Semiclassical and quantum production rates

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We discuss the effects of the off-shell nucleon propagation for the particle production. Results from nonequilibrium Kadanoff-Baym evolution modeling a heavy ion collision are presented. The production rates are compared to equilibrium emission rates obtained in the same approximation for the self-energy. The comparison of semiclassical and quantum in-medium production rates is performed within the T-matrix approximation. Self-consistent T-matrix resummation allows us to discuss also the effect of the off-shell propagation on the effective in-medium cross section.

1 Nonequilibrium evolution

In medium modifications of nucleon properties are expected to be important at normal nuclear densities and higher. Most of these effects can be taken into account by a modification of the effective nucleon mass, cross sections and mean-field. In a strongly interacting medium one expects that off-shell propagation of nucleons is crucial. On the other hand, transport equations using on-shell quasi-particles proved to be very successful in the description of intermediate energy heavy ion collisions.

Danielewicz in the first work comparing an actual solution of the nonequilibrium Kadanoff-Baym equations with its quasi-particle limit found a slower equilibration in the quantum evolution \(^1\). Thus the relaxation rate obtained from the Boltzmann collision term is larger than obtained from the Kadanoff-Baym collision term, including off-shell propagation and memory effects. The modification of such global transport coefficients due to off-shell propagation can be taken into account by an effective renormalization of the scattering amplitudes between quasi-particles. In a Fermi liquid \(^2\) this amounts to a multiplication of the cross section by the the factor \(z(p_F)^4 \left( z(p) = (1 - \partial \text{Re} \Sigma(p, \omega)/\partial \omega |_{\omega=\omega_p})^{-1} \right)\). For cold nuclear matter this leads to a reduction of the cross section by a factor 2 or more.

Production of very energetic mesons and photons has been observed in heavy ion reactions. This particle production is subthreshold, i.e. the energy available in single nucleon-nucleon collision is insufficient to produce such an energetic particle. Fermi motion of nucleon inside the colliding nuclei shifts
the effective threshold to higher energies but cannot explain the high momentum part of the spectra. The quasi-particle transport models invoke multiple collisions, or pion-nucleon collisions (with pions produced in earlier nucleon-nucleon collisions) to explain the production of energetic mesons or photons.

Subthreshold particle production could be a testing ground for the off-shell effects in nuclear propagation. Off-shell spectral function of nucleons in hot nuclear matter allows for meson production in the simplest one-loop approximation for the meson self-energy. Off-shell propagation of nucleons requires the solution of evolution equations for the two-time Green’s function, the Kadanoff-Baym equations. 16 years after the first calculations it is still limited to spatially homogeneous systems. A numerical solution of a homogeneous transport equation could be used in the estimate of the particle production in heavy ion collisions because of the approximate scaling of the number of produced particles with the number of nucleons participating in the collision, even for the subthreshold production. The production rate in real collision could be estimated by scaling the production rate in a homogeneous system by a geometric factor.

In previous work we studied the production in the nonequilibrium dynamics of nucleons. The evolution of the system follows the Kadanoff-Baym
equations, in the direct second Born approximation
\[
\left( \frac{i}{\hbar} \frac{\partial}{\partial t_1} - \omega_p \right) \langle G(p, t_1, t_2) \rangle = \int_C dt' \Sigma(p, t_1, t') G(p, t', t_2). \quad (1)
\]
The integration contour \( C \) involves an imaginary part giving, within the second Born approximation, the ground state of nuclear matter with two boosted Fermi spheres. The real part of the contour gives the collision integral with memory. In order to get the spectral function far from the quasi-particle pole, small steps in times \( t_1, t_2 \) have to be used \(^3\).

The meson production rate is obtained from the one-loop meson self-energy
\[
\frac{dN(p, t)}{dt^3} = 2Re \left( -\int_{t_0}^{t} + \int_{t_0}^{t_0 - i\tau_0} dt' \Pi^<(p, t, t') D^0(p, t, t') \right), \quad (2)
\]
where
\[
\Pi^<(p, t_1, t_2) = -i\lambda^2 \int \frac{d^3q}{(2\pi)^3} G^<(p - q, t_1, t_2) G^>(q, t_2, t_1) \quad (3)
\]
and \( D^0 \) is the vacuum meson propagator (we neglect the influence of the meson-fermion interaction on the nucleon evolution and we calculate only the production rate not the full meson transport equation). The results in Fig.1 show that the production rate is oscillating with large amplitude at initial times. It also changes sign. It is a manifestation of the oscillating time exponent, which gives the energy conservation in the limit \( t_0 \to -\infty \). In an interacting system with off-shell nucleons the positive oscillations dominate giving a finite production rate from one-loop diagram. It is however not possible to define a particle production rate at initial times. The initial correlations take into account only the fermion-fermion interaction via a two-body potential. There is no initial meson content in the collision, i.e. the initial state is not correlated with respect to the meson-fermion coupling. This and the previously discussed meson formation time (oscillations of the rate) dominate the meson production at initial times. The meson production rate can be defined only at large times \( t \approx 20 \text{fm/c} \) where the nucleon sector is basically equilibrated.

2 Equilibrium rates

Production rates in equilibrium finite temperature nuclear matter can be obtained with much less numerical effort and higher accuracy \(^4\). Equilibrium spectral functions for nucleons, at finite temperature and density, are obtained by iterative solution of the set of equations
\[
\pm i G^<(p, \omega) = A(p, \omega)f(\omega)[1 - f(\omega)],
\]

\^3\]
\[ A(p, \omega) = \frac{-2\Im \Sigma(p, \omega)}{(\omega - p^2/2m - \Re \Sigma(p, \omega))^2 + \Im \Sigma(p, \omega)^2}, \]

\[ \Sigma^{<|\rangle}(p, \omega) = \mathcal{F}(G^<, G^\gamma), \]

\[ \Re \Sigma(p, \omega) = \int \frac{d\omega'}{\pi} \frac{\Im \Sigma(p, \omega')}{\omega' - \omega}, \]

\[ -i \int \frac{d^3p d\omega}{(2\pi)^2} G^<(p, \omega) = \rho. \]

The functional \( \mathcal{F} \) in the equation for \( \Sigma^{<|\rangle} \) corresponds to the direct second Born approximation as in the nonequilibrium evolution. In Fig. 2 we present the spectral function obtained at \( T = 10\text{MeV} \). The spectral function is broad far from the Fermi momentum, but gets narrower for momenta close to it. As expected for a Fermi liquid the spectral function at the Fermi momentum gets very peaked when decreasing the temperature.

The quantum production rate of mesons with momentum \( p \) and energy \( \Omega_p = \sqrt{p^2 + M^2} \) in the one-loop approximation for the meson self-energy is

\[ \frac{dN(p)}{d^3p dt} = 4\lambda^2 \int \frac{d^3q d\omega}{(2\pi)^2} A(q, \omega) A(q - p, \omega - \Omega_p) (1 - f(\omega)) f(\omega - \Omega_p). \quad (4) \]

The semiclassical production rate is

\[ \frac{dN(p)}{d^3p dt} = \int \frac{d^3p_1}{(2\pi)^3} \cdots \frac{d^3p_4}{(2\pi)^3} |M|^2 (2\pi)^3 \delta(\sum p_i - p) 2\pi \delta(\sum \omega_i - \Omega_p) \]

\[ f(p_1) \cdots (1 - f(p_4)), \quad (5) \]

with the matrix element for particle emission given by

\[ |M|^2 = 4\lambda^2 V(p_2 - p_4)^2 \left( |G^+(\omega_{p_1} - \Omega_q, p_1 - q)|^2 \right. \]

\[ \quad + |G^+(\omega_{p_3} + \Omega_q, p_3 + q)|^2 \left. \right). \quad (6) \]

This matrix element is obtained by including self-energy corrections in the one-loop quasi-particle meson self-energy. The self-energy in the retarded propagators in \( |M|^2 \) as well as the single particle energies \( \omega_i \) are taken from the quantum calculation of finite temperature nuclear matter.

The production rate for particles of mass 140MeV is \( \sim 2 \) times larger in the quantum calculation. The self-consistent spectral function involves processes with multiple sequential nucleon-nucleon collisions which populate far
off-shell regions of the spectral function. Also the one-loop self-energy with off-shell fermion propagators includes contribution from 3-nucleon collisions corresponding to self-energy insertions on both propagators in the loop. These contributions are not taken into account by the 2-body matrix element (Eq. 6). The difference between the semiclassical and quantum production rates increases with the energy of the produced particle. On the other hand, the emission of soft particles is stronger in the semiclassical calculation. All the emission rates increase strongly with temperature. Quantum production rates are similar as obtained from the nonequilibrium evolution (Sec. 1) at large times. The semiclassical rates published in the work are different and obviously wrong. The overall collision rate is larger in the semiclassical collision integral, similarly as in the nonequilibrium calculation.

3 T-matrix approximation

In this section we discuss in-medium modification of the cross-sections. This requires the use of the T-matrix approximation instead of the Born self-energy. T-matrix ladder resummation with off-shell propagators has only recently been achieved. The equation for the T-matrix is

\[
< p | T^{\pm}(P, \omega) | p' > = V(p, p') + \int \frac{d^3 k}{(2\pi)^3} \int \frac{d^3 q}{(2\pi)^3} V(p, k) \langle k | G^{\pm}(P, \omega) | q \rangle < q | T^{\pm}(P, \omega) | p' > ,
\]
where the disconnected two-particle propagator is:

\[
\langle \mathbf{p} | G^\pm (\mathbf{P}, \Omega) | \mathbf{p}' \rangle = (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}') \int \frac{d\omega}{2\pi} \int \frac{d\mathbf{k}}{(2\pi)^3} \left( G^< (\mathbf{P}/2 + \mathbf{p}, \omega - \omega') G^> (\mathbf{P}/2 + \mathbf{p}, \omega - \omega') G^> (\mathbf{P}/2 - \mathbf{p}, \omega') / (\Omega - \omega \pm i\epsilon) \right) \]

(8)

The T-matrix equation has to be solved iteratively together with Eqs. (4). The functional defining the self-energy is now

\[
\text{Im} \Sigma^+(p, \Omega) = \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} A(k, \omega) \left( f(\omega) + g(\omega + \Omega) \right) - \langle (p - k)/2 | \text{Im} T^+(p + k, \Omega + \omega) | (p - k)/2 \rangle_A \]

(9)

instead of the Born approximation used in the previous section. The quasiparticle approximation in the T-matrix ladder consists in replacing the two fermion propagator (8) by quasiparticle propagators \(^6\). First applications of the self-consistent T-matrix scheme to finite temperature nuclear matter \(^7\) show important quantitative differences between the quasiparticle and self-consistent T-matrix results. Both at high temperatures where scattering is important and at lower temperatures close to the pairing transition.

The quasi-particle T-matrix approximation gives in-medium cross sections, but all the nucleon propagators are on-shell in the ladder resummation. The
Figure 4: (left panel) Single particle width obtained from Eq. (9) in the self-consistent T-matrix approximation (solid line) and from the Boltzmann collision integral (dashed line). (right panel) Production rate of mesons as function of their mass in the self-consistent T-matrix approximation. Quantum and semiclassical production rates are represented by solid and dashed lines respectively.

Single particle width and spectral function are the same as obtained using the Boltzmann collision integral with the cross section obtained from the in-medium T-matrix. Indeed the left panel in Fig. 3 shows the equivalence of the two calculations (the small differences are due to numerical inaccuracy and approximate treatment, angular averaging, of the energy conserving δ function in the collision term). The semiclassical production rate is defined as in the Born approximation except that the two-particle potential is replaced by the in-medium T-matrix. The quantum production rate differs from the semiclassical one only by the inclusion of 3-nucleon processes corresponding to having both propagators in the meson self-energy loop off-shell. This contribution is most important at large meson energy and increases with temperature.

The self-consistent T-matrix, using off-shell nucleon propagators in the ladder, leads to different spectral function and single particle energies. In a self-consistent calculation there is no simple relation between the single-particle width obtained from the Boltzmann equation and from the T-matrix scheme. As in the Born approximation we find that on-shell propagation leads to larger relaxation rates. Part of this effect can be understood as due to the different densities of states for the two-particle propagators in the quasi-particle and self-consistent T-matrix calculations. The optical theorem, if using the density of states corresponding to on-shell nucleons, leads to an overestimation of the cross-section.

The production rate is larger in the quantum calculation, especially for
energetic particles. The difference comes from the 3-body contribution discussed above and from the far off-shell regions of the spectral function. As in the Born approximation, the spectral function obtained in the self-consistent iteration is larger (wider) far from the shell. On the other hand, close to the quasi-particle pole the spectral function calculated from the Boltzmann collision term is wider, reflecting the systematically larger single particle width obtained with on-shell nucleons.

4 Summary

We have calculated particle production in nonequilibrium quantum transport framework. The production rate could not be defined in the initial stage of the evolution. It is possible only at large times, when the system is already equilibrated. The corresponding rates can be calculated using equilibrium spectral functions, obtained in an iterative solution of coupled equations (4). The quantum production rate is larger than the semiclassical one for energetic particles. This can be understood as due to 3 and sequential $n$-nucleon collisions.

In-medium T-matrix calculations allow to include in-medium modifications of the cross section in the picture. The cross section for Boltzmann type collision integrals and for the semiclassical production rates has to be modified by a factor describing a different density of states \(^2\). After such a renormalization of the cross sections the relaxation rates would be similar in the quantum and semiclassical transport equations, but the difference between the quantum and semiclassical production rates would increase.

Acknowledgments

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References