Confining Properties of Abelian-Projected Theories and Field Strength Correlators

D. Antonov\textsuperscript{a} and D. Ebert\textsuperscript{b}

\textsuperscript{a}Institute of Theoretical and Experimental Physics, RU - 117 218 Moscow, Russia

\textsuperscript{b}Theoretical Physics Division, CERN, CH - 1211 Geneva 23

\textsuperscript{b}Institut für Physik, Humboldt-Universität, D - 10115 Berlin, Germany

We review the string representations of Abelian-projected SU(2)- and SU(3)-gauge theories and their application to the evaluation of bilocal field strength correlators. The large distance asymptotic behaviours of the latter ones are shown to be in agreement with the Stochastic Vacuum Model of QCD and existing lattice data.

1. INTRODUCTION

The explanation and description of the confinement phenomenon in gauge theories is known to be one of the most fundamental challenges of the modern QFT (see e.g. [1]). Till now, an analytical description of this phenomenon is best of all elaborated on in the theories containing monopole ensembles. Those include, in particular, 3D compact QED [1], where monopoles form a dilute gas, and QCD-inspired Abelian-projected SU(\(N\)) gauge theories [2]. In the latter case, the dominance of Abelian degrees of freedom is usually realized by assuming that non-Abelian (charged) gauge bosons are heavy and do not propagate at a long-distance scale. In particular by partially fixing the gauge, one removes there as many non-Abelian degrees of freedom as possible, leaving the maximal Abelian subgroup \([U(1)]^{N-1}\) unbroken. The relevant IR degrees of freedom of the resulting Abelian-projected theories are then described by the field strengths of the \((N - 1)\) Abelian (neutral) gauge bosons supplemented by the same amount of magnetic Dirac strings generated by singular gauge transformations. It is further very convenient to reformulate these theories by means of a duality transformation, which leads to a theory of dual Abelian gauge fields interacting with \((N - 1)\) patterns of monopole currents. Finally, performing the summation over the grand canonical ensembles of such currents by making use of the Bardakci-Samuel formula [3], one obtains an equivalent effective Ginzburg-Landau type Lagrangian for disorder magnetic Higgs fields. Those belong to the maximal Abelian subgroup \([U(1)]^{N-1}\) [4] and describe the condensates of monopole Cooper pairs. The effective theory described by this Lagrangian allows for the formation of nonvanishing v.e.v.'s of magnetic Higgs fields mentioned above and thus realizes the beautiful dual Meissner scenario of confinement proposed many years ago by ‘t Hooft and Mandelstam [5].

Since confinement is usually associated with the formation of strings (tubes of electric flux between external quarks) [6], it seems natural to seek for the description of this phenomenon in terms of elementary string excitations. Obviously, the dynamical scheme of a dual superconductor mentioned above provides us with a very convenient theoretical framework for studying this problem by a derivation of string representations of the related effective Abelian-projected theories. In fact, due to the multivaluedness of the phase of the Higgs field, there should arise
singularities in the dual gauge fields, which are natural to be identified with electric Abrikosov-Nielsen-Olesen (ANO) type strings [7]. Moreover, by virtue of the Higgs mechanism, monopole condensation leads to the nonvanishing mass of the dual gauge bosons, which sets the scale of the resulting string tension.

In the present talk, we shall briefly review recent progress achieved in the construction of string representations of the above mentioned Abelian-projected $SU(N)$ theories. Possible consequences of the obtained results for the realistic QCD, based on the so-called Stochastic Vacuum Model (SVM) [8], will also be discussed. In our interpretation of this subject in the two subsequent sections, where the $SU(2)$- and $SU(3)$-cases will be successively considered, we shall follow Refs. [9] and [10] (see Ref. [11] for an extended review).

2. $SU(2)$-THEORY

As it is commonly argued, the Abelian-projected $SU(2)$-gluodynamics is just the Dual Abelian Higgs Model (DAHM), whose action in the limit (i.e. the limit of infinitely large coupling constant $\lambda$ of the magnetic Higgs field) has the form

$$S_{SU(2)^*} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}^2 + \frac{\eta^2}{2} (\partial_{\mu} \theta - 2 g B_{\mu})^2 \right].$$

(1)

Here, $F_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}$ is the field strength tensor of the dual vector potential $B_{\mu}$, $g$ is the magnetic coupling constant, and $\eta$ is the e.e.v. of the magnetic Higgs field. In the London limit under study, the radial part of the latter one has been integrated out, whereas its phase has the form $\theta = \theta^{\text{sing}} + \theta^{\text{reg}}$, where the multivalued part $\theta^{\text{sing}}$ describes a given electric string configuration, while $\theta^{\text{reg}}$ stands for a single-valued fluctuation around such a configuration. Owing to the fact that the singularities of the phase of the magnetic Higgs field occur at the world-sheets of closed electric ANO type strings [7], there exists a correspondence between $\theta^{\text{sing}}$ and string world-sheets, given by the equation

$$\varepsilon_{\mu\nu\lambda\rho} \partial_{\lambda} \partial_{\rho} \theta^{\text{sing}}(x) = 2\pi \Sigma_{\mu\nu}(x) \equiv 2\pi \int_{\Sigma} d\sigma_{\mu\nu}(x) \delta(x - x(x)).$$

(2)

Here, $x(x) \equiv x_{\mu}(x)$ is a vector parametrizing the world-sheet $\Sigma$ with $x \equiv (\xi^1, \xi^2)$ standing for the 2D coordinate. This correspondence eventually enables one to reformulate the integration over $\theta^{\text{sing}}$’s as an integration over $x_{\mu}(x)$’s. The resulting partition function has the form

$$Z_{SU(2)^*} = \int D\sigma_{\mu\nu}(x) \exp \left(-S_{\text{str}}^{SU(2)^*}\right),$$

where the string effective action reads [9]

$$S_{\text{str}}^{SU(2)^*} = \frac{g\eta^2}{2} \int d^4x \int d^4y \times \Sigma_{\mu\nu}(x) \left| K_1(|x - y|) \Sigma_{\mu\nu}(y) \right|^2.$$ 

(3)

Here, $m = 2g\eta$ is the mass of the dual gauge boson generated by the Higgs mechanism, and $K_1$ stands for the modified Bessel function. The reader is referred to the above cited papers for details of the so-called path-integral duality transformation, which leads to Eq. (3).

Performing the expansion of the action (3) in powers of the derivatives w.r.t. $\xi^a$’s, it has been shown that the first two terms of this expansion are the standard Nambu-Goto one and the so-called rigidity term, i.e.

$$S_{\text{str}}^{SU(2)^*} \simeq \sigma \int d^2\xi \sqrt{\hat{g}} + \frac{1}{\alpha_0} \int d^2\xi \sqrt{\hat{g}} \hat{g}^{ab} (\partial_a t_{\mu\nu}) (\partial_b t_{\mu\nu}).$$

(4)

Here, $\partial_a = \partial/\partial \xi^a$, $\hat{g} = \det |\hat{g}^{ab}|$ with $\hat{g}^{ab} = ((\partial^a x_{\mu}(\xi))(\partial^b x_{\mu}(\xi)))$ being the induced metric tensor of the world-sheet, and $t_{\mu\nu} = \frac{\hat{g}_{ab}}{\sqrt{\hat{g}}} (\partial_a x_{\mu}(\xi))(\partial_b x_{\nu}(\xi))$ standing for the extrinsic curvature tensor. The coupling constant of the Nambu-Goto term (also called string tension) with the logarithmic accuracy reads $\sigma \simeq \pi\eta^2 \ln \frac{\lambda}{\eta}$, while the inverse coupling constant of the rigidity term (considered first in Ref. [12]) has the form $\frac{1}{\alpha_0} = -\frac{\pi}{2\sqrt{g}}$. In particular, if external quarks are introduced into the system, the Nambu-Goto action yields a linearly rising quark-antiquark potential $V_{\text{conf}}(R) = \sigma R$. Notice also
the negative sign of the coupling \( a_0 \), which reflects the stability of strings.

Another important subject investigated in Ref. [9] is the evaluation of the irreducible biholomorphic field strength correlator (cumulant) \( \left\langle \langle \vec{F}_{\lambda \nu}(x) \vec{F}_{\mu \rho}(0) \rangle \right\rangle \), where \( \vec{F}_{\mu \nu} \equiv \frac{1}{2} \varepsilon_{\mu \nu \lambda \rho} F_{\lambda \rho} \), by a derivation of its string representation. Parametrizing the \( x_\mu \)-independent Lorentz structure of the cumulant according to the SVM [8] as \((\delta_{\lambda \mu} \delta_{\nu \rho} - \delta_{\lambda \rho} \delta_{\nu \mu}) D (x^2)\), we find the following IR asymptotic behaviour of the function \( D \) at the distances \( |x| \gg \frac{1}{m} \):

\[
D \rightarrow \frac{m^4}{4 \sqrt{2} \pi^2 (m|x|)^2} \tag{5}
\]

This behaviour is very similar to the one observed in the lattice simulations of QCD in Ref. [13]. In particular, one can see that the rôle of the so-called correlation length of the vacuum, \( T_g \), at which the cumulant in SVM decreases, is played in DAHM by the inverse mass of the dual gauge boson, \( m^{-1} \). The evaluation of the second coefficient function parametrizing the cumulant, which stands at the omitted \( x_\mu \)-dependent Lorentz structure, also matches the existing lattice data [13] with a good accuracy.

3. SU(3)-THEORY

The effective Abelian-projected theory of the SU(3)-gluodynamics [4] is also of the DAHM type, albeit with the \([U(1)]^2 \) gauge invariance w.r.t. to the maximal Abelian subgroup of the SU(3)-group. In the London limit, the action under study reads

\[
S_{\text{str.}^{\text{SU}(3)}^\gamma} = \int d^4 x \times
\left[ \frac{1}{4} \vec{F}_{\mu \nu}^2 + \frac{\eta^2}{2} \sum_{a=1}^{3} \left( \partial_\mu \theta_a - 2 g \varepsilon_a \vec{B}_\mu \right)^2 \right] , \tag{6}
\]

where \( \vec{F}_{\mu \nu} = \partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu \) is the field strength tensor of magnetic fields \( \vec{B}_\mu = (B_{\mu}^3, B_{\mu}^8) \), which are dual to the usual gluonic fields \( A_{\mu}^3 \) and \( A_{\mu}^8 \). Next, in Eq. (6) the so-called root vectors \( \vec{e}_1 = (1, 0) \), \( \vec{e}_2 = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \), \( \vec{e}_3 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \) have been introduced. These vectors naturally emerge during the Cartan decomposition as the structural constants in the commutation relations between the diagonal and (properly redefined) non-diagonal SU(3)-generators. Besides that, the action (6) is assumed to be supplied by the following constraint imposed on the phases \( \theta_a \) of magnetic Higgs fields, \( \sum \theta_a = 0 \), which is just the reflection of the fact that the original SU(3) group is special. Next, the relation (2) remains the same, with the substitution \( \theta^\text{sing} \rightarrow \theta^\text{Sing}, \Sigma_{\lambda \nu} \rightarrow \Sigma_{\mu \nu}, \Sigma \rightarrow \Sigma^a, \) and \( x(\xi) \rightarrow x^{(a)}(\xi) \). Consequently, there exist three different types of electric strings, among which, however, only two are independent of each other owing to the above-mentioned constraint. Performing the path-integral duality transformation of the theory (6) and integrating out one of the world-sheets (for concreteness, \( x_3^\mu \)), we arrive at the desired string representation for the partition function, which reads \( Z^{\text{SU}(3)} = \int D x_3^\mu(\xi) D x^{(a)}(\xi) \exp \left(-S_{\text{str.}^{\text{SU}(3)}^\gamma}\right) \).

Here, the string effective action has the form [10]

\[
S_{\text{str.}}^{\text{SU}(3)} = g \eta^3 \sqrt{\frac{3}{2}} \int d^4 x \int d^4 y \times \left[ \sum_{a=1}^{3} (\Sigma^a_{\mu \nu}(x)) \Sigma^a_{\mu \nu}(y) + \sum_{\mu \nu} (\Sigma^1_{\mu \nu}(x)) \Sigma^2_{\mu \nu}(y) + \sum_{\mu \nu} (\Sigma^2_{\mu \nu}(x)) \Sigma^3_{\mu \nu}(y) \right] K_1 \left( \frac{m_B |x - y|}{|x - y|} \right) , \tag{7}
\]

where \( m_B = \sqrt{6} g \eta \) is the mass of the fields \( \vec{B} \). One can see that, according to Eq. (7), the most crucial difference of the string effective theory corresponding to the Abelian-projected SU(3)-gluodynamics w.r.t. the SU(2)-case is the presence of two independent kinds of strings, which not only self-interact, but also interact with each other by the exchanges of the massive dual gauge bosons.

As far as the cumulants of the field strength tensors \( F_{3,8} \) are concerned, only for those of them, which are the diagonal ones, i.e. \( \left\langle \langle \vec{F}_{3 \lambda \nu}(x) \vec{F}_{3 \mu \rho}(0) \rangle \right\rangle \) and \( \left\langle \langle \vec{F}_{8 \lambda \nu}(x) \vec{F}_{8 \mu \rho}(0) \rangle \right\rangle \), the vacuum does exhibit a nontrivial correlation length \( T_g = \frac{1}{m_B} \). In particular, the IR asymptotics (5) of the function \( D \) remains valid for these two cumulants with the replacement \( m \rightarrow m_B \).
4. CONCLUSIONS

The obtained results demonstrate the relevance of the method of Abelian projection and the path-integral duality transformation to the description of the confining properties of the $SU(2)$- and $SU(3)$-gluodynamics. The approach considered above also provides us with a new field-theoretical status of SVM and sheds some light on the problem of finding an adequate string representation of QCD. Finally, it is worth mentioning that the field strength correlators in DAHM have been also investigated beyond the London limit in Refs. [14] and [11].

REFERENCES


Discussions

N. Brambilla (University of Vienna)

Why do you use the London limit? I think that this limit is appropriate for large transverse distances from the string, but not for large distances between an external quark and an antiquark.

D. Ebert

The London limit has been used here as a simplifying assumption leading to infinitely thin strings. If one includes an external quark-antiquark pair, this scheme leads to a confinement potential plus a Yukawa interaction (arising as a boundary term). Indeed, it would be interesting to go beyond the London limit in the sense of taking into account the vanishing of the Higgs field inside the finite core of strings. This is expected to yield, besides the usual confinement potential, a Coulomb potential instead of the Yukawa one.