A Note on Non-Commutative Orbifold Field Theories

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Abstract

We suggest that orbifold field theories which are obtained from non-commutative $\mathcal{N} = 4$ SYM are finite. In particular, non-supersymmetric orbifold truncations might be finite even at finite values of $N_c$. 

1 Introduction

Recently non-commutative Yang-Mills theory [1] attracted a lot of attention, mainly due to discoveries of new connections to string theory [2, 3, 4]. In a recent paper [5], it was suggested that the divergences of the non-commutative Yang-Mills theory are dictated by the large $N_c$ limit of the theory. Namely, that divergences occur only in planar diagrams (the observation that the planar commutative and non-commutative theories are the same up to phases in the Green functions, was already made in [6]. A careful analysis of the divergences was carried out in ref. [7]).

Another direction of research is the study of orbifold field theories - motivated by the AdS/CFT correspondence [8]. It was conjectured by Kachru and Silverstein [9] that orbifolds of $AdS_5 \times S^5$ which act on the $S^5$ part define a large $N_c$ finite theory, even when the R-symmetry is completely broken and the theory is not supersymmetric. This conjecture was later proven using both field theory [10] and string theory [11] techniques.

In this note we would like to consider non-commutative Yang-Mills theories which are obtained by an orbifold truncation of $\mathcal{N} = 4$ SYM. We suggest that these theories are finite, namely that there are no divergent Feynman diagrams, even when the theory under consideration is non-supersymmetric and the number of colors is finite.

2 Orbifold field theories

Orbifold field theories are obtained by a certain truncation of a supersymmetric Yang-Mills theory. Let us consider the special case of $\mathcal{N} = 4$ . The truncation procedure is as follows: consider a discrete subgroup $\Gamma$ of the $\mathcal{N} = 4$ R-symmetry group $SU(4)$. For each element of the orbifold group, a representation $\gamma$ inside $SU(|\Gamma|N_c)$ should be specified. Each field $\Phi$ transform as $\Phi \rightarrow r\gamma^|\gamma\Phi$, where $r$ is a representation matrix inside the R-symmetry group. The truncation is achieved by keeping invariant fields. The resulting theory has a reduced amount of supersymmetry, or no supersymmetry at all. It was conjectured [9], based on the AdS/CFT conjecture, that the truncated large $N_c$ theories are finite as the parent $\mathcal{N} = 4$ theory. Later it was proved [10] that the planar diagrams of the truncated theory and parent theories are identical. In particular it means that the perturbative beta function of the
large $N_c$ daughter theory is zero and that the theory is finite.

In the cases of $\mathcal{N} = 2$ truncations, there is only one-loop (perturbative) contribution to the beta function. Its vanishing indicates the perturbative finiteness of the daughter theory at finite $N_c$ as well.

In $\mathcal{N} = 1$ truncations the situation is more subtle. Indeed, the theory is finite at finite values of $N_c$, but the finiteness is due to Leigh-Strassler type of arguments[12]. In that case one should consider the $SU(N_c)$ version of the theory (and not the $U(N_c)$ theory which is obtained from the string theory orbifold), since the $U(1)$ beta function is always positive at the origin. In addition $\frac{1}{N_c^2}$ shifts of the Yukawa couplings are needed[13].

In the non-supersymmetric case there are no known examples of finite theories at finite $N_c$, though attempts in this direction using orbifolds were recently made[14]. As we shall see, due to non-commutativity such examples can be found.

3 Perturbative behavior of non-commutative Yang-Mills theories

Recently, several authors[7, 5] analyzed the renormalization behavior of non-commutative Yang-Mills theories (for an earlier discussion see[6]. Related works are [15, 16]). We briefly review their analysis. The Feynman rules of the commutative and non-commutative theories in momentum space are very similar. In fact the only difference is that each vertex of the non-commutative theory acquires a phase, $\exp i \Sigma_{ij} k_i \wedge k_j$ (the Moyal phase), with respect to the vertex of ordinary commutative theory[6]. For planar diagrams, this phase cancels at internal loops and the only remnant is an overall phase. Therefore the planar commutative and non-commutative theories are similar, in accordance with recent findings[17, 18, 19].

Another claim[5, 7] is that the oscillations of the Moyal phases at high momentum would regulate non-planar diagrams, namely that UV divergences of non-planar diagrams would disappear. There are two exceptional cases in which non-planar diagrams will still diverge[7]: (i). When the non-planar diagrams consist of planar sub-diagrams which might diverge. (ii). For specific values of momentum (a zero measure set) the Moyal phase can be zero. The later type of infinities should disappear in the non-compact limit[5]. If true,
it means that truly divergences in the non-commutative theory occur only in planar graphs.

4 Finiteness of non-commutative orbifold field theories

Let us now consider an orbifold truncation of non-commutative $\mathcal{N} = 4$ SYM. These theories can be defined perturbatively by a set of Feynman rules. The natural definition would be to attach to each vertex the corresponding Moyal phase[6]. According to ref.[5], the only potential divergences are the ones which arise in planar diagrams. Non-planar diagrams are expected to be finite (up to exceptions which were mentioned in the previous section). Moreover, according to the analysis of [10], the planar diagrams of an orbifold theory can be evaluated by using the corresponding diagrams of the parent non-commutative $\mathcal{N} = 4$. These diagrams are finite since they differ from the commutative $\mathcal{N} = 4$ only by an overall phase. In this way sub-divergences of non-planar diagrams will also be canceled. We therefore conclude that any orbifold truncation of non-commutative non-compact $\mathcal{N} = 4$ SYM is finite.

In particular, it means that we might have a rich class of non-supersymmetric gauge theories which are finite, even at finite $N_c$ (in contrast to ordinary non-supersymmetric orbifold field theories, where the two loop beta function is generically non-zero). Note that though these theories might be finite, they are certainly not conformal.

Examples of non-SUSY orbifold gauge theories are given in refs.[9, 20]. It might be interesting to understand how the finiteness of these theories arise from string theory orbifolds.

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References


