Neutrino Physics at Muon Colliders

Bruce J. King

Brookhaven National Laboratory
cmail: bking@bnl.gov

Abstract. An overview is given of the neutrino physics potential of future muon storage rings that use muon collider technology to produce, accelerate and store large currents of muons.

INTRODUCTION

This paper gives an overview of the neutrino physics possibilities at a future muon storage ring, which can be either a muon collider ring or a ring dedicated to neutrino physics that uses muon collider technology to store large muon currents. It summarizes a previous more detailed description of these topics by this author [1].

After a general characterization of the neutrino beam and its interactions, some crude quantitative estimates are given for the physics performance of a muon ring neutrino experiment (MURINE) consisting of a high rate, high performance neutrino detector at a 250 GeV muon collider storage ring.

NEUTRINO PRODUCTION AND EVENT RATES

Neutrinos are emitted from the decay of muons in the collider ring:

\[
\mu^- \rightarrow \nu_\mu + \bar{\nu}_e + e^-, \\
\mu^+ \rightarrow \nu_\mu + \nu_e + e^+.
\]

(1)

The thin pencil beams of neutrinos for experiments will be produced from long straight sections in either the collider ring or a ring dedicated to neutrino physics. From relativistic kinematics, the forward hemisphere in the muon rest...
frame will be boosted, in the lab frame, into a narrow cone with a characteristic opening half-angle, $\theta_\nu$, given in obvious notation by

$$\theta_\nu \simeq \sin \theta_\nu = \frac{1}{\gamma} \frac{m_\mu}{E_\mu} \simeq \frac{10^{-4} \ E_\mu}{E_\mu(\text{TeV})}. \quad (2)$$

The large muon currents and tight collimation of the neutrinos results in extremely intense beams – intense enough even to constitute a potential off-site radiation hazard [2].

For the example of 250 GeV muons, the neutrino beam will have an opening half-angle of approximately 0.4 mrad. The final focus regions around collider experiments are important exceptions to equation 2 since the muon beam itself will have an angular divergence in these regions that is large enough to significantly spread out the neutrino beam.

For TeV-scale neutrinos, the neutrino cross-section is approximately proportional to the neutrino energy, $E_\nu$. The charged current (CC) and neutral current (NC) interaction cross sections for neutrinos and antineutrinos have numerical values of [3]:

$$\sigma_{\nu N} \text{ for } \begin{pmatrix} \nu_{-CC} \\ \nu_{-NC} \\ \bar{\nu} - CC \\ \bar{\nu} - NC \end{pmatrix} \simeq \begin{pmatrix} 0.72 \\ 0.23 \\ 0.38 \\ 0.13 \end{pmatrix} \times \frac{E_\nu}{1 \text{ TeV}} \times 10^{-35} \text{ cm}^2. \quad (3)$$

These cross sections are easily converted into approximate experimental event rates for the example used in reference [1] of a 250+250 GeV collider with a 200 meter straight section. For a general purpose detector subtending the boosted forward hemisphere of the neutrino beam:

$$\text{Number of } \begin{pmatrix} \nu_{\mu} - CC \\ \nu_{\mu} - NC \\ \bar{\nu}_e - CC \\ \bar{\nu}_e - NC \end{pmatrix} \text{ events/yr} \simeq \begin{pmatrix} 2.6 \\ 0.8 \\ 1.4 \\ 0.5 \end{pmatrix} \times 10^7 \times l[\text{g.cm}^{-2}], \quad (4)$$

where $l$ is the detector length. For a long baseline detector in the center of the neutrino beam:

$$\text{Number of } \begin{pmatrix} \nu_{\mu} - CC \\ \nu_{\mu} - NC \\ \bar{\nu}_e - CC \\ \bar{\nu}_e - NC \end{pmatrix} \text{ events/yr} \simeq \begin{pmatrix} 1.4 \\ 0.4 \\ 0.7 \\ 0.2 \end{pmatrix} \times 10^7 \times \frac{M[\text{kg}]}{(L[\text{km}])^2}, \quad (5)$$

where $M$ is the detector mass and $L$ the distance from the neutrino source.

These event rates are several orders of magnitude higher than in today’s neutrino beams from accelerators.
A GENERAL PURPOSE NEUTRINO DETECTOR

Figure 1 is an example of the sort of high rate general purpose neutrino detector that would be well matched to the intense neutrino beams. Note the contrast with the kilotonne-scale calorimetric targets used in today’s high rate neutrino experiments.

The neutrino target is a 1 meter long stack of CCD tracking planes with a radius of 10 cm chosen to match the beam radius at approximately 200 meters from production for a 250 GeV muon beam. It contains 750 planes of 300 micron thick silicon CCD’s, corresponding to a mass per unit area of approximately 50 g.cm$^{-2}$, about 2.5 radiation lengths and 0.5 interaction lengths. Equation 4 predicts a very healthy $2 \times 10^9$ CC interactions per year for this target.

Besides providing the mass for neutrino interactions, the tracking target allows precise reconstruction of the event topologies from charged tracks, including event-by-event vertex tagging of those events containing charm or beauty hadrons or tau leptons. Given the favorable vertexing geometry and the few-micron typical CCD hit resolutions, it is reasonable to expect almost 100 percent efficiency for b tagging, perhaps 70 to 90 percent efficiency for
charm tagging and excellent discrimination between b and c decays.

The target in figure 1 is surrounded by a time projection chamber (TPC) tracker in a vertical dipole magnetic field. The characteristic dE/dx signatures from the tracks would identify each charged particle. Further particle ID is provided by the Cherenkov photons that are produced in the TPC gas then reflected by a spherical mirror at the downstream end of the tracker and focused onto a read-out plane at the upstream end of the target. The mirror is backed by electromagnetic and hadronic calorimeters and, lastly, by iron-core toroidal magnets for muon ID.

NEUTRINO INTERACTIONS AND THEIR EXPERIMENTAL INTERPRETATION

The dominant interaction of TeV-scale neutrinos is deep inelastic scattering (DIS) off nucleons (i.e. protons and neutrons). There are 2 types of DIS: neutral current (NC) and charged current (CC) scattering. In neutral current (NC) scattering, the neutrino is deflected by a nucleon ($N$) and loses energy with the production of several hadrons ($X$):

$$\nu + N \rightarrow \nu + X,$$  \hspace{1cm} (6)

This comprises about 25 percent of the total cross section and is interpreted as elastic scattering off one of the many quarks inside the nucleon through the exchange of a virtual neutral Z boson:

$$\nu + q \rightarrow \nu + q.$$  \hspace{1cm} (7)

Charged current (CC) scattering is similar to NC scattering except that the neutrino turns into its corresponding charged lepton:

$$\nu + N \rightarrow l^- + X,$$
$$\bar{\nu} + N \rightarrow l^+ + X,$$  \hspace{1cm} (8)

where $l$ is an electron/muon for electron/muon neutrinos. At the more fundamental quark level a charged W boson is exchanged with a quark ($q$), which is turned into another quark species ($q'$) whose charge differs by one unit.

$$\nu + q \rightarrow l^- + q',$$
$$\bar{\nu} + q' \rightarrow l^+ + q.$$  \hspace{1cm} (9)

The relativistically invariant quantities that are routinely extracted in DIS experiments are 1) Feynman $x$, the fraction of the nucleon momentum carried by the struck quark, 2) the inelasticity, $y = E_{\text{hadronic}}/E_\nu$, which is related to the scattering angle of the neutrino in the neutrino-quark CoM frame, and 3)
the momentum-transfer-squared, $Q^2 = 2M_{\text{proton}}E_{\nu xy}$. MURINE’s will have the further capability of reconstructing the hadronic 4-vector, resulting in a much better characterization of each interaction.

The final state quark always “hadronizes” at the nuclear distance scale, combining with quark-antiquark pairs to produce the several hadrons seen in the detector. Final state c and b quarks can be identified by vertex tagging of the decaying charm or beauty hadrons that contain them, and some statistically based flavor tagging will also be available for u, d or s final state quarks, using so-called “leading particle effect” [1].

**PHYSICS OPPORTUNITIES**

Neutrino interactions are interesting both in their own right and as probes of the quark content of nucleons, so a MURINE has wide-ranging potential to make advances in many areas of elementary particle physics. This section gives an overview for measurements involving the CKM quark mixing matrix, nucleon structure and QCD, electroweak measurements, neutrino oscillations and, finally, studies of charmed hadrons.

There is considerable theoretical interest in the mixture of final state quarks produced in CC interactions. The struck quark can be converted into any of the three final state quarks that differ by one unit of charge: a down (d), strange (s), or bottom(b) quark can be converted into an up (u), charmed (c), or top (t) quark and vice versa. In practice, production of the heavy top quark is kinematically forbidden at these energies and the production of other quark flavors is influenced by their mass. Beyond this, the Standard Model of elementary particle physics (SM) predicts the probability for the interaction to be proportional to the absolute square of the appropriate element in the so-called Cabbibo-Kobayashi-Maskawa (CKM) quark mixing matrix, a unitary matrix with 4 free parameters whose values are not predicted by the SM. Improved measurements involving the CKM matrix will test the SM hypothesis. It is of particular interest that one of the 4 parameters is a complex phase that is postulated as an explanation for CP violation – the intriguing experimental phenomenon that particles may have tiny deviations from the properties that mirror those of their antiparticles.

The experimentally determined values for the 9 mixing probabilities are given in table 1 [4], along with their current percentage uncertainties and speculative projections [1] for how the uncertainties could be reduced by a MURINE. From the large improvements in 4 of the 9 uncertainties it is clear that a MURINE has potential for tremendous improvements in measuring the quark mixing matrix, and more detailed studies are clearly desirable.

Another major motivation for MURINE’s is the potential for greatly improved measurements of nucleon structure functions – the momentum distributions of quarks inside the nucleon. This provides [5] important tests of
**TABLE 1.** Quark mixing probabilities. Threshold suppression due to quark masses has been neglected. In practice, this will reduce the mixing probabilities to the heavier c and b quarks to below the values given in the table and will prevent any mixing to the top quark. The second row for each quark gives current percentage uncertainties in quark mixing probabilities and speculative projections of the uncertainties after analyses from a MURINE. The two uncertainties in brackets have not been measured directly from tree level processes. The uncertainties assume that no unitarity constraints have been used.

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>s</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>0.95</td>
<td>0.05</td>
<td>0.00001</td>
</tr>
<tr>
<td></td>
<td>±0.1%</td>
<td>±1.6%</td>
<td>±50% → 1-2%</td>
</tr>
<tr>
<td>c</td>
<td>0.05</td>
<td>0.95</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>±15% → 0.2-0.5%</td>
<td>±35% → ~ 1%</td>
<td>±15% → 3-5%</td>
</tr>
<tr>
<td>t</td>
<td>0.0001</td>
<td>0.001</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>(±25%)</td>
<td>(±40%)</td>
<td>±30%</td>
</tr>
</tbody>
</table>

Quantum chromodynamics (QCD) – the theory of the strong interaction that is widely accepted for its elegance and simplicity but which has not been experimentally verified at the level of the electroweak theory. A MURINE might well be the best single experiment of any sort for the examination of perturbative QCD [1].

Neutrino physics has also had an important historical role in measuring the electroweak mixing angle, which is simply related to the mass ratio of the W and Z intermediate vector bosons:

\[
\sin^2 \theta_W \equiv 1 - \left( \frac{M_W}{M_Z} \right)^2.
\]  

(10)

Now that \( M_Z \) has been precisely measured at LEP, measurements of \( \sin^2 \theta_W \) in neutrino physics can be directly converted to predictions for the W mass. The comparison of this prediction with direct \( M_W \) measurements in collider experiments constitutes a precise prediction of the SM and a sensitive test for exotic physics modifications to the SM [5]. Reference [1] estimates that the predicted uncertainty in \( M_W \) from a MURINE analysis might be of order 10 MeV, which improves by more than an order of magnitude on today’s neutrino experiments [5,6] and is comparable with the projected best direct measurements from future collider experiments.

A neutrino property that is currently drawing much interest is the question of whether neutrinos have a non-zero mass. If they do then it is possible for the 3 neutrino flavors to mix, perhaps producing neutrino oscillations that can
be observed using a neutrino beam. The probability for an oscillation between two of the flavors is given by [7]:

\[
\text{Oscillation Probability} = \sin^2 \theta \times \sin^2 \left(1.27 \frac{\Delta m^2 [eV^2] \cdot L [km]}{E_{\nu} [GeV]} \right),
\]

where the first term gives the mixing strength and the second term gives the distance dependence.

Reference [1] obtains the following order-of-magnitude mass limit for an assumed long-baseline detector with reasonable parameters and with full mixing:

\[
\Delta m^2 |_{\text{min}} \sim O(10^{-4}) \ eV^2,
\]

independent of the distance to the detector. Similarly, a mixing probability sensitivity for $10^{10}$ events in a short-baseline detector is found to be as low as

\[
\sin^2 \theta |_{\text{min}} \sim O(10^{-7}),
\]

for the most favorable value of $\Delta m^2$. Both of these estimates apply generically to all 3 possible mixings between 2 flavors: $\nu_e \leftrightarrow \nu_\mu$, $\nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_\tau$. (See also reference [8] for another discussion of neutrino oscillations at a MURINE.)

The $\Delta m^2$ estimate is more than an order of magnitude better than any proposed accelerator or reactor experiments for $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_e \leftrightarrow \nu_\tau$, and is competitive with the best such proposed experiments for $\nu_e \leftrightarrow \nu_\mu$. The estimated value for $\sin^2 \theta |_{\text{min}}$ is even more impressive – orders of magnitude better than in any other current or proposed experiment for each of the three possible oscillation.

As an interesting final topic, MURINE’s should be rather impressive factories for the study of charm – with a clean, well reconstructed sample of several times $10^8$ charmed hadrons produced in $10^{10}$ neutrino interactions. There are several interesting physics motivations for charm studies at a MURINE [9]. As an example, particle-antiparticle mixing has yet to be observed in the charm sector [10], and it is quite plausible [1] that a MURINE would provide the first observation of $D^0 \rightarrow \bar{D}^0$ mixing.

**SUMMARY**

The intense neutrino beams at muon collider complexes should usher in an exciting new era of neutrino physics experiments, with great advances expected in both traditional and new areas of neutrino physics.
REFERENCES


3. See, for example, Chris Quigg, *Neutrino Interaction Cross Sections*, FERMILAB-Conf-97/158-T.


