &= \frac{1 - A_4}{1 + A_4} \frac{1 + A_3}{1 - A_3} \frac{1 - A_1}{1 + A_1} . \end{aligned} \quad (15)$$

If now the smallness of $A_{3,4}$ is utilized by keeping them to at most the first order, one gets

$$A_6 - A_5 = A_4(1 - \alpha) , \quad (16)$$

$$A_6 - A_5 = A_2 - A_1 , \quad (17)$$

$$A_2 - A_6 = A_3(1 - \alpha) , \quad (18)$$

where $\alpha = [(A_i)^2 \text{ for } i = 1, 2, 5, 6] \text{ or } [(A_j A_k) \text{ for } j \neq k, (j, k) = 1, 2, 5, 6]$. Thus, because of our tagging procedure, the relations

$$A_2 - A_5 = (A_3 + A_4)(1 - \alpha) , \quad (19)$$

$$A_1 - A_6 = (A_3 - A_4)(1 - \alpha) , \quad (20)$$

would be of help to obtain $A_{3,4}$ because

$$2A_4 = [(A_2 + A_6) - (A_1 + A_5)]/(1 - \alpha) \quad , \quad (21)$$

$$2A_3 = [(A_2 + A_1) - (A_5 + A_6)]/(1 - \alpha) \quad (22)$$

hold. The relation

$$A_1 A_6 = A_2 A_5 \quad (23)$$

may also help, as a consistency check.

In summary, the present data do not provide a direct and model- independent test of structural T- invariance for neutral kaons. We have emphasized the importance of measuring the known asymmetries $A_{3,4}$ for testing structural CPT- invariance and T- invariance respectively. Useful help is provided the remaining asymmetries $A_{1,2,5,6}$ through relations of the type Eqs. (15-23). While $A_{2,5}$ would avoid some of the strong- interaction - tagging uncertainties, $A_{1,6}$ avoid some of the possible normalisation problems. Our tests are general: no WWA and no involvement of the K_{l3} amplitudes. We hope that the difficulties associated with strong-interaction-tagging of strangeness of the final neutral kaon would be sorted out, and our asymmetries measured soon.

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