Quasi-Fixed Points and Charge and Colour Breaking in Low Scale Models

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Abstract
We show that the current LEP2 lower bound upon the MSSM lightest Higgs mass rules out quasi-fixed scenarios for string scales between $10^8$ and $10^{14}$ GeV. We consider the implications for charge and colour breaking (CCB) bounds in the MSSM, and demonstrate that CCB bounds from $F$ and $D$-flat directions are significantly weakened. For scales less than $10^{10}$ GeV these bounds become merely that degenerate scalar mass squareds are positive at the string scale.

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1 Introduction

For many years, string and unification scales were thought to be high ($\gtrsim 10^{16}$ GeV). The perturbative heterotic formulation of string theory had the fundamental string scale $\Lambda_s \sim O(10^{17})$ GeV close to $M_{\text{Planck}} \sim 10^{19}$ GeV because of its constrained description of the gravitational interaction. The grand unification scale was around $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV, motivated by the apparent convergence of the gauge couplings when they were evolved to this value. Recently however, attention has turned to models that have lower string and/or unification scales [1, 2, 3, 4, 5, 6] and this has raised some interesting questions to do with renormalisation group running of parameters.

The most immediate is of course whether gauge or Yukawa unification is still possible or even necessary with a lower string scale. One example that achieves gauge unification at the string scale [2] has the couplings experience power law ‘running’ [2, 4, 5] above a compactification scale due to the presence of additional Kaluza-Klein modes. A Kaluza-Klein spectrum with the same ratios of gauge beta functions as those in the MSSM leads to a logarithmic running upto the compactification scale with a rapid power law unification taking place very rapidly thereafter [2]. An example that does not achieve gauge unification is ‘mirage’ unification [6]. In mirage unification the gauge couplings at the string scale receive moduli dependent corrections that behave as if there were continued logarithmic running above the string scale up to unification at the usual $\Lambda_{\text{GUT}}$. ‘Mirage unification’ refers to this fictitious unification.

A particularly attractive choice for the string scale (albeit one that is not immediately accessible to experiment) is $\Lambda_s \sim 10^{11}$ GeV [3]. In this case the hierarchy between the weak scale and the Planck scale arises without unnaturally small ratios of fundamental scales. It was also noted in the first reference of [3] that $\Lambda_s \sim 10^{11}$ GeV gives neutrino masses of the right order. We return to this model below and refer to it as the Weak-Planck (WP) model.

In this paper we consider two other related issues in the Minimal Supersymmetric Standard Model (MSSM),

$$W_{\text{MSSM}} = h_U Q H_2 U^c + h_D Q H_1 D^c + h_E L H_1 E^c + \mu H_1 H_2,$$

with a low string scale. The first concerns the top quark Quasi-Fixed Point (QFP). The QFP is characterised by a focussing of some MSSM parameters to particular ratios as the renormalisation scale $\Lambda$ is decreased towards the top quark mass, $m_t$ [8, 9, 10]. Formally it is defined to be the point in parameter space where there is a Landau pole in the top Yukawa coupling $h_t$ at the string or GUT scale (whichever is the lower). In practice however this focussing behaviour can occur for a large but finite $h_t(\Lambda_s)$, still treatable by

\footnote{Note that, although there are problems [7] with the particular string realisation of mirage unification in Ref. [6], the idea may be realisable in other models and remains an interesting possibility.}
perturbation theory. The coupling $h_t$ focusses to some value at $m_t$ independent of $h_t(\Lambda_s)$ provided it is large enough. In low scale models, with their foreshortened logarithmic running, one naturally expects this behaviour to be very different. If the pole is at $10^{11}$ GeV for example, we expect the quasi-fixed value of the top Yukawa at $m_t$ to be larger than for the usual GUT unification. Conversely, for a given value of top mass and $\tan \beta$ at the weak scale the model will be further from the QFP. We shall determine the QFP prediction for $\tan \beta$, on which experimental constraints from LEP2 can be brought to bear in order to empirically constrain $\Lambda_s$ assuming the QFP scenario. In particular, we consider the Higgs mass which in the canonical GUT scenarios has been shown to be a strong restriction upon the QFP scenario [10].

The second issue we consider is the possibility of minima that break charge and colour lying along $F$ and $D$ flat directions [11, 12, 9, 13, 14, 15, 16]. The constraints found by requiring that there be no such (CCB) minima are dependent on the distance from the QFP. They are most severe at the QFP itself [9, 13, 14] and indeed, in the usual MSSM at the QFP, CCB constraints exclude half the parameter space. With a lower string scale it seems likely that such constraints will generally be less restrictive for two reasons. First, a given point in (weak-scale) parameter space will be further from the QFP as noted above. Second, the CCB minima are generated radiatively when the mass-squared parameter for $H_2$ becomes negative. When there is a lower string scale there is less ‘room’ for a minimum to form at vacuum expectation values (VEVs) much greater than the weak scale. (More specifically, there are positive mass-squared contributions to the potential along the flat direction that become dominant at lower VEVs.) We shall demonstrate that this is indeed the case and that for $\Lambda_s < \sim 10^{10}$ the CCB constraint (at least along the $F$ and $D$ flat directions) is merely that scalar mass squareds are positive.

We will throughout be discussing these aspects by assuming that there is the standard logarithmic running of the MSSM up to a scale, $\Lambda_s$, that we rather loosely refer to as the string scale. This scale may be much lower than $\Lambda_{GUT}$. We define the QFP to be where the top Yukawa has a Landau pole at this point, since any variation in the Yukawa couplings above $\Lambda_s$ is expected to be drastically changed by string physics. As for the CCB bounds, we derive them on the soft breaking parameters at $\Lambda_s$ since this is close to the scale at which we expect the supersymmetry breaking parameters to be derived in any fundamental string model (although we will have more to say on this in due course).

2 The Quasi-Fixed MSSM

The QFP [8, 9, 10] constraint, i.e. that the top Yukawa coupling $h_t$ has a Landau pole at the string scale, gives important predictions in terms of the couplings and masses of supersymmetric particles [8, 9, 10]. We now examine the prediction for $\tan \beta$ numerically paying special attention to its dependence on the string scale. We use the MSSM to two-
loop order as an effective theory above $m_t$. Fermions masses and gauge couplings are set to be at their central values in ref. [17] except for $\alpha_s(M_Z)$, which is varied to show the induced uncertainty. Below $m_t$, we run using a 3 loop QCD\(\otimes\)1 loop QED effective theory with all superpartners integrated out. Fig. 1 illustrates the quasi-fixed behaviour for two values of string scale. The dependence of the weak scale $h_t$ on its string scale value is shown for canonical QFP SUSY GUT framework with string/unification scale $\Lambda_s = 2 \times 10^{16}$ GeV. The almost horizontal part of the lines represent the QFP regime: where, for input values $h_t(\Lambda_s) > 1.5$, the prediction $h_t(m_t) = 1.1$ results. Lowering $\Lambda_s$ to $10^{11}$ GeV, as in the WP model, we see that the quasi-fixed behaviour is diminished somewhat, as indicated by the more positive slope of the relevant lines. However, for $h_t(\Lambda_s) > 2.5$ a QFP prediction of $h_t(m_t) = 1.2$ occurs. The figure shows that the variation due to uncertainties in $\alpha_s(M_Z)$ are smaller than the precision quoted in the prediction.

The $h_t(m_t)$ QFP prediction can be turned into a prediction of the MSSM parameter $\tan \beta$, the ratio of the two neutral CP-even Higgs vacuum expectation values through the relation

$$\sin \beta^{QFP} = \frac{\sqrt{2} m_t(m_t)}{v h_t(m_t)}$$

and the known value [17] of the top quark mass, $m_t(m_t) = 165 \pm 5$ GeV. $v$ refers to the Standard Model Higgs VEV of 246.22 GeV. Low values of $\tan \beta$, such as those predicted by Eq. 2, are constrained by the non-observation of the lightest MSSM Higgs boson at LEP2 [10]. The current limits [18] exclude the region

$$0.9 < \tan \beta < 1.6,$$

from the bound $m_{h^0} > 95$ GeV. If LEP2 produces a projected lower bound of $m_{h^0} > 105$ GeV, $0.7 < \tan \beta < 2.0$ will be excluded.

Fig. 2 displays the $\tan \beta$ prediction of the QFP for a varying string scale, $\Lambda_s$. Here, we have set $h_t(\Lambda_s) = 5$, close to its Landau pole and near the edge of perturbativity. The range between each pair of lines for a given value of $\alpha_s(M_Z)$ denotes the range of prediction due to the uncertainty in the top quark mass. Varying $\alpha_s(M_Z)$ within its 1\(\sigma\) error bounds produces a small variation (most pronounced for high $\Lambda_s$) as shown. In fact, we see from the figure that the QFP is ruled out for the range

$$10^8 < \Lambda_s/\text{GeV} < 10^{14}.$$  

If LEP2 reaches the limit $m_{h^0} > 105$ GeV, $\Lambda_s > 10^6$ GeV will be excluded if the MSSM is near to its QFP. In practice, these bounds imply that the top Yukawa coupling must not be too large at the string scale.
Figure 1: Prediction of low energy top Yukawa coupling $h_t$ for string scale input. Two string scales $\Lambda_s = 10^{11}, 2 \times 10^{16}$ GeV are shown. The pair of lines represent the range produced by varying $\alpha_s(M_Z) = 0.115 - 0.122$ (the upper lines corresponding to higher $\alpha_s(M_Z)$).

Figure 2: MSSM top quark quasi fixed point tan $\beta$ predictions with varying string scale $\Lambda_s$. The upper pair of lines are for $m_t(m_t) = 170$ GeV and the lower pair for $m_t(m_t) = 160$ GeV. The area between the experimental limits has been excluded for the MSSM by LEP2.
3 Analytic CCB Bounds at low string scales

We now turn to the discussion of CCB bounds. Unphysical CCB minima present some of the most severe bounds for supersymmetric models [11, 12, 9, 13, 14, 15, 16]. Indeed, for a number of models it has been found that they exclude much of the parameter space not already excluded by experiment; for example the MSSM where supersymmetry breaking is driven by the dilaton [12], SUSY GUTS at the low tan $\beta$ quasi-fixed point (QFP) [9], $M$-theory in which supersymmetry breaking is driven by bulk moduli fields [14, 15] and several other string/field theory scenarios [15, 16]. All of the above work, however, assumed a logarithmic evolution of the gauge couplings with unification at a high scale $\geq 10^{16}$ GeV.

In this section we shall be considering the effect of truncating this logarithmic evolution at a low string scale. For completeness, we first recall the three types of CCB minima that can occur in supersymmetric models:

- $D$-flat directions which develop a minimum due to large trilinear supersymmetry breaking terms.

- $F$ and $D$ flat directions corresponding to a single gauge invariant.

- $F$ and $D$ flat directions which correspond to a combination of gauge invariants involving $H_2$ [19].

Since the first type are important at low scales [11] and the second type are only important when there are negative mass-squared terms at the GUT scale, we shall concentrate on the constraints coming from the last type of minimum. These occur at intermediate scales due to the running $H_2$ mass-squared even if all the mass-squareds are positive at the GUT scale. Hence the resulting constraints are very dependent on renormalisation group running at high scales and are particularly interesting from the point of view of models with a lower string scale. As discussed above, our initial expectation is that the CCB bounds will be far less severe than in the usual versions of the MSSM.

We will consider the $F$ and $D$-flat direction in the MSSM corresponding to the operators

$$L_i L_3 E_3 \ ; \ H_2 L_i$$

where the suffices on matter superfields are generation indices. With the following choice of VEVs;

$$h_2^0 = -a^2 \mu / h_{E33}$$
$$\bar{e}_{L_3} = \bar{e}_{R_3} = a \mu / h_{E33}$$
$$\nu_i = a \sqrt{1 + a^2 \mu / h_{E33}}$$

(6)
the potential along this direction depends only on the soft supersymmetry breaking terms (neglecting a small D-term contribution);

\[ V = \frac{\mu^2}{h_{E_{33}}^2} a^2 (a^2 (m_2^2 + m_{L_{ii}}^2) + m_{L_{ii}}^2 + m_{E_{33}}^2 + m_{E_{33}}^2). \]  

(7)

In the usual MSSM we can reasonably assume that, since the CCB minimum forms at VEVs corresponding to \( a \gg 1 \), the largest relevant mass, and therefore the appropriate scale to evaluate the parameters at, is \( \phi = h_{U_{33}} \langle h_2^0 \rangle \equiv h_t \langle h_2^0 \rangle \). This minimises the top quark contributions to the effective potential at one-loop. Further corrections to the potential are assumed to be small. Once we lower the string scale however we encounter the problem that the CCB minimum moves towards low scales and that consequently this approximation breaks down. Evidently, from Eq.(7), this happens precisely where the positive \( m_2^2 + m_{L_{ii}}^2 + m_{E}^2 \) terms begin to dominate, and so we do not anticipate that CCB minima will be formed when \( a < 1 \). In order to check this however, our approach will be to construct the constraints using the above assumption on \( \phi \) and observe that they get far less restrictive as we move to moderately low string scales, say \( \Lambda_s \sim 10^8 \text{GeV} \). We then check the analytic results obtained with a more accurate numerical analysis at certain parameter points and observe numerically that CCB minima do not reappear as we move to very low string scales where \( a < 1 \).

In the above potentials, \( \langle h_2^0 \rangle = -a^2 \mu / h_{E_{33}} \) so that the Eq.(7) is of the form

\[ V = \frac{\Lambda^2}{h_{U_{33}}^2} \hat{\phi} \left( \hat{\phi} A + B/b \right) \]  

(8)

where \( A = m_2^2(\phi) + m_{L_{ii}}^2(\phi) \), \( B \) is the LLE combination of mass-squared parameters (also evaluated at \( \phi \)) that appears in the potential,

\[ \hat{\phi} = \phi / \Lambda \]  

(9)

and \( \Lambda \) is an arbitrary scale which we shall take to be the usual unification scale \( \Lambda_{GUT} \sim 10^{16} \text{GeV} \). The bound is therefore governed by \( A \), \( B \) and the parameter

\[ b(\phi) = \frac{\Lambda_{GUT} h_{E_{33}}}{h_{U_{33}} \mu} \]  

(10)

for the LLE, \( LH_2 \) direction described above, or

\[ b(\phi) = \frac{\Lambda_{GUT} h_{D_{33}}}{h_{U_{33}} \mu} \]  

(11)

for the equally dangerous \( LQD \), \( LH_2 \) direction.

To estimate the bound, we now adapt the results of Refs.[13, 14]. At large values of \( a \gg 1 \) the potential is governed by the first term. Whatever the string scale may be, we require that \( m_2^2 \) be positive there and negative at \( M_W \) (for successful electroweak
symmetry breaking). A CCB minimum forms radiatively close to the value $\phi_p$ where $A$ first becomes negative (typically at a scale of $\mu \approx h_{E_6}$) [13, 14].

In Refs.[13, 14] it was shown that once we are able to estimate $\phi_p$ the bound follows fairly easily, and this was done for models with degenerate gaugino masses. Bounds were derived for all non-universal scalar masses and couplings. In the present case however, the gauge couplings and the gaugino masses are also non-degenerate at the string scale $\Lambda_s$.

This makes a general analytic treatment of the RGEs extremely difficult, so in order to simplify matters we shall henceforth assume the ‘GUT gaugino relation’. That is we assume that at the scale $\Lambda_s$ we have the usual GUT expression for gaugino masses,

$$\frac{M_a}{M_b} = \frac{\alpha_a}{\alpha_b}. \quad (12)$$

This relationship has the useful property that the gaugino masses as well as the gauge couplings would be degenerate if we continued the evolution of the MSSM RGEs upto $\Lambda_{GUT}$. We shall call this fictitious degenerate value $M_a(\Lambda_{GUT}) = M_{1/2}$.

Although Eq.(12) may seem like a rather brutal requirement, it holds for a number of interesting cases, for instance in models with power law unification as shown in Ref.[5]. In these models the scale $\Lambda_s$ in our analysis should really be interpreted as the compactification scale at which the first Kaluza-Klein states appear in the spectrum, rather than the string scale which is where we expect the real gauge unification to take place after a short period of power law ‘running’.

Eq.(12) is also expected to hold in the mirage unification models of Ref.[6] when there is no $S/T$-mixing and in the limit $T + \overline{T} \rightarrow \infty$. In this limit we have

$$M_a \approx \sqrt{3}Cm_{3/2} \sin \theta \frac{\alpha_a}{\alpha_0} + O(1/(T + \overline{T})^2) \quad (13)$$

where we use the subscript-0 to represent values at the usual $\Lambda_{GUT}$ unification scale (i.e. $\alpha_0 \approx 1/25$), and where we have neglected terms of order $\alpha_a m_{3/2}$ which is consistent to one-loop accuracy. In this case we have $M_{1/2} = \sqrt{3}Cm_{3/2} \sin \theta$.

Eq.(12) allows us to adapt the expressions of Ref.[13] with only a modest amount of effort by writing the parameters at $\Lambda_s$ in terms of their values at $\Lambda_{GUT}$. In order to proceed, we next spend a little time discussing the analytic solutions to the renormalisation group running. The solutions of all the parameters may easily be expressed in terms of those combinations with infra-red QFPs; $R = h_t^2/y_3^2$, $A_t$ and $3M^2 = m_2^2 + m_{\nu_{33}}^2 + m_{Q_{33}}^2$. These may be written as functions of

$$r = \frac{\alpha_0}{\alpha_3} = \frac{1}{\tilde{\alpha}_3} = 1 + \frac{6\alpha_0}{4\pi} \log \frac{\Lambda}{\Lambda_{GUT}}, \quad (14)$$

so that

$$\frac{\alpha_0}{\alpha_2} = \frac{1}{\tilde{\alpha}_2} = \frac{3}{4 - r}; \quad \frac{\alpha_0}{\alpha_1} = \frac{1}{\tilde{\alpha}_1} = \frac{5}{16 - 11r}. \quad (15)$$
Taking $\alpha_3(m_t) = 0.108$ means that $0.37 < r < 1$ with $r = 1$ corresponding to the GUT scale. If the string scale is at $\Lambda_s = 10^{11}$ GeV as in the WP model, then the corresponding value of $r_s \equiv r(\Lambda_s)$ is $r_s = 0.82$. It is useful to define

$$\Pi(r) = \tilde{\alpha}^{16/9} \tilde{\alpha}_2^{-3} \tilde{\alpha}_1^{-13/99},$$

$$\tilde{J} = \frac{1}{r \Pi(r)} \int_r^1 \Pi(r')dr'.$$  \hspace{1cm} (16)

Solving for $R$ in terms of its value $R_s$ at the string scale (we use subscript-$s$ to denote string-scale values) we find

$$\frac{1}{R} = \frac{\Pi_s r_s}{R_0 \Pi r} + \frac{1}{R^{QFP}}$$  \hspace{1cm} (17)

where the QFP value (where the Yukawa couplings blow up at the string scale) is given by

$$\frac{1}{R^{QFP}} = 2\tilde{J}(r) - 2\tilde{J}(r_s) \frac{\Pi_s r_s}{\Pi r}. $$  \hspace{1cm} (18)

We also, for later use, define the distance from the real QFP,

$$\sigma = \frac{R}{R^{QFP}}.$$  \hspace{1cm} (19)

This can be rewritten in terms of a fictitious renormalisation of $R$ down from a $\Lambda_{GUT}$ scale value of $R_0$; i.e. defining

$$\frac{1}{R^{QFP}} = 2\tilde{J}$$  \hspace{1cm} (20)

we have

$$\frac{1}{R} = \frac{1}{R_0 \Pi r} + \frac{1}{R^{QFP}}$$

$$\frac{1}{R_s} = \frac{1}{R_0 \Pi_s r_s} + \frac{1}{R_s^{QFP}}.$$  \hspace{1cm} (21)

This is the usual expression for $R$ (c.f. Ref.[14]); however it should be noted that $R_0$ is here merely a parameter that is negative in the region $1/R^{QFP} > 1/R_s > 0$. In the usual MSSM with unification at the GUT scale, this would of course be an unphysical (non-perturbative) region. For $A_t$ and $M^2$ we now define the distance from the usual QFP (i.e. where couplings blow up at the usual unification scale $\Lambda_{GUT}$)

$$\rho = \frac{R}{R^{QFP}} $$  \hspace{1cm} (22)

and also

$$\xi = \frac{1-r}{r\tilde{J}} - 1.$$  \hspace{1cm} (23)

We then obtain expressions for $\tilde{A}_t = A_t/M_{1/2}$ and $\tilde{M}^2 = M^2/M_{1/2}^2$ in terms of their fictitious values, $\tilde{A}_0$ and $\tilde{M}_0^2$, at $\Lambda_{GUT}$;

$$\tilde{A}_t = (1-\rho)\tilde{A}_0 + \rho \xi - \Gamma $$

$$\tilde{M}^2 = (1-\rho)\tilde{M}_0^2 - \frac{1}{3} \rho \tilde{K} + \frac{2}{3} (1-r) \gamma $$  \hspace{1cm} (24)

9
\gamma = \frac{16}{9} \tilde{\alpha}_3 (1 + \tilde{\alpha}_3 (1 - r)/2) + \tilde{\alpha}_2 (1 - \tilde{\alpha}_2 (1 - r)/6) + \frac{13}{45} \tilde{\alpha}_1 (1 - 11 \tilde{\alpha}_1 (1 - r)/10) \\
\Gamma = (1 - r) \left( \frac{16}{9} \tilde{\alpha}_3 + \tilde{\alpha}_2 + \frac{13}{45} \tilde{\alpha}_1 \right) \\
\tilde{K} = (1 - \rho)(\xi - \tilde{A}_0)^2 - \xi^2 + (\xi + 1) \Gamma \\
\tilde{A}_0 = \left( \tilde{A}_s - \rho_s \tilde{\xi}_s + \Gamma_s \right) / (1 - \rho_s) \\
\tilde{M}_0^2 = \left( \tilde{M}_s^2 + \frac{1}{3} \rho_s \tilde{K}_s - \frac{2}{3} (1 - r_s) \gamma_s \right) / (1 - \rho_s). \quad (25)

It is important to note that, since

\begin{equation}
1 - \rho = (1 - \sigma)(1 - \rho_s), \quad (26)
\end{equation}

$A_t$ and $M^2$ retain their QFP behaviour since when $\sigma = 1$ (or $R_s \to \infty$) they are both independent of their values at the string scale, $\Lambda_s$. In addition, factors of $1/(1 - \rho_s)$ cancel so that there is no divergent behaviour at the usual QFP. Also note that this QFP is at lower tan $\beta$ than in the usual MSSM unification. We can estimate the difference in tan $\beta$ at the QFP by using

\begin{equation}
R = \frac{m_t^2}{4 \pi \alpha_3 v^2 \sin^2 \beta}, \quad (27)
\end{equation}

so that

\begin{equation}
\sin^2 \beta_{QFP} = \frac{R_{QFP}}{R_{QFP}^Q} \sin^2 \beta_{QFP}. \quad (28)
\end{equation}

Eqs.(18,20) then give tan $\beta_{QFP} \approx 1.2$ in the WP model with $\Lambda_s = 10^{11}$ GeV, in agreement with the full two-loop numerical result presented in Fig. 1.

With all parameters expressed in terms of GUT scale parameters, we are now simply able to apply the bounds derived in Ref.[14] for non-universal SUSY breaking directly. Consider for example the LH$_2$, LLE direction. The cosmological bounds in this case are

\begin{equation}
(2 \tilde{m}_{Lii}^2 + \tilde{m}_2^2 - \tilde{m}_{U_{33}}^2 - \tilde{m}_{Q_{33}}^2) |_{0} \gtrsim f(\tilde{B}|_{0}) + (\rho_p - 1) \left( g(\tilde{B}|_{0}) + 3 \tilde{M}_0^2 |_{0} - \rho_p (1 - \tilde{A}_0)^2 \right), \quad (29)
\end{equation}

where $\rho_p$ is the value of $\rho$ at the scale $\phi_p$ and

\begin{align*}
f(x) &= 1.20 - 0.14x + 0.02x^2 \\
g(x) &= 2.77 - 0.18x + 0.02x^2 \\
B &= m_{Lii}^2 + m_{U_{33}}^2 + m_{E_{33}}^2, \quad (30)
\end{align*}

for $\mu = 500$ GeV. (The small dependence of $f$ and $g$ on $\mu$, which we must choose by hand, is discussed in Ref.[14].) To a good approximation the value of $\rho_p$ is given by [14]

\begin{equation}
\frac{1}{\rho_p} = 1 + \frac{1}{2 R_0} = 1 + 3.17(\sin^2 \beta - \sin^2 \beta_{QFP}). \quad (31)
\end{equation}
Figure 3: Charge and colour breaking bounds with a lower string scale, $\Lambda_s$, for $\mu = 500$ GeV and degenerate trilinear terms, $A = -M_{1/2}$, and scalar masses $m_s$, at the string scale. The figure shows bounds on $\tilde{m}_s^2 = m_s^2/M_{1/2}^2$ for varying $\Lambda_s$ and $\tan \beta$. The contours are $\tilde{m}_s^2 > 0$ (black), $\tilde{m}_s^2 > 0.25$ (medium dark), $\tilde{m}_s^2 > 0.33$ (medium), $\tilde{m}_s^2 > 0.5$ (medium light), $\tilde{m}_s^2 > 0.66$ (light), $\tilde{m}_s^2 > 0.75$ (white).

In order to relate the quantities to their string scale values, we use the one loop RGE solutions for $A$ and $B$;

$$
(2\tilde{m}_{Lii}^2 + \tilde{m}_2^2 - \tilde{m}_{U33}^2 - \tilde{m}_{Q33}^2)|_s \approx \frac{16}{9} \delta_i^{(2)} - 3\delta_{2s}^{(2)} - \frac{5}{99} \delta_{1s}^{(2)} + f(\tilde{B}|_0) + (\sigma_p - 1) \left\{ (1 - \rho_s) g(\tilde{B}|_0) + 3M^2|_s - 2(1 - r_s)\gamma_s \right. \\
+ \rho_s \left[ -1 + \rho_s(\xi_s - 1)^2 - \Gamma_s(\xi_s - 3) - 2\tilde{A}_s(\xi_s - 1) \right] - \sigma_p(\tilde{A}_s - \rho_s\xi_s + \Gamma_s + \rho_s - 1)^2 \right\},
$$

where

$$
\delta_i^{(n)} = \frac{\alpha_i^n}{\alpha_0^n} - 1
$$

$$
\tilde{B}|_s = \tilde{B}|_0 - 3\delta_{2s}^{(2)} - \frac{1}{11} \delta_{1s}^{(2)},
$$

and where

$$
\sigma_p = 1 - \frac{\Pi_s r_s}{\Pi_s r_s(1 - \rho_s) + 2R_s}.
$$

The general behaviour of the bounds is clearly similar to that in the usual unification scenario. The bounds are on the particular combination $(2\tilde{m}_{Lii}^2 + \tilde{m}_2^2 - \tilde{m}_{U33}^2 - \tilde{m}_{Q33}^2)|_s$.
Figure 4: Charge and colour breaking bounds with a lower string scale at the quasi fixed point (QFP) for $\mu = 500$ GeV. The figure shows lower bounds on the string scale values of $\tilde{m}_s^2 = m_s^2/M_{1/2}^2$ for varying $\Lambda_s$.

and are most restrictive at the QFP, decreasing as $\tan \beta$ increases. Away from the QFP there is a quadratic dependence on $\tilde{A}_s$ with a minimum at $\tilde{A}_s = O(1)$.

We can now see why the bounds at low scales are far less severe than in the MSSM with unification at the GUT scale. First, close to the QFP, the bound is

$$
(2\tilde{m}_{Lii}^2 + \tilde{m}_2^2 - \tilde{m}_{U33}^2 - \tilde{m}_{Q33}^2) \gtrsim \frac{16}{9} \delta_{3s}^{(2)} - 3\delta_{2s}^{(2)} - \frac{5}{99} \delta_{1s}^{(2)} + f(\tilde{B}|0) \\
= -0.48 + f(\tilde{B}|0),
$$

for $\Lambda_s = 10^{11}$ GeV. Thus the non-degeneracy of gauge couplings and gauginos contributes negatively to the bound even at the QFP. Second, away from the QFP, the bound asymptotes to the values with

$$
\rho_p = \frac{1}{1 + 3.17 \cos^2 \beta_{\text{QFP}}} \sim 0.57.
$$

However, the quantity multiplying $\tilde{M}_s^2$ in the bound is now $(\sigma_p - 1)$ which is a larger negative factor than $(\rho_p - 1)$.

We now further specialise to the mirage unification models with $V_0 = 0$, which have degenerate $A$-terms and degenerate scalar masses at the string scale;

$$
\tilde{A}_s = -1 \\
\tilde{m}_s^2 = \text{unconstrained}.
$$

Contours of the $LH_2$, $LLE$ bound are shown in Fig. 3, for varying $\tan \beta$ and $\Lambda_s$. The diagram shows that a lower string scale removes the dangerous minima. Indeed, for the
WP model value of $\Lambda_s \sim 10^{11}$ GeV, there are no CCB minima appearing along the $LH_2$, $LLE$ direction except close to the QFP ($\tan \beta \lesssim 3$) or for negative $m_s^2$. At the QFP we find that the bound at $\Lambda_s = \Lambda_{GUT}$ is $\tilde{m}_s^2 \gtrsim 0.95$ but drops rapidly towards smaller values of $\Lambda_s$, as shown in Fig. 4. A full numerical determination of the bounds for specific points in parameter space is in accord with Figs. 3 and 4. It also shows that the bounds are in fact not overly sensitive to the precise values of $\alpha_1$ and $\alpha_2$ at $\Lambda_s$ since the running is dominated by $\alpha_3$.

Moreover, this behaviour is expected to be a general feature resulting from the low string scale pushing the CCB minimum to low scales. For example we can analyse the bound at large $\tan \beta$ where Eq.(36) holds. Choosing $M_s^2 = 0$ and adjusting $A_s$ to make $A_0 = M_{1/2}$, one finds that, away from the QFP, there are no CCB minima for any positive choice of non-universal mass-squared parameters at the string scale for $\Lambda_s \lesssim 10^{10}$ GeV. In other words, for these intermediate and low string scales one may always adjust $A_s$ to remove CCB minima. Conversely, choosing a large enough value of $A_s$ forms a CCB minimum at any $\Lambda_s$.

For $\Lambda_s \lesssim 10^7$ GeV the analytic approximations we have been using break down for reasons outline above. Specifically, instead of evaluating the parameters at the renormalisation scale $\phi = h t(h^0_2)$, it is now more accurate to evaluate them at the scale $\phi = g_2(l)$ (in the $LLE, LH_2$ direction) since this would be the largest relevant mass. Using this definition for $\phi$ we find numerically that minima do not reappear when $\Lambda_s$ is lowered still further, as expected due to the dominance of the positive $m_L^2 + m_L^2 + m_E^2$ contribution to the potential at low VEVs.

4 Summary

To summarise, we have examined constraints on the MSSM coming from the QFP scenario and CCB bounds when the string scale is lower than the canonical unification value of $10^{16-17}$ GeV. The quasi-fixed behaviour is weakened somewhat as the scale is reduced, i.e. weak MSSM parameters retain more information about their high energy boundary conditions. Very strict bounds upon the string scale are obtained from the LEP2 lower bound upon the lightest Higgs mass in the QFP scenario. Current limits exclude the QFP scenario for string scales between $10^8$ and $10^{14}$ GeV. This range of exclusion will increase by the end of running of LEP2, as the bounds improve. The currently most restrictive bounds upon the QFP are the Higgs mass constraint and CCB bounds. We then provided an analytic treatment of CCB bounds with lower string scales which we confirmed numerically. It is clear from our results that lowering the string scale significantly weakens the CCB bounds. As an example, we considered the most restrictive case of the QFP. In this case the lower bound upon string-scale, degenerate, scalar mass-squareds $\tilde{m}_s^2$ is weakened by 30% in the WP model, $\Lambda_s = 10^{11}$ GeV. Remarkably, for $\tan \beta > 2$ and


$\Lambda_s < 10^{10}$ GeV, the CCB bound is merely $m^2 > 0$ for any non-universal pattern of supersymmetry breaking.

Although we have concentrated on a particular subset of models (i.e. those that preserve the ‘GUT gaugino relation’), we argue that this is generally true. As the string scale is lowered, provided that all mass-squareds are initially positive, the CCB minima are inevitably pushed to lower VEVs. At these low scales, the negative $m^2$ term no longer dominates the potential along the most dangerous $F$ and $D$-flat directions.

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References


