Lorentz gauge fixing and the Gribov problem: the fermion correlator in lattice compact QED with Wilson fermions

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For the Lorentz gauge the influence of Gribov copies on the fermion propagator is investigated in quenched lattice compact QED. In the Coulomb phase zero-momentum modes of the gauge fields are shown to be the main reason for a significant deviation from ordinary perturbation theory.

Numerical lattice computations of gauge-dependent gauge or fermion field correlators can give us detailed information about nonperturbative properties of quantum fields and allow a direct comparison with perturbation theory in the continuum. However, most of the known iterative gauge fixing procedures are faced with the existence of Gribov copies. For compact lattice QED within the Coulomb phase the standard Lorentz gauge fixing causes the transverse photon correlator to deviate significantly from the expected zero-mass behaviour \cite{1,2}. Also the fermion correlator depends strongly on the achieved gauge copies \cite{3}. Numerical \cite{2,4–6} and analytical \cite{7} studies have shown that the main gauge field excitations responsible for the occurrence of disturbing gauge copies are double Dirac sheets (DDS) and zero-momentum modes (ZMM).

In the present talk we are going to discuss the influence of the ZMM on the fermion correlator. We shall demonstrate that only a proper account of them will allow us to determine the renormalized fermion mass.

We consider 4d compact QED in the quenched approximation on a finite lattice ($V = N^3_s \times N_t$). We employ the standard Wilson (plaquette) gauge and fermionic action, respectively. The latter is given by

\[ S_F = \sum_{x,y} \bar{\psi}_x M_{xy}(\theta) \psi_y, \quad M = 1 - \kappa \mathbf{D}, \tag{1} \]

\[ D_{xy} = \sum_{\mu=1}^4 \left\{ U_{x,\mu} P^-_{\mu} \delta_{y,x+\hat{\mu}} + U_{x-\hat{\mu},\mu}^* P^+_{\mu} \delta_{y,x-\hat{\mu}} \right\}, \]

where $P^\pm_{\mu} = \hat{1} \pm \gamma_{\mu}$. The $U_{x,\mu} = e^{i \theta_{x,\mu}}$ denotes the gauge link degrees of freedom. Periodic boundary conditions (b.c.) are implied. The fermion fields are anti-periodic in $x_4$. The fermion correlator for a given gauge field $\theta$ reads

\[ \Gamma(\tau; \theta) = \frac{1}{V} \sum_{x,x_4} \sum_y M_{x,x_4,y,x+\tau}^{-1}(\theta). \tag{2} \]

We shall restrict ourselves to the vectorial part

\[ \Gamma_V(\tau; \theta) = \frac{1}{4} \text{Re} \text{Tr} \left( \gamma_4 \Gamma(\tau; \theta) \right), \tag{3} \]

with the trace taken with respect to the spinor indices.

LATERON, we shall compare the average $\langle \Gamma_V \rangle_\theta$ within the approximation where only constant gauge fields are taken into account. The correlator in a uniform background $\theta_{x,\mu} = \phi_\mu$, $-\pi < \phi_\mu \leq \pi$, $\mu = 1, \cdots, 4$ is given by

\[ \Gamma_V(\tau; \phi) = \frac{(1 - \delta_{\tau,0} - \delta_{\tau,N_t}) (1 + \mathcal{M})^{-1}}{2 (1 + \mathcal{E}^{2N_t} - 2 \mathcal{E}^N \cos(\phi_4 N_t))} \times \left\{ \mathcal{E}^{\tau + \mathcal{E}^{2N_t} - \tau} \cos(\phi_4 \tau) \right\} \tag{4} \]

\[ - [\mathcal{E}^{N_t} \tau + \mathcal{E}^N \tau] \cos[\phi_4 (N_t - \tau)] \]

\[ \quad \times \left\{ \mathcal{E}^{\tau + \mathcal{E}^{2N_t} - \tau} \cos(\phi_4 \tau) \right\} \tag{4} \]

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\[ - [\mathcal{E}^{N_t} \tau + \mathcal{E}^N \tau] \cos[\phi_4 (N_t - \tau)] \]

\[ \quad \times \left\{ \mathcal{E}^{\tau + \mathcal{E}^{2N_t} - \tau} \cos(\phi_4 \tau) \right\} \tag{4} \]
\[ \mathcal{E} = \left[ 1 + \frac{\sqrt{M^2 + K^2}^3}{2(1 + M)} \right] \left( \frac{\sqrt{M^2 + K^2} + \sqrt{(M + 2)^2 + K^2}}{1 + \cos \theta} \right)^{-1}, \]

\( \mathcal{M} = m_0 + \sum_{l=1}^{3} (1 - \cos \phi_l) \), \quad \mathcal{K} = \sum_{l=1}^{3} \sin^2 \phi_l. \)

For \( \phi_\mu = 0, \ \mu = 1, \cdots, 4 \) the free fermion correlator for finite lattice size \([8]\) is reproduced. Note, that the bare mass \( m_0 \) is related to the hopping-parameter by \( \kappa = 1/(8 + 2m_0) \).

In numerical simulations the Lorentz gauge is fixed by iteratively maximizing the functional

\[ F[\theta] = \frac{1}{4V} \sum_{x,\mu} \cos \theta_{x,\mu} = \text{Max.} \tag{5} \]

with respect to (periodic) gauge transformations.

The algorithm is called standard Lorentz gauge fixing, if it consists only of local maximization and overrelaxation steps. It is well-known that the Gribov problem has to be solved by searching for the global maximum providing the best gauge copy. In [5] we have shown that in order to reach the global maximum we have necessarily to remove both the DDS and the ZMM from the gauge fields.

DDS can be identified as follows. The plaquette angle \( \theta_{x,\mu} \) is decomposed into the gauge invariant (electro-) magnetic flux \( \theta_{x,\mu} \in (-\pi, \pi] \) and the discrete gauge-dependent contribution \( 2\pi n_{x,\mu} \), \( n_{x,\mu} = 0, \pm 1, \pm 2 \). The latter represents a Dirac string passing through the given plaquette if \( n_{x,\mu} = \pm 1 \) (Dirac plaquette). A set of Dirac plaquettes providing a world sheet of a Dirac string on the dual lattice is called Dirac sheet. DDS consist of two sheets with opposite flux orientation extending over the whole lattice and closing themselves by the periodic b.c. They can easily be identified by counting the total number of Dirac plaquettes \( N^b_{(\mu,\nu)} \) for each choice \((\mu;\nu)\). DDS can be removed by periodic gauge transformations. For standard Lorentz gauge fixing DDS have been seen to occur independently of the lattice size and \( \beta [2,5,6] \). As a consequence the non-zero momentum transverse photon correlator significantly deviates from the expected zero-mass behaviour [5–7].

The ZMM of the gauge field

\[ \phi_\mu = \frac{1}{V} \sum_x \theta_{x,\mu} \]

do not contribute to the pure gauge field action either. For gauge configurations representing small fluctuations around constant modes it is easy to see, that the global maximum of the functional (5) requires \( \phi_\mu = 0 \). The latter condition can be achieved by non-periodic gauge transformations

\[ \theta_{x,\mu} \rightarrow \theta_{x,\mu}^c = c_\mu + \theta_{x,\mu} \mod 2\pi, \quad c_\mu \in (-\pi; \pi]. \]

We realize a proper gauge fixing procedure as proposed in [5]. The successive Lorentz gauge iteration steps are always followed by non-periodic gauge transformations suppressing the ZMM. Additionally we check, whether the gauge fields contain yet DDS. The latter can be excluded by repeating the procedure with initial random gauges. We call the combined procedure zero-momentum Lorentz gauge (ZML gauge). It provides with very high accuracy the global maximum of the gauge functional. The photon propagator perfectly agrees with the expected perturbative result throughout the Coulomb phase [5].

Our Monte Carlo simulations were carried out with a filter heat bath method. In order to extract the pure ZMM effect, we first apply the standard Lorentz gauge procedure modified by initial random gauges in order to suppress DDS. We denote this modified standard Lorentz gauge procedure by LG. We compare the latter with the ZML gauge described above. For both these gauges we have computed the averaged fermion correlator employing the conjugate gradient method and point-like sources. In the upper part of Fig. 1 we have plotted \( \Gamma_V(\tau; \theta) \) (normalized to unity at \( \tau = 1 \)). The situation seen is typical for a wide range of parameter values within the Coulomb phase. Obviously, there is a strong dependence of the fermion propagator on the gauge fixing procedure resulting in the presence or absence of ZMM. The masses to be extracted seem to have differ-
ent values. Let us determine the effective mass \( m_{\text{eff}}(\tau) \) in accordance with

\[
\frac{\langle \Gamma(\tau + 1; \theta) \rangle_{\theta}}{\langle \Gamma(\tau; \theta) \rangle_{\theta}} = \frac{\cosh[E(\tau)(N_t/2 - \tau - 1)]}{\cosh[E(\tau)(N_t/2 - \tau)]}
\]  
where \( E(\tau) = \ln(m_{\text{eff}}(\tau) + 1) \). See the lower part of Fig. 1. In the LG case no plateau is visible, whereas the ZML case provides a very stable one. Thus, only the ZML gauge yields a reliable mass estimate, whereas the standard method to fix the Lorentz gauge obviously fails. To get deeper insight into the effect of ZMM for the LG case (with DDS suppressed) we measure the probability distributions \( P(\phi) \) for the space- and time-like components of ZMM according to Eq. (6). The distributions turn out to be flat up to an effective cutoff at \( |\phi_\mu| \simeq \pi/N_\mu \) and to be independent of \( \beta \). In accordance with Eq. (4) we compute the fermion propagator for constant modes in the LG case and average

\[
\langle \Gamma_V(\tau; \phi) \rangle_\phi = \int [d\phi] P(\phi) \Gamma_V(\tau; \phi).
\]  

The results for several parameter sets are presented in Fig. 2 together with the corresponding free (i.e. zero-background) propagator.

The model describes qualitatively the influence of ZMM on the (full) fermion propagator in the LG case very well. It shows that the ZMM effect should not be expected to become weaker with increasing \( \beta \) and/or lattice size.

REFERENCES


Figure 1. The fermion vector propagator (a) and the effective mass (b) at \( \beta = 2 \) and \( \kappa = 0.122 \) on a 12\( \times \)6\(^3\) lattice for LG and ZML gauges as explained in the text.
Figure 2. Free fermion propagator (dashed line) and averaged constant-mode propagator in the LG case (full line) for $\beta = 2, 10$, $\kappa = 0.120, 0.124$, lattice size $12 \times 6^3, 16 \times 8^3$.