ANALYTIC MODEL FOR ADBECTION-DOMINATED ACCRETION FLOWS IN A GLOBAL MAGNETIC FIELD

Osamu Kaburaki

Astronomical Institute, Graduate School of Science, Tohoku University,
Aoba-ku, Sendai, 980-8578, Japan; okabu@astr.tohoku.ac.jp

Received ________________; accepted ________________
ABSTRACT

A model for advection-dominated accretion flows (ADAFs) in a global magnetic field is proposed. In contrast to the well known ADAF models in which the viscosity of a fluid determines both angular momentum transfer and energy dissipation in the flow, the magnetic field and the electric resistivity, respectively, control them in this model. A manageable set of analytic solutions for the flow and the magnetic field is obtained to vertically non-integrated basic equations. This set describes mathematically a fully advective accretion flow and, in physically plausible situations for most AGNs, it is also confirmed that the radiation cooling estimated on this solution is really negligible compared with the internal energy of the flow.

Subject headings: accretion, accretion disks — magnetohydrodynamics: MHD — galaxies: active — black hole physics
1. INTRODUCTION

In recent years, advection-dominated accretion flows (ADAFs) are drawing much attention in astrophysics because the radiation spectrum of Sgr A* is well reproduced in the framework of such models (Narayan, Yi & Mahadevan 1995; Manmoto, Mineshige & Kusunose 1997; Narayan et al. 1998). The attempts are further made to apply them also to some other AGNs and soft X-ray transients (for a review, see Narayan, Mahadevan & Quataert 1999). Such a flow may be realized when the rate of mass accretion is smaller than the Eddington accretion rate, and the flow becomes optically thin while geometrically thick. Almost whole energy dissipated in the flow is advected down to the central object owing to an inefficient radiation cooling.

The basic ideas of such ADAF models has been developed by a number of researchers (e.g., Ichimaru 1977; Rees et al. 1982; Narayan & Yi 1994, 1995; Abramowicz et al. 1995). We call them the “viscous” ADAF models because both angular-momentum transfer and energy dissipation in the flow is undertaken by the kinematic viscosity whose size is usually specified by the \( \alpha \) parameter: \( \nu = (2/3) \alpha C_S H \), where \( C_S \) is the sound velocity and \( H \) is the half-thickness of the flow. Another parameter \( \beta \) enters into this model through the assumed presence of turbulent magnetic fields (Narayan & Yi 1995): \( p_m = 3(1 - \beta) C_S^2 \rho \), where \( p_m \) is the turbulent magnetic pressure and \( \rho \) is the mass density. The spectrum of Sgr A* in a range from radio up to hard X-ray has been explained by the synchrotron emission from high temperature electrons, its inverse-Compton scattered component and bremsstrahlung. For the presence of synchrotron emission, the inclusion of turbulent magnetic field is crucial in this model.

Although the above model completely ignores the presence of an ordered magnetic field, it is natural to expect the presence of such fields in most AGNs and especially in the central region of our Galaxy (e.g., Yusef-Zadeh, Morris & Chance 1984). Taking this fact
into account, we propose another ADAF model which shall be called the “resistive” ADAF model in order to distinguish it from the viscous ADAF. This name reflects the facts that angular momentum is transported by a global magnetic field and energy is released as a resistive dissipation of the electric current driven by a rotational motion of the accreting plasma. Originally, this model appeared (Kaburaki 1986, 1987) as a magnetic counterpart of the standard model (usually called the $\alpha$-disk model) for optically thick, geometrically thin accretion disks (Shakura & Sunyaev 1973).

Analogously to the $\alpha$-disk model, this model contains a parameter $\Delta$ which specifies the efficiency of dissipation: $\sigma^{-1} \propto \Delta^2$, where $\sigma^{-1}$ is the electric resistivity. The parameter $\Delta$ in this model also has a geometrical meaning of the half opening-angle of a disk, which is assumed for simplicity to be constant throughout the disk. The set of analytic solutions for the flow and the magnetic field was obtained there under the assumption of geometrically thin disk ($\Delta \ll 1$), and it describes an accretion flow which has a reduced Keplerian rotation and a magnetically confined vertical structure.

Although the heating process in the disk has been discussed in the above papers, the cooling process has not been discussed and implicitly assumed to be in local balance with the heating. Since the energy equation is not included explicitly, the solution may be inconsistent from a viewpoint of energy transport. In principle, the fraction of advected part of the released energy can be calculated from that set of solutions. Actually, however, the inclusion of a vertical motion in the model of Kaburaki (1987, hereafter referred to as K 87) has made this task rather difficult and obscure. Therefore, in the present paper, we intend to obtain a more manageable set of solutions to a similar problem by adding further simplifying assumptions of no vertical flows and electric currents. Although such a solution has already been obtained in a previous paper (Kaburaki 1986), it is unsatisfactory because the effect of pressure gradient has been omitted without justification.
Our new solution obtained in the present paper turns out \textit{a posteriori} to describe a fully advective accretion flow (as a preliminary report, see Kaburaki 1999). In other words, the requirements for dynamical balances and the geometry of a flow specify in our model also the way of energy transport. The consistency of this solution as an ADAF model in most physical situations expected in typical AGNs is also confirmed. Namely, it is shown by using this solution that the flow is actually optically thin and the radiation cooling is negligible compared with the heating. Therefore, our resistive ADAF model may be useful in calculating spectra form various AGNs and in considering stabilities of realistic ADAFs. Some results in such applications will appear elsewhere.

This paper is organized as follows. In §2, a set of resistive MHD equations is first introduced as our starting point, and then various assumptions to simplify these equations are stated. The resulting equations in spherical polar coordinates are cited in §3, and the effects of the individual assumptions and the policy for solving these equations are discussed. A set of approximate solutions which we propose as a model of resistive ADAFs is written out in §4 and its properties, including energy budget, are examined. §5 is devoted to determining the disk edges which are defined conceptually as the limiting radii for the validity of this solution. In §6, various quantities predicted by this model are suitably scaled to typical situations in most AGNs and the physical consistency of this solution as an ADAF model is also checked. Finally, main issues which may be raised about our model are discussed in §7. Some remarks concerning a relation between the rotation law and geometry of a disk are also described in Appendix.
2. BASIC EQUATIONS AND SIMPLIFYING ASSUMPTIONS

The basic equations adopted in the resistive ADAF model are those of resistive MHD which are written in usual notation as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p - \frac{1}{4\pi\rho} [\mathbf{B} \times (\nabla \times \mathbf{B})] + \mathbf{g}, \quad (\nu = 0) \tag{2}
\]

\[
\nabla \cdot \mathbf{B} = 0, \tag{3}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{c^2}{4\pi\sigma} \Delta \mathbf{B}. \quad (\sigma = \text{const.}) \tag{4}
\]

In the equation of motion (2), the viscosity of the fluid is completely neglected in order to make the contrast between the resistive and viscous ADAF models clear. Instead, the term of magnetic diffusion is retained in the induction equation (4) assuming the constancy of electrical conductivity \(\sigma\) for simplicity. The above set of equations forms an apparently closed set in the sense that the number of equations and unknowns are the same. In fact, however, the number of equations is insufficient by one. This is because Maxwell’s equations guarantee that \(\text{div} \mathbf{B} = 0\) always holds once it is satisfied initially. This point will become more clear in §3. Other related quantities of our interest are calculated from the following subsidiary equations: the current density, electric field and charge density, from

\[
j = \frac{1}{4\pi} \nabla \times \mathbf{B}, \quad \mathbf{E} = \frac{j}{\sigma} - \frac{1}{c} \mathbf{v} \times \mathbf{B}, \quad q = \frac{1}{4\pi} \nabla \cdot \mathbf{E}, \tag{5}
\]

respectively. The temperature of the fluid is simply assumed to be common to electrons and ions, and is calculated from the ideal gas law

\[p = \frac{R}{\bar{\mu}} \rho T, \tag{6}\]

neglecting the radiation pressure (\(\bar{\mu}\) is the mean molecular weight and \(R\) is the gas constant).

Hereafter, we employ spherical polar coordinates \((r, \theta, \varphi)\) since the gravity is spherically symmetric. The main assumptions used in simplifying the above basic equations are, 1)
stationarity, 2) axisymmetry, 3) geometrically thin disk, 4) well-developed magnetic disk, 5) reduced Keplerian rotation and 6) vanishing of $v_\theta$ and $j_\theta$. All these assumptions except the last one are the same as employed in K 87. Although the solution has been obtained there without 6), we add it here in order to get a more manageable set of solutions.

The first two of the above assumptions imply that $\partial/\partial t = 0$ and $\partial/\partial \phi = 0$ in every component equation. The most essential assumption is the third one which demands that $\Delta \ll 1$. This fact allows us to treat $\Delta$ as a smallness parameter in the following discussion. In such situations, a derivative with respect to $\theta$ results in a large quantity (introduce $\xi \equiv (\theta - \pi/2)/\Delta$ then $\partial/\partial \theta = \Delta^{-1} \partial/\partial \xi$). It is worth emphasizing, however, that the thin disk condition is satisfied even by rather geometrically thick disks since 1 radian is about 60° of arc.

The fourth assumption states that, when the total magnetic field is divided into the externally-given seed field $B_0$ and the component produced in the disk $b$, we expect $|b| \gg |B_0|$ inside the disk except near a disk edge where $|b| \sim |B_0|$. The actual implication of the fifth assumption is that of large magnetic Reynolds numbers $\mathcal{R}$ (strictly speaking, $\mathcal{R}^2(r) \gg 1$) in the disk except near its inner edge. Namely, as it will turn out a posteriori from the resulting solutions, the assumption of reduced Keplerian rotation ($v_\phi = \text{const.} \times v_K < v_K$, where $v_K \equiv \sqrt{GM/r}$ represents the Kepler velocity) is justified when $\mathcal{R}^2(r) \gg 1$ (see §4). The sixth assumption results in an $r$-independent accretion rate $\dot{M}$ and the dependence $b_\phi \propto r^{-1}$.

3. LEADING ORDER EQUATIONS IN $\Delta$

The set of basic equations is simplified first according to the above assumptions except 5) and 6). Since the flow is assumed to be geometrically thin, only the leading order terms
in $\Delta$ are retained in each component of the basic equations. Regarding $b_r$, $b_\varphi$, $v_r$, $v_\varphi$, $\rho$ and $p$ as quantities of order unity in $\Delta$, we obtain the following component equations, within the approximation $\sin \theta \sim 1$.

**mass continuity:**

$$\frac{\partial}{\partial r} (r^2 \rho v_r) + r \frac{\partial}{\partial \xi} (\rho v_\theta) = 0.$$  \hspace{1cm} (7)

**magnetic flux conservation:**

$$\frac{\partial}{\partial r} (r^2 b_r) + r \frac{\partial b_\theta}{\partial \xi} = 0.$$  \hspace{1cm} (8)

**equation of motion:**

$$\left[ v_r \frac{\partial}{\partial r} + v_\theta \frac{\partial}{\Delta r \frac{\partial}{\partial \xi}} \right] v_r - \frac{v_\varphi^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{GM}{r^2} + \frac{1}{4\pi \rho r} \left[ b_\theta \frac{\partial b_r}{\partial \xi} - b_\varphi \frac{\partial}{\partial r} (rb_\varphi) \right],$$  \hspace{1cm} (9)

$$\frac{\partial p}{\partial \xi} + \frac{1}{8\pi} \frac{\partial}{\partial \xi} (b_r^2 + b_\varphi^2) = 0,$$  \hspace{1cm} (10)

$$\left[ v_r \frac{\partial}{\partial r} + \frac{v_\theta}{\Delta r \frac{\partial}{\partial \xi}} \right] v_\varphi + \frac{v_\varphi v_r}{r} = \frac{1}{4\pi \rho r} \left[ b_r \frac{\partial}{\partial r} (rb_\varphi) + b_\theta \frac{\partial b_\varphi}{\partial \xi} \right].$$  \hspace{1cm} (11)

**induction equation:**

$$\frac{\partial}{\partial \xi} \left[ v_r b_\theta - v_\theta b_r + \frac{c^2}{4\pi \sigma \Delta r} \frac{1}{\partial \xi} \frac{\partial b_r}{\partial \xi} \right] = 0,$$  \hspace{1cm} (12)

$$\frac{\partial}{\partial r} \left[ r \left( v_r b_\theta - v_\theta b_r + \frac{c^2}{4\pi \sigma \Delta r} \frac{1}{\partial \xi} \frac{\partial b_r}{\partial \xi} \right) \right] = 0,$$  \hspace{1cm} (13)

$$\frac{\partial}{\partial r} [r(v_\varphi b_r - v_r b_\varphi)] + \frac{1}{\Delta} \frac{\partial}{\partial \xi} \left[ v_\varphi b_\theta - v_\theta b_\varphi + \frac{c^2}{4\pi \sigma \Delta r} \frac{1}{\partial \xi} \frac{\partial b_\varphi}{\partial \xi} \right] = 0.$$  \hspace{1cm} (14)

From the equations of mass continuity and flux conservation, it turns out that $v_\theta$ and $b_\theta$ are quantities of order $\Delta$. Further, the assumption of $v_\theta = 0$ results in an $r$-independent radial mass flux, $r^2 \rho v_r$.

At this stage of approximation, there remain so many terms in the $r$-component of the equation of motion. Only the last term on the the right-hand side can be dropped by
a further assumption of $j_\theta = 0$ (for the component expressions of the current density to the leading order in $\Delta$, see K 87). As confirmed retrospectively, however, $\rho, p \propto R^2$ while $v_r \propto R^{-1}$. Therefore, when $R^2 \gg 1$ (the assumption 5), equation (9) reduces to

$$\frac{GM}{r^2} = \frac{v_r^2}{r} - \frac{1}{\rho} \frac{\partial p}{\partial r}.$$  

(15)

Owing to the presence of the outward pressure-gradient, the effect of gravity is somewhat reduced. Anticipating the same $r$- and $\theta$-dependences for the gravity and pressure terms, we have a reduced Keplerian rotation from this equation. The validity of this anticipation is again justified \textit{a posteriori} from the resulting solution.

The $\theta$-component of the equation of motion can be integrated to give the “vertical” pressure balance for the disk:

$$p + \frac{b_\theta^2}{8\pi} = \tilde{p}(r),$$

(16)

where $\tilde{p}(r)$ represent a function of $r$ only. In the above procedure, the magnetic pressure due to $b_r$ has been omitted in expectation of the relation $b_\phi/b_r \propto R$, which is also confirmed retrospectively. This equation implies the confinement of the accreting flow into a disk structure by the induced toroidal magnetic field which changes sign at the equatorial plane.

The $\phi$-component of the equation of motion, on the other hand, reduces to

$$v_r \frac{\partial (rv_\phi)}{\partial r} = \frac{1}{4\pi \Delta} \frac{b_\theta \partial b_\phi}{\rho \partial \xi}.$$  

(17)

when simplified by the assumption 6). This equation describes the transfer of angular momentum by the magnetic stress. The extracted angular momentum is carried away from the disk along the poloidal lines of force.

Both $r$- and $\theta$-components of the induction equation reduce to

$$v_r = -\frac{c^2}{4\pi \sigma \Delta} \frac{1}{rb_\theta} \frac{\partial b_r}{\partial \xi},$$

(18)

since the quantities in the brackets of these equations are proportional to the $\phi$-component of the electric field $E_\phi$ which should vanish in axisymmetric situations (see Ohm’s law in
Thus, the degeneracy of the two equations have become evident. However, the degeneracy itself is not a consequence of symmetry but of the nature inherent in Maxwell’s equations as stated before. We can expect from this equation that $v_r \propto R^{-1}$.

As for the $\varphi$-component of the induction equation, it is also simplified by the assumption of $v_\theta = 0$ and by the fact that $v_\varphi$ is independent of $\xi$ since it is proportional to the Kepler velocity. Further eliminating $b_\theta$ with the aid of magnetic flux conservation, we obtain

$$r^2 b_r \frac{\partial}{\partial r} \left( \frac{v_\varphi}{r} \right) - \frac{\partial}{\partial r} (rv_r b_\varphi) + \frac{c^2}{4\pi\sigma\Delta^2} \frac{1}{r} \frac{\partial^2 b_\varphi}{\partial \xi^2} = 0.$$  \hspace{1cm} (19)

In solving this equation, we assume intuitively the proportionality of the first and second terms. Afterwards, from the resulting solution, the consistency of such an assumption is confirmed. This is the same technique as used in solving equation (15).

4. AN ADAF SOLUTION

The above set of partial differential equations is incomplete in the sense that the number of equations is insufficient by one. Usually, this shortage is supplemented by the energy equation or the polytropic one, and it makes the process of solving the set of equations rather difficult. Fortunately, however, we can get a solution here without introducing another such equation. The set is solved by the method of approximate variable separation. Starting from the assumption of $b_\varphi \propto \tanh \xi$, we can obtain almost automatically the angular dependences of other quantities according to the equations. The only approximation needed is to accept the relation $(d^3/d\xi^3) \tanh \xi \simeq -2 \text{sech}^4 \xi$ which hold fairly good around the disk’s midplane. Including the method of obtaining the radial parts of the unknowns, the process is essentially the same as described in K 87. Therefore, we cite below only the final results.
The boundary value of the external magnetic field $B_0$ is taken into account at a reference radius $r_0$, which may be the inner or outer edge of the accretion disk (they will be defined explicitly in the next section). Then the set of solutions for various quantities are written as

$$b_r(r, \xi) = \tilde{b}_r(r) \tanh \xi, \quad \tilde{b}_r(r) = B_0 \left( \frac{r}{r_0} \right)^{-3/2}, \quad (20)$$

$$b_\theta(r, \xi) = \tilde{b}_\theta(r) \tanh \xi, \quad \tilde{b}_\theta(r) = \frac{\Delta}{4} B_0 \left( \frac{r}{r_0} \right)^{-3/2}, \quad (21)$$

$$b_\varphi(r, \xi) = -\tilde{b}_\varphi(r) \tanh \xi, \quad \tilde{b}_\theta(r) = \Re(r_0) B_0 \left( \frac{r}{r_0} \right)^{-1}, \quad (22)$$

$$v_r(r, \xi) = -\tilde{v}_r(r) \tanh \xi, \quad \tilde{v}_r(r) = \frac{v_K(r_0)}{\sqrt{3} \Re(r_0)} \left( \frac{r}{r_0} \right)^{-1}, \quad (23)$$

$$v_\theta(r, \xi) = 0, \quad (24)$$

$$v_\varphi(r, \xi) = \tilde{v}_\varphi(r), \quad \tilde{v}_\varphi(r) = \frac{v_K(r_0)}{\sqrt{3} \Re(r_0)} \left( \frac{r}{r_0} \right)^{-1/2}, \quad (25)$$

$$p(r, \xi) = \tilde{p}(r) \tanh \xi, \quad \tilde{p}(r) = \frac{\Re^2(r_0) B_0^2}{8\pi} \left( \frac{r}{r_0} \right)^{-2}, \quad (26)$$

$$\rho(r, \xi) = \tilde{\rho}(r) \tanh \xi, \quad \tilde{\rho}(r) = \frac{3 \Re^2(r_0) B_0^2}{8\pi \nu_K(r_0)} \left( \frac{r}{r_0} \right)^{-1}, \quad (27)$$

$$j_r(r, \xi) = -\tilde{j}_r(r) \tanh \xi, \quad \tilde{j}_r(r) = \frac{c}{4\pi \Delta} \frac{\Re(r_0) B_0}{r_0} \left( \frac{r}{r_0} \right)^{-2}, \quad (28)$$

$$j_\theta(r, \xi) = 0, \quad (29)$$

$$j_\varphi(r, \xi) = -\tilde{j}_\varphi(r) \tanh^4 \xi, \quad \tilde{j}_\varphi(r) = \frac{c}{4\pi \Delta} \frac{B_0}{r_0} \left( \frac{r}{r_0} \right)^{-5/2}, \quad (30)$$

$$T(r, \xi) \equiv \frac{\tilde{\mu}}{\tilde{\rho}} p(r, \xi) = \frac{\tilde{\mu}}{\tilde{\rho}} \frac{v_K(r_0)}{3} \left( \frac{r}{r_0} \right)^{-1}. \quad (31)$$

The definition and its $r$-dependence of the magnetic Reynolds number in our model are

$$\Re(r) \equiv \frac{\tilde{b}_\theta}{\tilde{b}_r} = \frac{\tilde{v}_\varphi}{\tilde{v}_r} = \Re(r_0) \left( \frac{r}{r_0} \right)^{1/2}, \quad \Re(r_0) = \frac{\pi \sigma \Delta^2 r_0 v_K(r_0)}{\sqrt{3} c^2}. \quad (32)$$

From the latter expression, it turns out that our magnetic Reynolds number is reflecting the vertical structure of the disk since $\Delta r$ and $\Delta v_K$ represent typical sizes of the height and the velocity in the $\theta$-direction, respectively.
Here, we summarize the characteristic features of the above set of solutions. The plasma density and pressure are large quantities of order $\mathcal{R}^2$, reflecting the development of a disk-like structure of the flow. In order to vertically support this configuration, a large toroidal magnetic field $b_\phi$ of order $\mathcal{R}$ is required to appear, and this fact also guarantees sufficient extraction of angular momentum from the accreting matter. Controlled by this extraction, the plasma falls inwardly. This inflow is understood also as a result of resistive diffusion across the poloidal magnetic field lines. Reflecting this fact, $v_r$ is inversely proportional to $\mathcal{R}$. The presence of a factor $\mathcal{R}$ in $j_r$ represents that the rotation dominated accretion flow acts mainly as a poloidal current generator, from an electrodynamic point of view.

Since the poloidal and toroidal components of vector quantities have different $r$-dependences, our solution is not such a similarity solution as obtained in the framework of viscous ADAF (Narayan & Yi 1994). Reflecting the disk’s geometry, the rotation velocity is a reduced Keplerian (see Appendix for further details) at the points on a surface of constant $r$. However, the plane of rotation is always parallel to the disk’s midplane so that the center of rotation generally does not coincide with the center of gravitational attraction. It is also interesting to see that a polytrope-like relation holds only for the radial functions, i.e., $\tilde{p} \propto \tilde{\rho}^{\gamma}$ with $\gamma = 2$. The assumption of $v_\theta = 0$ assures an $r$-independent mass accretion rate

$$\dot{M} = -4\pi \Delta \int_0^\infty r^2 \rho v_r \, d\xi = \frac{\Delta \mathcal{R}(r_0) B_0^2 r_0^2}{\sqrt{3} \, v_K(r_0)}.$$  \hspace{1cm} (33)$$

A similar thing is assured by the assumption of $j_\phi = 0$ for the total poloidal current flowing in the disk. In order to fulfill the requirement of poloidal current closure, however, non-zero $j_\phi$ should be included. Our model, therefore, is shifting this problem to the regions both beyond the outer edge and within the inner edge of the disk.

Based on the above cited solution, we can discuss the energy budget in the accretion flow. Since the electric current vector is dominated by its $r$-component, we have for the
local Joule dissipation rate

\[ q^+_J(r, \xi) \equiv \frac{j^2}{\sigma} \sim \frac{\Re(r_0)}{2\sqrt{3}} \frac{B_0^2}{8\pi} \Omega_K(r_0) \left( \frac{r}{r_0} \right)^{-4} \text{sech}^4 \xi, \quad (34) \]

where \( \Omega \equiv v_\varphi/r \) denotes the angular velocity (within the approximation of \( \sin \theta \sim 1 \)) and \( \sigma \) has been eliminated by equation (32). Although the origin of the resistivity need not be specified in our model, it is very likely to be of anomalous type since a large current density in the disk may cause a plasma turbulence. On the other hand, the advection cooling, which is defined as

\[ q_{\text{adv}} (r, \xi) = \Re(r_0) \frac{2}{\sqrt{3}} \frac{B_0^2}{8\pi} \Omega_K(r_0) \left( \frac{r}{r_0} \right)^{-4} \text{sech}^4 \xi, \quad (36) \]

is calculated to be

\[ q_{\text{adv}}(r, \xi) = 4q^+_J(r, \xi). \quad (37) \]

In addition to the heat generation discussed above, the pressure gradient along the flow also increases the enthalpy of the fluid. For a unit volume, this amounts to

\[ w(r, \xi) \equiv v_r \frac{\partial p}{\partial r} = 2 \frac{\Re(r_0)}{\sqrt{3}} \frac{B_0^2}{8\pi} \Omega_K(r_0) \left( \frac{r}{r_0} \right)^{-4} \text{sech}^4 \xi = 4q^+_J(r, \xi). \quad (37) \]

However, it is more convenient to discuss the energy budget in terms of the quantities per unit mass of a fixed fluid element. Then, the Bernoulli sum becomes of our interest. Neglecting higher order terms in \( \Re^{-1} \) in the definition of the sum, we can show that

\[ K \equiv \frac{v_\varphi^2}{2} - \frac{GM}{r} + h = 0 \quad (38) \]

from our solution. Generally the left-hand side may be a function of position, but in the present case it is constant throughout the flow. Therefore, in particular, \( K \) is constant along the stream lines. This fact also implies that the flow is fully advective. The magnetic field plays only a catalytic role in the energy budget.
5. INNER AND OUTER EDGES

The ADAF solution described in the previous section naturally has a finite range of validity in the radial direction. The limiting radii for this validity are called the edges. The inner edge exists at a radius where the infall velocity becomes comparable with the rotation velocity. This means that, at around the inner edge, $\Re$ becomes of order unity and the assumption 5) in §2 becomes invalid. Therefore, we fix the inner edge at the radius where $\Re(r_{\text{in}}) = 1$. As seen from equation (32), this guarantees the relation $\sigma^{-1} \propto \Delta^2$ and that $\Re(r) > 1$ for $r > r_{\text{in}}$. Thus the definition of the inner edge is conceptually clear, but the calculation of its explicit expression is generally not so simple.

On the other hand, the definition of the outer edge is rather vague even in conceptual level. There is no general way of defining it. Both the definition of the outer edge and the explicit calculation of the inner edge depend on the configurations of external magnetic fields. Therefore, we discuss only two typical cases separately. Although similar problems have been discussed in K 87, we need to repeat and refine the discussions since the $r$-dependences of the present solution is slightly different from the previous one.

The first example to be considered is a dipolar external field, because such a configuration may well be conceivable around a neutron star or a white dwarf in a mass exchanging binary system. In this case, all the advected energy is released finally at the surface of the central object. Anyway, from the $r^{-3}$-dependence of the external dipolar field, its strength exceeds that of the induced field in the region within the inner edge. Therefore the place at which the boundary value for $\tilde{b}_0$ should be fixed is the inner edge, i.e. $r_0 = r_{\text{in}}$, and the boundary value is

$$B_0 = \frac{\mu}{r_{\text{in}}^3},$$

(39)

where $\mu$ is the magnitude of a dipole moment and its direction has been assumed to be vertical to the midplane of the accretion disk. Substituting this into equation (33), we
obtain the explicit expression for \( r_{in} \) as
\[
\frac{r_{in}}{\Delta^2} = \left( \frac{\Delta^2}{3} \frac{\mu^4}{GMM^2} \right)^{1/7}.
\] (40)

The position of the outer edge is determined from a requirement of global magnetic-flux conservation. Namely, we require that the unperturbed magnetic flux penetrating the equatorial plane on the outside of the inner edge is compressed by the presence of an accretion flow within the region between the inner and outer edges, i.e.,
\[
B_0 \int_{r_{in}}^{\infty} \frac{r_{in}^3}{r^3} 2\pi r \, dr = \int_{r_{in}}^{r_{out}} \tilde{b} \theta \, 2\pi r \, dr.
\] (41)

In this case, therefore, the outer edge corresponds to a screening radius. The ratio of \( r_{out} \) to \( r_{in} \) is obtained from this equation as
\[
\zeta \equiv \frac{r_{out}}{r_{in}} = (1 + \Delta^{-1})^2 \simeq \Delta^{-2}.
\] (42)

The second example, which is of our present interest, is a uniform external field perpendicular to the equatorial plane. This is an idealization of the configurations expected in various AGNs. In this case, the external field dominates over the induced field at large radii. Therefore, the place at which the boundary value for \( \tilde{b}_\theta \) should be fixed is the outer edge, i.e. \( r_0 = r_{out} \). From equation (33) and the relation \( \Re(r_{out}) = \zeta^{1/2} \), we have an implicit expression for \( r_{out} \)
\[
\frac{r_{out}}{r_{in}} = \left( \frac{3}{\Delta^2 \zeta} \frac{GMM^2}{B_0^4} \right)^{1/5}.
\] (43)

Also in this case, the distance ratio of the outer to inner edges is obtained from a consideration of global magnetic-flux conservation. We assume that all the poloidal flux generated by the disk and penetrating the disk region should close in the region outside the outer edge. Then, the decrease in the flux of the external field in the outer region \((r > r_{out})\) is balanced by the increase in the inner region \((r < r_{in})\), i.e.,
\[
\pi r_{in}^2 \tilde{b}_r(r_{in}) = \int_{r_{in}}^{r_{out}} \tilde{b}_\theta \, 2\pi r \, dr,
\] (44)
where the typical field strength in the inner region has been approximated by $\tilde{b}_r$. From this equation, we obtain the same result as (42) again. Therefore, when $\Delta^2 \ll 1$, equation (43) reduces to

$$r_{\text{out}} = \left( \frac{3GM\dot{M}^2}{B_0^4} \right)^{1/5}, \quad (45)$$

Here, we briefly return to the energy budget in the disk and discuss it, this time, from a global point of view. It has been shown in the previous section that the rate of increase of the enthalpy in a unit volume through the current dissipation and the plasma compression is $5q_J^+(r, \xi)$. Integrating $q_J^+$ over the whole volume of the disk region, we obtain

$$Q_J = \int_{r_{\text{in}}}^{r_{\text{out}}} dr \int_{-\infty}^{\infty} d\xi \ 2\pi \Delta \alpha^2 q_J^+(r, \xi) = \frac{GM\dot{M}}{6} \left[ \frac{1}{r_{\text{in}}} - \frac{1}{r_{\text{out}}} \right], \quad (46)$$

where $B_0$ has been eliminated by equation (33). The result depends neither on the type of the external field discussed above nor on the explicit expressions for $r_{\text{in}}$ and $r_{\text{out}}$. As expected for a fully advective flow, the increase of the enthalpy in the whole disk region is exactly balanced by the net enthalpy flux coming out of the disk region:

$$F_{\text{enth}} \equiv \int h\rho v \cdot dS = \frac{5GM\dot{M}}{6} \left[ \frac{1}{r_{\text{in}}} - \frac{1}{r_{\text{out}}} \right] = 5Q_J, \quad (47)$$

where $dS$ is the surface element of the disk region.

On the other hand, the kinetic energy flux coming out of the disk region is

$$F_{\text{rot}} = \frac{\dot{M}}{2} \left[ v_{\phi}^2(r_{\text{in}}) - v_{\phi}^2(r_{\text{out}}) \right] = \frac{GM\dot{M}}{6} \left[ \frac{1}{r_{\text{in}}} - \frac{1}{r_{\text{out}}} \right]. \quad (48)$$

Thus, it has turned out that the net input of the gravitational energy in the disk region per unit time, $-GM\dot{M} (r_{\text{in}}^{-1} - r_{\text{out}}^{-1})$, is converted into the kinetic energy of quasi-Keplerian rotation (by $1/6$) and the enthalpy which has been added as heat (by $1/6$) and also as compression (by $2/3$).
6. SCALING FOR ACTIVE GALACTIC NUCLEI

Since we are now interested in the AGN activities, the quantities in our model are suitably scaled here to such circumstances. The model is completely specified by three boundary values and one parameter, i.e., mass of the central black hole $M$, mass accretion rate $\dot{M}$, strength of the global magnetic field $B_0$ and the dissipation parameter $\Delta$. They are normalized, respectively, by $10^8 M_\odot$, the Eddington accretion rate $\dot{M}_E$, 1 Gauss and 0.1:

$$m \equiv \frac{M}{10^8 M_\odot}, \quad \dot{m} \equiv \frac{\dot{M}}{\dot{M}_E}, \quad b_0 \equiv \frac{B_0}{1\text{G}}, \quad \delta \equiv \frac{\Delta}{0.1}. \quad (49)$$

The explicit expression for the Eddington accretion rate is

$$\dot{M}_E = 1.4 \times 10^{25} \left( \frac{M}{10^8 M_\odot} \right) \text{g s}^{-1}. \quad (50)$$

Since the reference radius should be fixed at the outer edge, i.e. $r_0 = r_{\text{out}}$, we have $\mathcal{R}(r_{\text{out}}) = \Delta^{-1}$.

In terms of the above non-dimensional quantities, the boundary values of other quantities appearing in the model are expressed as

$$r_{\text{out}} = 0.96 \times 10^{17} b_0^{4/5} \dot{m}^{2/5} m^{3/5} \text{ cm}, \quad (51)$$

$$\tilde{b}_\nu(r_{\text{out}}) = 10 \delta^{-1} b_0 \text{ G}, \quad (52)$$

$$v_K(r_{\text{out}}) = 3.7 \times 10^8 b_0^{2/5} \dot{m}^{-1/5} m^{1/5} \text{ cm s}^{-1}, \quad (53)$$

$$\tilde{p}(r_{\text{out}}) = 4.0 \delta^{-2} b_0^2 \text{ dyne cm}^{-2}, \quad (54)$$

$$T(r_{\text{out}}) = 2.7 \times 10^8 b_0^{4/5} \dot{m}^{-2/5} m^{2/5} \text{ K}, \quad (55)$$

$$\tilde{\rho}(r_{\text{out}}) = 8.8 \times 10^{-17} \delta^{-2} b_0^{6/5} m^{2/5} m^{-2/5} \text{ g cm}^{-3}. \quad (56)$$

All other induced quantities can be calculated from the set of analytic solutions given in §4 and their boundary values given above. For example, integrating the density over the
disk height, we obtain the surface density which becomes independent of $r$ in our model:

$$\Sigma(r) \equiv \int_{-\infty}^{\infty} \rho(r, \xi) r \Delta \xi = \text{const.} \equiv \Sigma(r_{\text{out}}),$$

$$\Sigma(r_{\text{out}}) = 1.7 \times 10^{-1} \delta^{-2/5} \dot{m}^{4/5} m^{1/5} \text{ g cm}^{-2}. \quad (57)$$

The dependence of $\Sigma \propto \dot{M}^{4/5}$ suggests that the disk is secularly stable (e.g., Kato, Fukue & Mineshige 1998). Further integrating $\Sigma(r)$ over the radius, we have the total mass in the disk as

$$M_D \simeq \pi r_{\text{out}}^2 \Sigma = 4.9 \times 10^{35} \delta^{-1/2} \dot{m}^{-6/5} m^{8/5} m^{7/5} \text{ g}. \quad (58)$$

In order for our analytic solution to serve well as a model of ADAF in a global magnetic field, the radiation energy which is expected to come out of the disk should be really negligible in the physical situations of our interest. This point will be checked here. The opacity due to the electron scattering is $\kappa_{\text{es}} = 0.4 \text{ cm}^2 \text{ g}^{-1}$ and the free-free opacity is written as $\kappa_{\text{ff}} = 0.6 \times 10^{23} \rho T^{-7/2} \text{ cm}^2 \text{ g}^{-1}$. In view of the scaled values for $\rho$ and $T$, one can safely conclude that the former opacity is dominant over the latter. Then, the optical depth of the disk becomes $r$-independent:

$$\tau(r) = \int_{-\infty}^{\infty} \kappa(r, \xi) \rho(r, \xi) r \Delta \xi \simeq \kappa_{\text{es}} \Sigma(r_2)$$

$$= 3.4 \delta^{-1} \dot{m}^{-6/5} \dot{m}^{4/5} m^{1/5}. \quad (59)$$

Provided that $\dot{m} \ll 1$, it can be confirmed from this expression that the disk is everywhere optically thin unless its opening angle is extremely small.

As a representative of radiation losses, we evaluate the Bremsstrahlung from the disk. This is because it can be evaluated by a simple formula and other mechanism such as the synchrotron loss seems to have a similar order of magnitude (Narayan et al. 1995, Manmoto et al. 1997, Narayan et al. 1998). The radiation flux from the disk surface is

$$F_B^{-}(r) = \int_{-\infty}^{\infty} g_B(r, \xi) r \Delta \xi = F_B^{-}(r_{\text{out}}) \left( \frac{r}{r_{\text{out}}} \right)^{-3/2},$$
where \( q_B(r, \xi) \) is the volume emissivity. On the other hand, height integrating the rate of enthalpy increase, we obtain for energy input rate at a radius \( r \) as

\[
Q^{+}_{\text{enthalpy}}(r) = \int_{-\infty}^{\infty} \left[ q_J(r, \xi) + w(r, \xi) \right] r \Delta \xi = Q^{+}_{\text{enthalpy}}(r_{\text{out}}) \left( \frac{r}{r_{\text{out}}} \right)^{-3/2},
\]

\[
Q^{+}_{\text{enthalpy}}(r_{\text{out}}) = 2.9 \times 10^7 b_0^{12/5} \dot{m}^{-1/5} m^{1/5} \text{ ergs cm}^{-2} \text{ s}^{-1}.
\]

Comparing this with the expression (60), we can see that the radiation loss is actually negligible under the circumstances with \( \dot{m} \ll 1 \) unless the disk is extremely thin.

Although our model assumes, for simplicity, a common temperature for both the electron and ion components, actually it may take different values for different components as is the case in the viscous ADAF models. Such a difference in temperature, however, may be less important in the resistive ADAFs because the (effective) Joule dissipation will heat mainly the electron component (Bisnovati-Kogan & Lovelace 1997) which is easier to cool down by radiation than the ion component. In any case, that is beyond the scope of the present paper.

7. DISCUSSION

In this section, we discuss main issues which may be raised about our model, introducing speculations to some extent. The first is about the accuracy of our set of analytic solutions. As described in §4, the separation of variables are performed only approximately. It is accurate at the midplane of a disk, but is invalid for large \( \xi \). The errors in the angular dependence may exceed 50% for \( |\xi| > 0.5 \), and certain inconsistencies appear beyond \( \tanh |\xi| = 1/\sqrt{3} \) where \( (d^3/d\xi^3) \tanh \xi \) changes its sign. One may therefore identify there (i.e., \( |\xi| \sim 1 \)) as the surfaces of a disk. Within this range, the set of solutions serves
well for most of our purposes because we are usually interested not in the exact angular
dependences but in their qualitative behavior.

Angular momentum is expected to be carried away from the disk by magnetic stresses
along the externally given poloidal magnetic lines of force. In the case of a dipole type
external field it is transferred to the central star, and in the case of a uniform external
field, to distant regions. The rate of angular momentum extraction from the upper and
lower surfaces of a disk can be calculated from our solution. The result is proportional to
\[ \dot{M}[\sqrt{GMr_{\text{out}}} - \sqrt{GMr_{\text{in}}}] \], and the proportionality constant depends on the definition of the
surfaces \( \xi = \pm \xi_0 \). Such an ambiguity is also a reflection of the approximate nature of the
solution.

Although a definite answer cannot be obtained in the present stage of approximation,
the above picture for angular momentum extraction seems to be physically plausible also
from a global consideration of the accretion process. A schematic picture is shown in Fig.
1 for a uniform external field. This field is an idealization of such a large-scale field of
galactic origin whose sources are in distant regions. The rotation of the accreting plasma in
this poloidal field acts as a dynamo in a DC-circuit and drives a radial current in the disk.
We expect this current to close its circuit around distant regions and reaching to the polar
regions (as for the supply of plasma to this regions, see below). The presence of such a
globally circulating poloidal current system guarantees the generation of toroidal magnetic
field outside the disk, which is necessary for the transfer of angular momentum by magnetic
stresses.

Thus, the angular momentum can be extracted through a nearly vacuum space, but
we do not intend to deny the possibility of the extraction also by wind type outflows (e.g.,
Blandford & Payne 1982). Rather, an accretion flow with a divergent component in the
\( \theta \)-direction has been obtained also in our scheme when the simplifying assumption of \( v_\theta = 0 \)
is relaxed (Kaburaki 1987).

The $\mathbf{j}_p \times \mathbf{b}_\varphi$ force acting on the poloidal current $\mathbf{j}_p$ whose origin has been discussed above contributes to the confinement of accreting plasma towards the equatorial plane and, possibly, also to the collimation and acceleration of a bipolar jet system (see, Kaburaki & Itoh 1987). The former effect has been described in the present paper as the magnetic confinement of an accretion disk (the $\theta$-component of the equation of motion). The latter two effects are interpreted as the pinch effect on the return current near the polar axis and the acceleration by magnetic pressure-gradient force due to $b_\varphi$, respectively.

Therefore, the development of a toroidal magnetic field in the middle-latitude regions seems to naturally explain such an association of disk-like structure and bipolar-jet structure as is frequently observed in many astrophysical situations. Interestingly enough, essentially the same story as the above for the formation of a disk-like structure and for the collimation and acceleration of a jet applies even to the case of disk-like outflows which are expected to exist around rapidly rotating, magnetized stars including pulsars (Kaburaki 1989). This is because the sense of the $\mathbf{j}_p \times \mathbf{b}_\varphi$ force does not change even in that case, in spite of the change in the sense of $b_\varphi$ (and therefore that of $\mathbf{j}_p$) which is required to realize an outflow (i.e., angular momentum should be added to the disk). Thus, the disk-jet association is understood naturally in the same scheme even in the case of outflows.

The source of plasmas in the polar regions is of course an accretion disk. Within the inner edge of an accretion disk, there develops a strong poloidal magnetic field swept by the accretion flow. This field may form a core of a bipolar-jet system. The accretion flow would be decelerated within the inner edge by the strong magnetic field perpendicular to it and, although main fraction of it may be eventually swallowed by the central black hole, some part may be turned its direction to follow the core field and then accelerated along it (see Fig. 1).
It may seem at first rather curious that, as far as our solution is concerned, no energy is extracted from the disk associated with the angular momentum extraction by magnetic stresses. As confirmed by substituting our solution into the leading-order expressions in $\Delta$ of the electric field,

$$
E_r = \frac{c}{4\pi\sigma\Delta} \frac{1}{r} \frac{\partial b_\phi}{\partial \xi} + \frac{1}{c} \nu_r b_\theta, \quad E_\theta = -\frac{1}{c} (\nu_r b_r - \nu_r b_\phi), \quad E_\phi = 0, \quad (62)
$$

we have $\mathbf{E} = 0$ in the disk and therefore the vanishing Poynting flux. The situation, however, is quite analogous to that in the viscous ADAF models in which angular momentum is transported radially outwards by viscous stresses but energy is dissipated locally in the flow.

The vanishing of the Poynting flux is a consequence of idealization that the external electric loads in which some part of energy may be consumed is negligible in determining the structure of an accretion flow. In such a situation, there is no need to transport energy to the outside. This is an opposite extreme of ideal MHD disks. Since no energy dissipation can occur in an ideal MHD disk, the whole power input is carried away in terms of the Poynting flux to distant regions where the dissipation is assumed to occur. In view of the above discussion of global configurations, at least, the jet should be included as an external load in a more realistic determination of the current in the disk. However, of course this is beyond the scope of the present work.

The global MHD stability of accretion disks in the resistive ADAF model may be an important issue. This is because the ratio of the toroidal to the poloidal magnetic fields which is represented by $\mathcal{R}$ is assumed to be large ($\mathcal{R} > 1$ and $\mathcal{R}^2 \gg 1$) in the model, while the configurations with too large $\mathcal{R}$ are generally believed to be unstable. Since $\mathcal{R}$ increases from $\sim 1$ at the inner edge like $r^{1/2}$ towards the outer edge ($\mathcal{R}(r_{\text{out}}) = \Delta^{-1}$ in the case of uniform external field), the outermost regions are apt to be unstable to the instabilities of helical type. Although the determination of the critical value for $\mathcal{R}$ in actual resistive ADAFs is again beyond the scope of this paper, it can be said qualitatively that only not
so widely extended disks and not so thin disks are expected to be maintain safely.

As for the local MHD stabilities, we have already performed an analysis based on the background solution obtained in the present paper and have found that there are actually growing modes. Although further investigations including non-linear evolutions are needed for a definite conclusion, the instabilities are expected to be not so strong as to destroy the whole structure. Rather, they may cause a turbulence in the flow which is the origin of some types of anomalous resistivity or viscosity (e.g., Balbus & Hawley 1991). The results of the local analysis will be reported elsewhere.

A. APPENDIX

Although a reduced Keplerian velocity has appeared in our resistive ADAF model, the rotation law may be different for different geometries of the accretion flows. Here, this point will be demonstrated by considering two typical cases: one is the disk-like flow of constant height (i.e., $H$ = const.) and the other is that of constant opening angle (i.e., $\Delta$ = const.). For simplicity, we assume that the disk structure is maintained only by the gravity and that the infall velocity is negligibly small compared with the rotation velocity.

When the disk height is constant, the pressure force is almost vertical to the equatorial plane and it is convenient to adopt cylindrical coordinates ($\varpi$, $\varphi$, $z$). The equations of force balance in a poloidal plane are, therefore,

\[ \frac{GM}{r^2} \frac{\varpi}{r} = \frac{v_r^2}{\varpi}, \]
\[ \frac{GM}{r^2} \frac{z}{r} = \frac{1}{\rho} \frac{\partial p}{\partial z}, \]

where $r = (\varpi^2 + z^2)^{1/2}$ and the pressure term in the $\varpi$-component has been neglected. Equation (A2) implies a gravitational confinement of the accreting plasma.
Equation (A1) yields the Kepler angular velocity

$$\Omega(\varpi, z) = \left(\frac{GM}{r^3}\right)^{1/2} \equiv \Omega_K(r).$$  \hfill (A3)

However, the plane of rotation is always parallel to the equatorial plane, so that the center of rotation does not coincide with the center of gravitational attraction except for the rotation in the disk’s midplane. Since the angular velocity is Keplerian, the centrifugal force is generally smaller than the gravity at an arbitrary $r$:

$$v_\varphi^2 \varpi = GM\sin \theta \leq GM\frac{r}{r}.$$  \hfill (A4)

From equation (A2) we have a rough estimate for the ratio of sound velocity to the Kepler velocity:

$$\frac{C_S^2}{v_K^2} \sim \frac{H^2}{r^2}.$$  \hfill (A5)

Since $H$ is a constant, this suggests a temperature variation of the form $T(r) \propto r^{-3}$.

When the opening angle of a disk is constant, the pressure force is almost along the $\theta$-direction and it is convenient to adopt spherical polar coordinates $(r, \theta, \varphi)$. The force balance equations are

$$\frac{v_\varphi^2}{r} = \frac{GM}{r^2},$$  \hfill (A6)

$$\frac{v_\varphi^2}{r} \cot \theta = \frac{1}{r} \frac{\partial p}{\partial \theta},$$  \hfill (A7)

where the pressure term in $r$-component has been neglected compared with the gravity. Equation (A7) implies a centrifugal confinement of the accreting plasma.

Equation (A6) yields the Kepler rotational velocity

$$v_\varphi(r, \theta) = \left(\frac{GM}{r}\right)^{1/2} \equiv v_K(r).$$  \hfill (A8)

The plane of rotation is again parallel to the equatorial plane, so that the center of rotation does not coincide with the center of gravitational attraction except for the rotation in the
disk’s midplane. Since the rotational velocity is Keplerian, the centrifugal force generally exceeds the gravity at an arbitrary $r$:

$$\frac{v^2}{\omega} = \frac{GM}{r} \frac{1}{\sin \theta} \geq \frac{GM}{r}.$$  \hspace{1cm} (A9)

From equation (A7) we have a rough estimate for the ratio of sound velocity to the Kepler velocity:

$$\frac{C_s^2}{v_K^2} \sim \left| \frac{\cot \Delta}{\Delta} \right|.$$  \hspace{1cm} (A10)

Since $\Delta$ is a constant, this suggests a temperature variation of the form $T(r) \propto r^{-1}$.

From the above discussions, we can see that the geometry of disks has essential effects on their properties. The rotation law is different between the disks of constant $H$ and constant $\Delta$. The radial dependences of the temperature is also quite different. The same dependence as the virial temperature is obtained only in the disks of constant $\Delta$, but thin disks can be realized there only when the force of confinement other than gravity is present (i.e., equation (A7) cannot hold when $\Delta \ll 1$ even if the temperature is nearly virial). In the case of constant $H$, on the other hand, thin disks are realized if the temperature is much smaller than the virial one.
REFERENCES


This manuscript was prepared with the AAS \LaTeX{} macros v4.0.
Fig. 1.— Schematic drawing of the global geometries of magnetic field and plasma flow. A poloidally circulating current system ($j_p$) driven by the rotational motion of accreting plasma generates a toroidal magnetic field $b_\phi$ in addition to a nearly uniform external field. The presence of this toroidal field outside the disk guarantees the magnetic extraction of angular momentum from the disk. This field acts to confine the accreting plasma towards the equatorial plane and also has a tendency to collimate and accelerate the plasma in the polar regions. If the condition is favorable, the plasma in the polar regions may form a set of bipolar jets.