Testing quark mass matrices with right-handed mixings

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In the standard model, several forms of quark mass matrices which correspond to the choice of weak bases lead to the same left-handed mixings $V_L = V_{CKM}$, while the right-handed mixings $V_R$ are not observable quantities. Instead, in a left-right extension of the standard model, such forms are ansatze and give different right-handed mixings which are now observable quantities. We partially select the reliable forms of quark mass matrices by means of constraints on right-handed mixings in some left-right models, in particular on $V_{cb}^R$. Hermitian matrices are easily excluded.

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In the framework of the Standard Model (SM), based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, the right-handed mixings are not observable quantities, but they become observable in extensions of the SM such as the Left-Right Model (LRM) $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [1], the Pati-Salam model $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ [2], and the grand unified model $SO(10)$ [3]. Right-handed mixings are the most direct tool to test models of quark mass matrices. Let us explain how this may happen. In the LRM the quark mass and charged current terms are

$$
u_L M_u u_R + \bar{d}_L M_d d_R + g_L \bar{u}_L d_L W_L + g_R \bar{u}_R d_R W_R.$$  

Diagonalization of $M_u, M_d$ by means of the biunitary transformations

$$U_u^\dagger M_u U_u = D_u, \quad U_d^\dagger M_d U_d = D_d$$

gives (renaming the quark fields)

$$\nu_L D_u u_R + \bar{d}_L D_d d_R + g_L \bar{u}_L d_L W_L + g_R \bar{u}_R d_R W_R.$$  

where

$$V_L = U_u^\dagger U_d = V_{CKM}, \quad V_R = U_u^\dagger V_d$$

are the left- and right-handed mixing matrices of quarks, and $D_u, D_d$ have non-negative matrix elements. In the SM the last term in Eqns. (1), (2) is absent and it is possible to perform, without physical consequences, that is without changing the observable quantities appearing in Eqn. (2), the following unitary transformations on the quark fields:

$$u_L \rightarrow \mathcal{U} u_L, \quad d_L \rightarrow \mathcal{U} d_L,$$  

$$u_R \rightarrow \mathcal{V}_u u_R, \quad d_R \rightarrow \mathcal{V}_d d_R.$$  

In the LRM Eqn. (4) must be replaced by

$$u_R \rightarrow \mathcal{V} u_R, \quad d_R \rightarrow \mathcal{V} d_R.$$
that is also \( u_R \) and \( d_R \) must transform in the same way because of the last term in Eqns.(1),(2). From the point of view of quark mass matrices, the consequences of replacing Eqn.(4) with Eqn.(5), keeping Eqn.(3), are the following. In the SM we can use the freedom in \( U \) and \( V_u \) to choose \( M_u = D_u \). Further we can use the freedom in \( V_d \) to choose \( M_d \) to be hermitian or to have three zeros [5–10]. In the LRM, the second freedom is not there because both the diagonalizing matrices of \( M_d \) are physical observables. This fact means that bases in the SM become ansatze in the LRM, giving the same \( V_L \) but different \( V_R \).

The aim of this Letter is to begin a selection of quark mass matrices in the LRM by using informations on right-handed mixings. In fact, if without loss of generality one sets \( M_u = D_u \), then

\[
V_R^\dagger M_d V_R = D_d,
\]

permits to calculate the right-handed mixing matrix \( V_R \) (values of quark masses at the scale \( M_Z \) and of the mixing \( V_L \) are extracted from refs. [11] and [12]). It is well-known that if \( M_d \) is hermitian or symmetric then \(|V_R| = |V_L|\). These conditions correspond to manifest and pseudomanifest left-right symmetry, respectively [4]. In the general case, however, \( V_R \) is not related to \( V_L \) [4]. Notice that different quark mass and mixing matrices, which correspond to bases in the SM, are connected by a suitable unitary \( U_R \), because

\[
V_R^\dagger M_d V_R = V_L^\dagger M_d U_R^\dagger V_R = V_L^\dagger M_d' V_R',
\]

where \( M_d' = M_d U_R \) and \( V_R' = U_R^\dagger V_R \). Therefore, they give different right-handed mixings in the LRM. For example, let us consider the simple case of the first two generations with real mass matrices (in the LRM mass matrices are complex in general, even for only two generations). The left-handed mixings are given by

\[
V_L \simeq \begin{pmatrix} 1 & \lambda \\ -\lambda & 1 \end{pmatrix}, \ \lambda = 0.22.
\]
In the SM, using a right-handed rotation $\mathcal{V}_d$ it is possible to put one zero in $M_d$ in any position. In the LRM such four forms give different $V_R$. We can get all forms from just one, for example from [10]

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix} \Rightarrow V_R \simeq \begin{pmatrix} -1 & \lambda \\ \lambda & 1 \end{pmatrix},$$

by using in Eqn.(7) $U_R$ like

$$U_R \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix}.$$

In fact

$$M_d U_R = \begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{m_d m_s} & 0 \\ m_s & \sqrt{m_d m_s} \end{pmatrix} = M'_d,$$

$$U_R^\dagger V_R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & \lambda \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} \lambda & 1 \\ -1 & \lambda \end{pmatrix} = V'_R.$$

The mixing $V_{us}^R$ is small on the first basis and large on the second. Moreover,

$$\begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix} = \begin{pmatrix} -\sqrt{m_d m_s} s & \sqrt{m_d m_s} c \\ \sqrt{m_d m_s} c - m_s s & \sqrt{m_d m_s} s + m_s c \end{pmatrix},$$

and imposing the element 2-1 to vanish, we have

$$c = \sqrt{\frac{m_s}{m_s + m_d}} \simeq 1, \quad s = \sqrt{\frac{m_d}{m_s + m_d}} \simeq \lambda,$$

and the third basis

$$\begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix} \begin{pmatrix} \sqrt{\frac{m_s}{m_s + m_d}} & \sqrt{\frac{m_d}{m_s + m_d}} \\ -\sqrt{\frac{m_d}{m_s + m_d}} & \sqrt{\frac{m_s}{m_s + m_d}} \end{pmatrix} \simeq \begin{pmatrix} -m_d & \sqrt{m_d m_s} \\ 0 & m_s \end{pmatrix},$$

$$\begin{pmatrix} 1 & -\lambda \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} -1 & \lambda \\ \lambda & 1 \end{pmatrix} \simeq \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$
And again we can get the fourth basis, from the third, through

\[
\begin{pmatrix}
-m_d & \sqrt{m_d m_s} \\
0 & m_s
\end{pmatrix}
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
= 
\begin{pmatrix}
\sqrt{m_d m_s} - m_d \\
m_s & 0
\end{pmatrix},
\]

\[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix}
= 
\begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}.
\]

Mixing is nearly zero on the third basis and nearly one on the fourth. In this way, also for more than two generations, one can construct different bases in the SM, which are different ansatze in the LRM, from just few of them.

Therefore, let us consider now three generations and label elements in $M_d$ as

\[
\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{pmatrix}.
\]

There are several SM bases with three zeros in $M_d$ [10]. For example zeros can be put in positions 137 [8] and 236 [7], 478 [9], 124. The last form can be obtained from 137 by just relabeling the family indices 2,3. From bases 124, 137, 478 we can calculate fifty-four bases by means of the six special rotations

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix},
\begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix},
\begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix},
\begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{pmatrix},
\begin{pmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix},
\begin{pmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix}.
\]
which produce permutations of columns in $M_d$ and of rows in $V_R$, and suitable unitary transformations of the type

$$\begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\alpha} c & e^{i\beta} s \\
0 & -e^{i\gamma} s & e^{i\delta} c
\end{pmatrix}, \quad \alpha + \gamma = \beta + \delta.$$ 

For each of the three starting forms under examination we have calculated the matrix $M_d$ by the relation [8]

$$M_d M^\dagger_d = V_L D_d^2 V_L^\dagger$$

and $V_R$ by Eqn.(6). For $V_L$ we use the standard parametrization [12]. Moreover, to keep arbitrary representation of $V_L$ one must put three phases (not just one SM observable) in $M_d$ [6]. Their positions for our starting bases are 356, 256, 236, respectively. Putting three phases and their position is part of the ansatze in the LRM, because for three generations of quarks there are seven observable phases, one can be inserted in $V_L$ and six in $V_R$ [13]. However, due to the position of the three zeros, putting the three phases in another position, or more than three phases, up to six, does not change the moduli of $V_R$. From the three starting bases, by means of the six rotations we get eighteen bases and with the help of the unitary transformation we get other six bases (imposing one element in the second column to vanish) which become thirty-six by using again the six rotations, making a total of fifty-four. These are all the SM bases with $M_u = D_u$ and $M_d$ containing three zeros, out of eighty-four possibilities. We try to understand if some of the fifty-four bases satisfy constraints coming from $B$ decay, $K_L - K_S$ mass difference and $B - \bar{B}$ mixing, within the LRM.
Table 1. Position of zeros and phases in $M_d$ and corresponding prediction for $|V_{cb}^R|$.  

<table>
<thead>
<tr>
<th>zeros</th>
<th>124</th>
<th>236</th>
<th>146</th>
<th>256</th>
<th>149</th>
<th>259</th>
<th>127</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>phases</td>
<td>356</td>
<td>145</td>
<td>589</td>
<td>479</td>
<td>568</td>
<td>467</td>
<td>356</td>
<td>346</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}^R</td>
<td>$</td>
<td>0.896</td>
<td>0.896</td>
<td>0.789</td>
<td>0.789</td>
<td>0.984</td>
<td>0.984</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>zeros</th>
<th>167</th>
<th>268</th>
<th>479</th>
<th>589</th>
<th>347</th>
<th>358</th>
<th>467</th>
<th>568</th>
</tr>
</thead>
<tbody>
<tr>
<td>phases</td>
<td>589</td>
<td>479</td>
<td>235</td>
<td>134</td>
<td>256</td>
<td>146</td>
<td>235</td>
<td>134</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}^R</td>
<td>$</td>
<td>0.785</td>
<td>0.785</td>
<td>0.999</td>
<td>0.999</td>
<td>0.875</td>
<td>0.875</td>
</tr>
</tbody>
</table>

In fact, a recent analysis [14] of right-handed currents in B decay within the LRM suggests that $|V_{cb}^R|$ is large and perhaps near unity. In such analysis $M_{W_R} \gtrsim 720$ GeV [15] is supposed. Actually, this experimental bound is obtained by manifest left-right symmetry. From inclusive semileptonic decays of B mesons ref. [14] gives $|V_{cb}^R| \gtrsim 0.782$. Moreover, if, as suggested in ref. [16], right-handed currents can help to solve the $B$ semileptonic branching fraction and charm counting problems, then ref. [14] gives $|V_{cb}^R| \geq 0.908$ and $M_{W_R} \lesssim 1600$ GeV. For our purposes we assume $|V_{cb}^R| > 0.750$. We have used this constraint to select quark mass matrices, taking $\delta = 75^\circ$ in $V_L$. This value is well inside the experimentally favoured region [12]. A moderate variation of $\delta$, say from 60$^\circ$ to 90$^\circ$, has not a relevant effect. Of course, the hermitian form of $M_d$ (and in general of both mass matrices) is excluded because it yields $|V_{cb}^R| = |V_{cb}^L| \simeq \lambda^2$. The successful ansatze are in Table 1, where the position of phases can be changed by a diagonal phase matrix $U_R$. It should be noted that ansatze 149, 259, 167, 268, 347, 358 give the quite particular exact result that some element in $M_d^T M_d = V_R D_d^2 V_R^T$ is zero.
As an example of a successful ansatz we report $|M_d|$ and $|V_R|$ for model 124:

$$ |M_d| = \begin{pmatrix} 0 & 0 & 0.023 \\ 0 & 0.106 & 0.104 \\ 0.541 & 2.687 & 1.213 \end{pmatrix}, \quad |V_R| = \begin{pmatrix} 0.958 & 0.221 & 0.180 \\ 0.205 & 0.393 & 0.896 \\ 0.199 & 0.892 & 0.405 \end{pmatrix}. $$

The near equality of elements 5 and 6 in $|M_d|$ is due to the specific value $\delta \simeq 75^\circ$ [10]. The matrix $|V_R|$ has an approximate symmetric expression

$$ |V_R| \simeq \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 2\lambda & 1 \\ \lambda & 1 & 2\lambda \end{pmatrix}. $$

Further constraints on the form of $V_R$ come from $K_L - K_S$ mass difference [17] and $B - \bar{B}$ mixing. Assuming that each row and column of $V_R$ contains only one large element and forbidding fine-tuning, these constraints give [14] $|V_{us}^R| \lesssim \lambda^2$, $|V_{ub}^R| \lesssim \lambda$, $|V_{td}^R| \lesssim \lambda$, when $|V_{ts}^R|$ is large, and $|V_{ud}^R| \lesssim \lambda$, $|V_{ub}^R| \lesssim \lambda$, $|V_{tb}^R| \lesssim \lambda^3$, when is $|V_{td}^R|$ large. Out of the sixteen models in Table 1, the 128 satisfies quite well the second set of constraints:

$$ |M_d| = \begin{pmatrix} 0 & 0 & 0.023 \\ 0.103 & 0.021 & 0.104 \\ 2.741 & 0 & 1.213 \end{pmatrix}, \quad |V_R| = \begin{pmatrix} 0.199 & 0.892 & 0.405 \\ 0.088 & 0.395 & 0.914 \\ 0.976 & 0.217 & 0.003 \end{pmatrix}. $$

The approximate expression for $|V_R|$ is

$$ |V_R| \simeq \begin{pmatrix} \lambda & 1 & 2\lambda \\ 2\lambda^2 & 2\lambda & 1 \\ 1 & \lambda & \lambda^5 \end{pmatrix}. $$

Model 124 gives $|V_{us}^R| \simeq \lambda$ rather than $|V_{us}^R| \lesssim \lambda^2$. Also models 479 and 589 are reliable, with very small mixings.

To better explain the physical content of the foregoing calculation we present some comments. We have considered fifty-four forms of quark mass matrices with
three zeros and three phases inside $M_d$ and a diagonal $M_u$. As we said, $|V_{R}|$ does not change if we put more than three phases. On the other hand the existence of three zeros in $M_d$ is a strong restriction because in the LRM just one or two zeros settle an ansatz. Nevertheless, we have found a few models that satisfy the constraints from $K$ and $B$ physics. Other ansatze can be obtained starting from a diagonal $M_d$.

We stress the simple result that, if $|V_{cb}|$ is large, hermitian or symmetric quark mass matrices [18] are not reliable [19]. Non-symmetric mass matrices have important applications in the leptonic sector, mainly in connection with the large mixing of neutrinos [20].

Using Eqns.(3),(5) it is possible to change the structure of both $M_u$ and $M_d$. For example, from model 128, multiplying to the right by a simple unitary transformation in the 2-3 sector, it yields $M_d$ with one zero in position 1 and $M_u$ with four zeros in positions 2347. Although such forms can be more interesting to discover an underlying theory of fermion masses and mixings, they lead to the same parameters of Eqn.(2), and we need other observable quantities to make a selection of such models with non diagonal mass matrices. Such new physical parameters exist in extensions of the LRM. Actually, in the SM one can get $M_u$ diagonal and $M_d$ with three zeros; in the LRM $M_u$ can be diagonalized but $M_d$ is fixed. In the Pati-Salam model, due to the relation between quarks and leptons, we cannot choose $M_u$ diagonal in the general case. In $SO(10)$ also $u_L$ and $u_R$ transform in the same way and then it is never possible to choose $M_u$ diagonal. Non-symmetric mass matrices can be obtained by using also the antisymmetric Higgs $120$ in the Yukawa couplings with fermions, or if one allows for effective non-renormalizable couplings of the light generations [21].
In conclusion, for the first time, we have performed, within the LRM, a systematic study of quark mass matrices which have a general structure in the SM. Constraints on right-handed mixings, coming from various experimental and theoretical sources, permit to select three reliable forms, apart from phases.

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