Neutrino masses and mixing angles from leptoquark interactions

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Abstract

In this paper we show that the mixing between leptoquarks (LQ’s) from different $SU(2)$ multiplets can generate a non-trivial Majorana mass matrix for neutrinos through one loop self energy diagrams. Such mixing can arise from gauge invariant and renormalizable LQ-Higgs interaction terms after EW symmetry breaking. We use the experimental indication on neutrino oscillation to find constraints on specific combinations of LQ couplings to quark-lepton pairs and to the SM higgs boson. These constraints are compared with the ones from $\pi \rightarrow e\bar{\nu}_e$. 
I. Introduction

The recent neutrino data from Superkamiokande (SK) [1] has provided strong evidence for $\nu_\mu$ oscillating into $\nu_\tau$ or some species of sterile neutrinos as an explanation of the atmospheric neutrino anomaly. The observed solar neutrino deficit [2] is also probably an indication of $\nu_e$ oscillating into some other species of neutrinos. Results from the laboratory experiment by the LSND collaboration [3] can also probably be considered as yet another type of neutrino oscillation. Neutrino oscillation imply a non-trivial structure of the neutrino mass matrix. The extreme smallness of neutrino masses compared to quarks and charged lepton masses naturally suggests that they are zero at the tree level but are generated through higher order loop corrections. The coupling constants involved in this radiative mass generation must be small in order reproduce the small neutrino masses. In this paper we shall adopt this viewpoint and show how the experimentally suggested neutrino masses and mixing angles could be generated at one loop level through leptoquark (LQ) interactions.

Leptoquarks (LQ’s) [4] occur naturally in many extensions of the SM namely grand unified models, extended technicolor models and models of quark lepton substructure which contain quarks and leptons in the same multiplet. If LQ’s couple both to quark pairs and quark-lepton pairs then they mediate too rapid proton decay. To avoid this disaster and to keep the LQ interpretation interesting for low energy phenomenology it is necessary that LQ couplings to quark pairs must vanish. In order to enable LQ’s to couple to quark-lepton pairs in the context of the $SU(3)_c \times SU(2)_l \times U(1)_y$ theory, they must be color triplets or antitriplets. Further they can form a singlet, doublet or triplet representation of $SU(2)_l$. Just like quarks and leptons come in different generations LQ’s can also carry a generation index. In this work we shall assume that the number of LQ generations is equal to that of fermions ($n_g = 3$). This happens in the superstring inspired E(6) model and also in the E(8) $\sigma$ models. The coupling constant of LQ’s to q-l pairs in general can depend on the the generation indices of the LQ, the quark and the lepton. Such a general flavor structure
for LQ couplings was first considered in ref. [4] for singlet LQ’s. Here we shall extend this structure to doublet and triplet LQ’s also. The existing flavor changing neutral current data however strongly constrains the flavor off diagonal couplings of LQ’s to q-l pairs. Besides coupling to q-l pairs, LQ’s can also couple to the SM gauge bosons and to the higgs doublet. Of particular importance for this paper are the Higgs-LQ interaction terms which lead to mixing between LQ’s from different $SU(2)_l$ multiplets after EW symmetry breaking. We shall show that this mixing among different LQ multiplets gives rise to a non-trivial lepton number violating Majorana mass matrix for neutrinos through loop corrections.

The contents of this article are divided into the following subsections. In Sec. II we present the interaction Lagrangians of singlet, doublet and triplet LQ’s with q-l pairs and with the higgs doublet. In Sec. III we derive the neutrino mass matrix that arises from the mixing of doublet and singlet LQ’s. In Sec. IV we show that the mixing between doublet and triplet LQ’s can also generate a non-trivial mass matrix for neutrinos. In Sec. V we use the experimental indication on neutrino oscillation to derive some constraints on specific combinations of LQ couplings and compare them with the existing constraints. Finally in Sec. VI we present the main conclusions of our study.

II. Interaction Lagrangians for Leptoquarks

We shall assume that the light LQ’s present in the low energy theory arise from an underlying theory that breaks down into the SM gauge group at some high energy scale $\Lambda \gg v$. To discuss the low energy LQ phenomenology in a model independent way we shall construct its Lagrangian based on invariance under the SM gauge group and renormalizability. To satisfy the strong constraints [4] of the helicity suppressed decay $\pi \rightarrow e\bar{\nu}_e$ on LQ’s with non chiral couplings we shall assume that each LQ couples to quarks of a particular chirality only. To obtain a non-trivial flavor structure for the neutrino mass matrix we shall retain the flavor off diagonal LQ couplings to q-l pairs but assume that they are small enough to satisfy the constraints of FCNC data. Consider three scalar LQ
fields $D$, $S$ and $T$ with the following $SU(3)_c \times SU(2)_l \times U(1)_y$ assignments: $D \sim (3^*, 2, \frac{1}{3})$, $S \sim (3^*, 1, \frac{1}{3})$ and $T \sim (3^*, 3, \frac{1}{3})$. The general flavor structure of their Yukawa couplings to $q$-$l$ pairs are given by

$$L_1 = \left[ \sum \lambda^k_{ij} \bar{q}_i \tau_2 q_j S_k + \sum \lambda^{nk}_{ij} \bar{d}_R i \tau_2 q_j T^{n}_k \right] + h.c. = \left[ \sum \lambda^k_{ij} \bar{\nu}^c_{L_i} d_{L_j} - \bar{e}_{L_i}^c u_{L_j} S_k + \sum \lambda^{nk}_{ij} \bar{D}_R (\nu_{L_j} D^{+1}_k + e_{L_j} D^{+2}_k) \right] + \sum \lambda^{nk}_{ij} \bar{e}_{L_i}^c d_{L_j} T^k + \left[ \sum \lambda^{nk}_{ij} \bar{\nu}^c_{L_i} u_{L_j} T^k \right] + h.c. \quad (1)$$

In the above $i,j,k$ refer to the generation index of the relevant field. Repeated indices are all summed over. $q$ and $l$ are LH quark and lepton fields. $D_R$ is the RH down quark field. $\psi^c = C\bar{\psi}^T$ is the charge conjugated fermion field. $D^{+1}_k$ is the $I_3 = \frac{1}{2}$ component of $D^{+}_k$. $T^{+}_k = T^{+1}_k + i T^{+2}_k$ and $T^{-}_k = T^{+1}_k - i T^{+2}_k$. $T^{+}_k$, $T^{-}_k$ and $T^3_k$ have the following $I_3$, and $Q$ assignments: $T^{+}_k \sim (1, \frac{4}{3})$, $T^{-}_k \sim (0, \frac{1}{3})$ and $T^{3}_k \sim (-1, -\frac{2}{3})$. Since $T^a_k$ carries $U(1)_y$ charge $T^a_k \neq (T^a_k)^+$. The above interaction terms are written in the gauge eigenstate basis for the relevant fields. From the above Lagrangian it follows that $S_i$, $T_i$ and $D_i$ have lepton numbers of -1, -1 and +1 respectively. Note that since the Yukawa couplings of $D_i$, $S_i$ and $T_i$ are chiral in nature they cannot generate any neutrino mass through radiative corrections unless we add new interactions.

The Higgs-LQ interaction can be expressed by the following Lagrangian

$$L_2 = \left[ \sum K_i (D^{+}_i \phi_c) S_i + \sum K^i_\alpha (\bar{\phi}_c \tau_\alpha D_i) T^{+\alpha}_i \right] + h.c. = \left[ \sum \mu^2_\alpha (\chi^{+\alpha}_i \chi^{\alpha}_i) + \sum h^i_\alpha (\phi^+ \phi) (\chi^{+\alpha}_i \chi^{\alpha}_i) \right] = v + h \left[ \sum K_i D^{+\ast}_i S_i + K^i_\alpha D^{2\ast}_i T^{-\alpha}_i + K'_i D^{1\ast}_i T^{3\ast}_i \right] + h.c. = \sum \mu^2_\alpha (\chi^{+\alpha}_i \chi^{\alpha}_i) + \sum h^i_\alpha (\phi^+ \phi) (\chi^{+\alpha}_i \chi^{\alpha}_i) \quad (2)$$
where $\phi_c = i\tau_2\phi^*$. $\chi^i_\alpha$ is a collective symbol for all the LQ fields with $\alpha = 1, 2, 3$ referring to D, S and T respectively. Note that the first two terms of $L_2$ violate lepton number by two units. This will be the source of the lepton number violating Majorana masses for neutrinos. If the SM Higgs doublet $\phi$ and the LQ’s belong to the same multiplet of some higher symmetry group then such interaction terms can naturally arise when the higher symmetry group breaks down into $SU(3)_c \times SU(2)_l \times U(1)_y$ at the high energy scale $\Lambda$. The same symmetry breaking mechanism might break lepton number also. After EW symmetry breaking at the Fermi scale $L_2$ will generate mixing between different LQ multiplets. In the following we shall consider the mixing between two different LQ multiplets at a time. However our results will not be very different from the more general case where all the mixing terms are present provided there is no accidental cancellation among different contributions. Third the neutrino mass matrix vanishes if $m_{Di}^2 = m_{Si}^2$ for all $i$. The reason being under this condition the dimensional parameter $K_i$ that mixes $D_i$ with $S_i$ vanishes. The couplings of $D_i$ and $S_i$ are then purely chiral and therefore no neutrino mass is generated.

**Neutrino mass matrix from doublet-singlet mixing**

After EWSB in unitary gauge the doublet-singlet mixing term can be written as

$$L_3 = \sum K_i \left( \frac{v + h}{\sqrt{2}} \right) (D_i^1 S_i^* + S_i^* D_i^1)$$

The mass matrix for LQ’s mixes $D_i^1$ with $S_i$ which carry the same charge and color quantum numbers. It can be shown that the LQ fields under consideration in the mass eigenstate basis are given by $D_i^{1'} = \cos \theta_i D_i^1 + \sin \theta_i S_i$ and $S_i^*' = -\sin \theta_i D_i^1 + \cos \theta_i S_i$. The mixing angle $\theta_i$ is given by $\sin \theta_i = \frac{v K_i}{\sqrt{2b_i^2 + v^2}}$. $a_i = m_{Di}^2 + m_{Si}^2$ and $b_i = m_{Di}^2 - m_{Si}^2$. $m_{Di}$ and $m_{Si}$ are the shifted masses after EWSB and are given by $m_{Di}^2 = \mu_{Di}^2 - h_D^i \frac{v^2}{2}$ and $m_{Si}^2 = \mu_{Si}^2 - h_S^i \frac{v^2}{2}$. The primed fields refer to the mass eigenstate basis. Their masses are given by $m_{Di}^2 = \frac{1}{2} [a_i + b_i + \frac{v^2 K_i^2}{b_i}]$ and $m_{Si}^2 = \frac{1}{2} [a_i - b_i - \frac{v^2 K_i^2}{b_i}]$. All phenomenological
implications of LQ interactions must be derived in terms of fields in the mass eigenstate basis. When the interactions of $D_1^i$ and $S_i$ with the $d$-$\nu$ pair are written in terms of mass eigenstate fields we get

$$L_4 = \sum \lambda^{k}_{ij} \bar{\nu}_{L_i} d_{L_j} \left( \sin \theta_k D_k' + \cos \theta_k S_k' \right)$$

$$+ \lambda'^{k}_{ij} \bar{d}_{R_i} \nu_{L_j} \left( \cos \theta_k D_k' - \sin \theta_k S_k' \right)$$

(4)

Note that the mixing between $D_1^i$ and $S_i$ induced by the higgs doublet introduces non-chiral couplings for $D_1'^i$ and $S_i'$ which enables them to generate neutrino masses through radiative corrections. It can be shown that one loop self energy diagrams involving the exchange of $D_1'^i$ and $S_i'$ gives rise to the following Majorana mass matrix for neutrinos

$$M_{ik} \approx \frac{N_c}{16\pi^2} \sum \lambda'^{l}_{ji} \lambda'^{l}_{kj} m_{d_l} \ln \frac{m_{D_l}^2}{m_{S_l}^2} \sin \theta_l \cos \theta_l$$

(5)

In the above we have neglected the matrices $D_L$ and $D_R$ that connect the quark gauge eigenstates $d_L$ and $d_R$ to their respective mass eigenstates. We would like to note first that the individual diagrams for $D_1'^i$ and $S_i'$ exchange are separately log divergent. But the log divergences cancel each other in the sum yielding a finite result. Second the extension of the SM considered here does not include any right handed neutrino and therefore the seesaw mechanism does not play any role here. The smallness of neutrino masses in our case has to follow from the smallness of $\lambda$, $\lambda'$ and $K_i$.

**IV. Neutrino masses from doublet triplet mixing**

The mixing between doublet and triplet LQ’s can also give rise to a non-trivial Majorana mass matrix for neutrinos. The mixing arises from the following LQ-Higgs interaction term

$$L_5 = \sum K'_n (\phi^+_c \tau_a D_n) T_n^{a+} + h.c.$$  

$$= \frac{v + h}{\sqrt{2}} \left[ \sum K'_n D_n^1 \tau_n^{3+} + \sum K'_i D_i^2 T_i^- \right] + h.c.$$  

(6)
The above Lagrangian implies that $D_n^1$ mixes with $T_n^3$ and $D_n^2$ mixes with $T_n^-$. The general flavor structure of the Yukawa couplings of $T_n^a$ to q-l pairs is given by

$$L_6 = \sum \lambda''^n_{ij} \bar{l}_i \tau_i \tau_2 q_j T_n^a + h.c.$$ 

$$= \left[ \sum \lambda''^n_{ij} (\bar{\nu}_{L_i} d_{L_j} + e_{L_i} c_{L_j}) T_k^3 + \ldots \right] + h.c.$$ 

$$= \left[ \sum \lambda''^n_{ij} (\bar{\nu}_{L_i} d_{L_j} + e_{L_i} c_{L_j}) (T_k'^3 \cos \theta'_k - D_k'^1 \sin \theta'_k) + \ldots \right] + h.c. \quad (7)$$

where $\theta'_k$ is the mixing angle between $T_k^3$ and $D_k'^1$. Proceeding as in Sec. III it can be shown that one loop self energy diagrams involving the exchange of $D_k'^1$ and $T_k'^3$ generates the following mass matrix for neutrinos

$$M_{ik} \approx \frac{N_c}{16\pi^2} \sum \lambda''^m_{ij} \lambda''^l_{kj} m_{d_j} \ln \frac{m_{T_k'^1}}{m_{D_k'^1}^2} \sin \theta'_l \cos \theta'_l \quad (8)$$

V. Implications of neutrino oscillation data on LQ couplings

The experimental data indicating neutrino oscillation can be used to find constraints on specific combinations of LQ couplings. To be specific let us consider the doublet-singlet mixing case. In general the neutrino mass matrix $M_{ik}$ is not symmetric. In the following we shall assume that $\lambda''^m_{ij} = \lambda''^l_{ij}$ for all $i,j$ so that the matrix $M_{ik}$ is symmetric and hence diagonalizable by a $3 \times 3$ orthogonal matrix $U$. The matrix $U$ can be completely specified in terms of three angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$. It has been shown [5] that the recent SK data, the CHOOZ data [6] and the solar neutrino data can be accomodated in a three neutrino oscillation model at 99% C.L. for $\sin \theta_{12} = .63$, $\sin \theta_{23} = .71$, $\sin \theta_{13} = .45$, $\delta m_{32}^2 = m_3^2 - m_2^2 = 8 \times 10^{-4} \text{eV}^2$ and $\delta m_{21}^2 = m_2^2 - m_1^2 = 1 \times 10^{-4} \text{eV}^2$. If we take $m_1 = .01 \text{eV}$, the elements of the neutrino mass matrix in the flavor basis will be given by $M_{11} = .015$ ev, $M_{12} = M_{21} = .007$ ev, $M_{13} = M_{31} = .004$ ev, $M_{22} = .020$ ev, $M_{23} = M_{32} = .008$ ev and $M_{33} = .021$ ev. We would like to emphasize that the value of $m_1$ cannot be chosen arbitrarily. It must satisfy the constraint from neutrinoless double beta decay which provides a bound on $M_{11}$. The present experimental upper bound on the effective
Majorana neutrino mass in the flavor basis is given by $M_{11} < 0.2 - 0.4 \text{ eV}$ at 90% CL [7]. The range of the upper bound is mainly due to the uncertainty in the theoretical calculation of the nuclear matrix elements. Besides the resulting values of $m_2$ and $m_3$ must satisfy the bounds $m_2 < .17 \text{ Mev}$ and $m_3 < 18 \text{ Mev}$ [8]. Using the expression of $M_{ik}$ derived earlier we can now find constraints on particular combinations of LQ couplings. Since the values of $M_{ik}$ for different i,k do not differ much among them we shall consider the constraints that arise from only one of them namely $M_{11}$. We shall assume that the value of $\ln \frac{m_D^2}{m_S^2}$ do not change appreciably with generation. For $m_D' = 300 \text{ Gev}$ and $m_S' = 200 \text{ Gev}$ we get $\sum (\lambda_{i1}^j)^2 m_{d_j} \sin \theta_i \cos \theta_i \approx 0.74 \times 10^{-9} \text{ Gev}$. Further if we assume that only one product coupling is non-zero at a time we get $(\lambda_{11}^j)^2 \sin \theta_1 \cos \theta_1 \approx 0.74 \times 10^{-7}$, $(\lambda_{12}^j)^2 \sin \theta_1 \cos \theta_1 \approx 5.36 \times 10^{-9}$ and $(\lambda_{13}^j)^2 \sin \theta_1 \cos \theta_1 \approx 1.48 \times 10^{-10}$ indicating a flavor dependent hierarchy in LQ couplings. If we set $M_{11} = .4 \text{ eV}$ which is the present upper bound from neutrinoless double beta decay we get $(\lambda_{11}^j)^2 \sin \theta_1 \cos \theta_1 \approx 1.97 \times 10^{-6}$. For a LQ mass of 200 Gev, HERA and Tevatron will soon be able to probe flavor diagonal LQ couplings for first generation down to .1. If we set $\lambda_{11}^j \approx .1$ we get $K_1 \approx 2.85 \text{ Mev}$ which is close to the light quark masses. It is interesting to compare the constraints on LQ couplings derived from neutrino oscillation data with those derived from other low energy experiments e.g. $\pi \rightarrow e\bar{\nu}e$. The mixing between different multiplets of LQ’s introduces non-chiral couplings for LQ’s. Such non-chiral LQ couplings can give rise to helicity unsuppressed contributions to $\pi \rightarrow \bar{\nu}e$. For the exchange of S we have $g_R = \lambda_{11}^j \sin \theta_1$ and $g_L = \lambda_{11}^j \cos \theta_1$. From ref[4] we then get $g_R g_R = (\lambda_{11}^j)^2 \sin \theta_1 \cos \theta_1 < \frac{m_S^2}{(100 \text{ Tev})^2} \approx 4 \times 10^{-6}$. Actually the contributions due to both $D_i^1$ and $S_i$ must be taken into account which raises the bound to $8 \times 10^{-6}$. This bound is nearly two orders of magnitude greater than the value provided by the neutrino oscillation data for $m_1 = .01 \text{ eV}$. However it agrees with the value provided by the neutrino oscillation data for $M_{11} \sim 0.4 \text{ eV}$. The next generation of neutrinoless double beta decay is expected to be sensitive to values of $M_{11}$ in the range $10^{-2} - 10^{-1} \text{ eV}$. It will enable us to decide whether the neutrino data gives more stringent bounds on LQ couplings than the
pion decay rate.

Coming next to other bounds on flavor off-diagonal LQ couplings, we note that flavor changing radiative decays like $\mu \rightarrow e\gamma$ require non-chiral couplings for LQ’s and hence mixing between different multiplets of LQ’s. To be specific consider the higgs induced mixing between $D_1^{l_i}$ and $S_i$. But this mixing does not contribute to $\mu \rightarrow e\gamma$ since $D_1^{l_i}$ does not couple to $e_i - q$ pair. Similarly the mixing between $D_1^{l_i}$ and $T_3^{l_i}$ or $D_2^{l_i}$ and $T_{-i}$ does not contribute to radiative muon decay because $D_1^{l_i}$ and $T_{-i}$ does not couple to charged lepton-quark pair. The decay $\mu \rightarrow e\gamma$ therefore does not impose any bound on the parameters of our model. We would also like to note that the LQ masses chosen by us are consistent with the latest bounds from HERA and Tevatron [8]. The HERA bounds depend on the value of $g_{lq}$. For $g_{lq} = e$ the HERA bounds are 237 Gev for first generation and 73 Gev for second generation. The Tevatron bounds are most stringent for first generation ($> 225$ Gev) and become progressively weaker for second ($> 131$ Gev) and third generation ($> 95$ Gev). The Tevatron and HERA bounds however depend on several assumptions the most crucial of which is the branching ratio of the LQ into e-q or $\nu$-q pair. The bounds get lower if the branching ratios are lower than those usually assumed.

**VII. Conclusion**

In conclusion in this paper we have shown that the interaction between LQ’s and the the SM higgs doublet induces mixing between LQ’s from different $SU(2)_l$ multiplets after EW symmetry breaking. This mixing introduces small non-chiral couplings for LQ’s in the mass eigenstate basis. and generates a non-trivial Majorana mass matrix for neutrinos through one loop radiative correction. We have determined the neutrino mass matrix in terms of LQ couplings and masses for the singlet-doublet mixing and the doublet-triplet mixing. Using the recent experimental data indicating neutrino oscillation data we have derived constraints on specific combinations of LQ couplings to q-l pairs and to the higgs boson. Such constraints are comparable with those derived from $\pi \rightarrow e\bar{\nu}_e$ if the value of $M_{11}$ is close to the present upper bound provided by the neitriinoless double beta decay.
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