Abstract

We construct non-standard interactions between exterior form gauge fields by gauging a particular global symmetry of the Einstein-Maxwell action for such fields. Further gauge theories of the same type with couplings to scalar fields are then obtained by dimensional reduction. The construction uses an appropriate tensor calculus.

Introduction

Exterior form gauge fields $A_p = (1/p!) dx^\mu_1 \cdots dx^\mu_p A_{\mu_1 \cdots \mu_p}$ generalize naturally the electromagnetic gauge potential and are therefore interesting on general grounds in the context of gauge theories. In particular they play a substantial role in supergravity models and string theory. It is therefore natural to study consistent interactions of such gauge fields, both the interactions between themselves and their couplings to other fields.

In this paper we construct non-standard interactions between exterior form gauge fields, both without and with couplings to scalar fields. This is done in curved spacetime, i.e., the (standard) coupling to the gravitational field is included as well. Our starting point is the Einstein-Maxwell action for a $p$-form gauge field $A_p$ and an $(n-p-1)$-form gauge field $A_{n-p-1}$ in $n$-dimensional spacetime (for arbitrary $p$ and $n$). This action has a global symmetry which shifts $A_p$ by the hodge dual of the field strength of $A_{n-p-1}$, and $A_{n-p-1}$ by the hodge dual of the field strength of $A_p$ (with an appropriate sign factor, see below). We gauge this global symmetry, using an additional 1-form gauge field $V = dx^\mu V_\mu$. This yields inevitably interactions which are non-polynomial in the $V_\mu$. It would therefore be cumbersome to construct these interactions in a pedestrian way via the standard Noether procedure. Instead, we employ an appropriate tensor calculus which is analogous to the one introduced in [1] in flat four-dimensional spacetime. By formulating this tensor calculus in the differential form language we simplify the construction considerably as compared to the formulation in terms of components used in [1]. From the resulting models we then derive further ones by standard Kaluza-Klein
type dimensional reduction. This yields models with non-standard interactions between $p$-form gauge fields and scalar fields.

Our work is linked to recent progress in four-dimensional supersymmetric gauge theories, and theories with exterior form gauge fields in general. To our knowledge, gauge theories of the type constructed here appeared for the first time in [2]. There four-dimensional $N = 2$ supersymmetric gauge theories were constructed in which the central charge of the vector-tensor multiplet is gauged. This central charge is a global symmetry of the type described above and therefore the models found in [2] contain interactions of the same non-standard type as those we shall obtain. Further four-dimensional $N = 2$ supersymmetric gauge theories of the same type were constructed in [3]. Independently of these developments related to the $N = 2$ vector-tensor multiplet, models of the type considered here, and generalizations thereof, were discovered in [4] (along with further new interactions of exterior form gauge fields) within a systematic classification of possible consistent interaction vertices of exterior form gauge fields. Four-dimensional $N = 1$ supersymmetric versions of some of the models found in [4] were constructed in [5]. Our work thus adds to the list of new gauge theories in these references.

The Model

The properly normalized Einstein-Maxwell action for $A_p$ and $A_{n-p-1}$ is\footnote{Operators such as the exterior derivative and the Hodge star are understood to act only on the object to their immediate right!}

$$S_0 = \frac{1}{2} \int \left[ (-)^{(p+1)(n-1)} dA_p \star dA_p + (-)^p (-)^{(n-1)} dA_{n-p-1} \star dA_{n-p-1} \right].$$

Here we use the following conventions for a $p$-form $\omega_p$ and its Hodge dual:

$$\omega_p = \frac{1}{p!} dx^{\mu_1} \ldots dx^{\mu_p} \omega_{\mu_1 \ldots \mu_p}$$

$$\star \omega_p = \frac{1}{p!(n-p)!} dx^{\mu_1} \ldots dx^{\mu_{n-p}} \varepsilon_{\mu_1 \ldots \mu_n} \omega^{\mu_{n-p+1} \ldots \mu_n},$$

where indices are raised with the inverse metric $G^{\mu\nu}$, and the curved Levi-Civita tensor is defined by ($x^1$ labels the time coordinate)

$$\varepsilon_{12 \ldots n} = -\sqrt{G}, \quad G = -\det G_{\mu\nu}.$$

One should of course add the Einstein action to $S_0$, which however we shall not write explicitly.

$S_0$ has among others a global symmetry generated by

$$\Delta A_p = \star dA_{n-p-1}, \quad \Delta A_{n-p-1} = -(-)^{p(p+1)} \star dA_p.$$

We will now try to gauge this symmetry. To do this, we search for a modified generator $\Delta'$, such that $\Delta' A_p$ and $\Delta' A_{n-p-1}$ transform covariantly under gauge transformations

$$\delta \epsilon A_p = g \epsilon \Delta' A_p, \quad \delta \epsilon A_{n-p-1} = g \epsilon \Delta' A_{n-p-1},$$

where indices are raised with the inverse metric $G^{\mu\nu}$, and the curved Levi-Civita tensor is defined by ($x^1$ labels the time coordinate)
where $\epsilon$ is an arbitrary scalar field and $g$ a coupling constant of mass dimension $-1$. A reasonable ansatz is to replace the exterior derivative in (2) with a covariant one. Let us therefore introduce a connection 1-form $V$ with the standard transformation law

$$\delta \epsilon V = d\epsilon \tag{4}$$

and a covariant derivative

$$D = d - gV\Delta', \tag{5}$$

where $\Delta'$ is the covariant version of (2),

$$\Delta' A_p = \star DA_{n-p-1}, \quad \Delta' A_{n-p-1} = -(-)^{n(p+1)} \star DA_p. \tag{6}$$

These equations give the action of $\Delta'$ only implicitly because $D$ involves $\Delta'$. We now determine the covariant derivatives $DA_p$ and $DA_{n-p-1}$. Starting with the former, we have

$$DA_p = dA_p - gV \star DA_{n-p-1} = dA_p - gV \star [dA_{n-p-1} + (-)^{n(p+1)} gV \star DA_p].$$

Using the identity

$$\star(V \star \omega_p) = (-)^{np} i_V \omega_p, \quad i_V = V^\mu \frac{\partial}{\partial (dx^\mu)}, \tag{7}$$

which holds for any $p$-form $\omega_p$ (where $V^\mu = G^\mu\nu V_\nu$), this gives

$$DA_p = dA_p - gV \star dA_{n-p-1} - g^2 i_V DA_p.$$ 

To solve for $DA_p$, we have to invert the operator $1 + g^2 i_V$. With $(Vi_V)^2 = V^\mu V_\mu V_i V$, it is easily verified that

$$(1 + g^2 i_V)^{-1} = 1 - g^2 E^{-1} V i_V, \quad E = 1 + g^2 V^\mu V_\mu, \tag{8}$$

and we obtain

$$DA_p = dA_p - g E^{-1} V \left[ \star dA_{n-p-1} + g i_V dA_p \right]. \tag{9}$$

Analogously, one finds

$$DA_{n-p-1} = dA_{n-p-1} - g E^{-1} V \left[ -(-)^{n(p+1)} \star dA_p + g i_V dA_{n-p-1} \right]. \tag{10}$$

Note that due to the appearance of $E^{-1}$ the covariant derivatives, and thus the gauge transformations, of $A_p$ and $A_{n-p-1}$ are non-polynomial in the connection $V_\mu$ and the coupling constant $g$.

As can be checked, $DA_p$ and $DA_{n-p-1}$ are indeed covariant, i.e. their gauge transformations do not involve derivatives of $\epsilon$, and one has

$$\delta \epsilon DA_p = g \epsilon D\Delta'A_p = g \epsilon D \star DA_{n-p-1}$$
\[ \delta_e D_{n-p-1} = g \epsilon D A_{n-p-1} = -(-)^{n(p+1)} g \epsilon D * DA_p . \]  

(11)

Now we can proceed to construct the gauge invariant action. To do this, we use the following fact: let \( X \) be a covariant volume form which transforms according to \( \delta_e X = g \epsilon DK \), with \( K \) a covariant \((n-1)\)-form. Then \( X + g V K \) transforms into a total derivative,

\[ \delta_e X = g \epsilon DK , \quad \delta_e K = g \epsilon D' K \quad \Rightarrow \quad \delta_e (X + g V K) = d(g \epsilon K) . \]  

(12)

In particular, thanks to (11) this applies to

\[ X = (-)^{(p+1)(n-1)} DA_p * DA_p + (-)^{(n-1)} DA_{n-p-1} * DA_{n-p-1} \]  

(13)

with

\[ K = -2(-)^{n-p} * DA_p * DA_{n-p-1} . \]  

(14)

Adding a kinetic term for the connection \( V \), the action thus reads

\[ S = \frac{1}{2} \int (X + g V K + dV * dV) , \]  

(15)

with \( X \) and \( K \) as in (13) and (14). By construction, \( \delta_e S \) is a surface term, with the transformations as in (3), (4) and \( \delta_e G_{\mu\nu} = 0 \). Furthermore, the action is invariant under the standard abelian gauge transformations of \( A_p \) and \( A_{n-p-1} \),

\[ \delta A_p = d\Lambda_{p-1} , \quad \delta A_{n-p-1} = d\Lambda_{n-p-2} , \]  

(16)

since these fields enter the covariant derivatives (9), (10) only via their exterior derivatives.

Alternatively, one may consider a first order formulation. Introducing auxiliary fields \( \beta_p \) and \( \beta_{n-p-1} \), with the form degree as indicated, the action is given by

\[ S = \frac{1}{2} \int \left[ dV * dV + (-)^{(p-1)(n-1)} \beta_p * \beta_p + (-)^{(p+1)(n-1)} \beta_{n-p-1} * \beta_{n-p-1} \\
+ 2 \beta_p dA_{n-p-1} - 2(-)^{(p+1)} \beta_{n-p-1} dA_p - 2(-)^p g V \beta_p \beta_{n-p-1} \right] , \]  

(17)

while the gauge transformations read

\[ \delta_e V = d\epsilon , \quad \delta_e A_p = g \epsilon \beta_p , \quad \delta_e A_{n-p-1} = g \epsilon \beta_{n-p-1} \]
\[ \delta_e \beta_p = \delta_e \beta_{n-p-1} = \delta_e G_{\mu\nu} = 0 . \]  

(18)

The equations of motion for the auxiliary fields are coupled in exactly the same way as the equations that determine the covariant derivatives of \( A_p \) and \( A_{n-p-1} \) (the relation \( \approx \) denotes on-shell equality),

\[ -(-)^{(p-1)(n-1)} \beta_p \approx dA_{n-p-1} - g V \beta_{n-p-1} \]
\[ -(-)^p \beta_{n-p-1} \approx dA_p - g V \beta_p . \]  

(19)
and the solutions are thus
\[ \beta_p \approx \Delta' A_p , \quad \beta_{n-p-1} \approx \Delta' A_{n-p-1} . \] (20)

The action (17) may also be used to derive a dual version of our model. To that end one solves the equations of motion for \( A_p \) and \( A_{n-p-1} \) through \( \beta_p \approx dA_{p-1} \) and \( \beta_{n-p-1} \approx dA_{n-p-2} \) and inserts the solution back into the action. The interaction vertex \( V \beta_p \beta_{n-p-1} \) then turns into a Chern-Simons term.

Remark. If \( n = 1 + 4k \) and \( p = 2k \), one may identify \( A_p \) with \( A_{n-p-1} \). For instance, in the case \( n = 5 \), \( p = 2 \) one then gets a 5-dimensional theory involving only one 2-form gauge field besides \( V \) and the gravitational field.

### Dimensional Reduction

The model introduced above admits some generalization. In what follows, we show how a dimensional reduction from \( n \) to \( n-1 \) dimensions gives rise to couplings to additional gauge fields and scalars.

We denote the coordinates by
\[ x^\mu = (x^a, x^n) , \quad a = 1, \ldots, n-1 , \] (21)
and take all fields to be constant along the \( x^n \)-direction. Then the metric decomposes in the usual manner,
\[ G_{\mu\nu} \, dx^\mu \otimes dx^\nu = g_{\alpha\beta} \, dx^\alpha \otimes dx^\beta + e^{2\varphi}(W + dx^n) \otimes (W + dx^n) , \] (22)
where \( W = dx^n W_a \) is a 1-form in \( n-1 \) dimensions.

Upon dimensional reduction, \( A_p \) gives rise to a \( p \)-form \( \hat{A}_p \) and a \( (p-1) \)-form \( \hat{A}_{p-1} \), while \( A_{n-p-1} \) decomposes into an \( (n-p-1) \)-form \( \hat{A}_{n-p-1} \) and an \( (n-p-2) \)-form \( \hat{A}_{n-p-2} \). The connection \( V \) introduces in addition to a 1-form \( \hat{V} \) a scalar field \( \phi \),
\[ A_p = \hat{A}_p + \hat{A}_{p-1} \, dx^n , \quad A_{n-p-1} = \hat{A}_{n-p-1} + \hat{A}_{n-p-2} \, dx^n , \quad V = \hat{V} + \phi \, dx^n . \quad (23)\]

In the following, one should keep in mind that the descendants \( \hat{A}_p \), \( \hat{A}_{n-p-1} \) and \( \hat{V} \) all transform nontrivially under the abelian gauge transformation associated with \( W \), while \( \hat{A}_{p-1} \), \( \hat{A}_{n-p-2} \) and \( \phi \) are invariant,
\[ \delta_\lambda W = d\lambda , \quad \delta_\lambda \hat{A}_p = \hat{A}_{p-1} \, d\lambda , \quad \delta_\lambda \hat{A}_{n-p-1} = \hat{A}_{n-p-2} \, d\lambda , \quad \delta_\lambda \hat{V} = \phi \, d\lambda \]
\[ \delta_\lambda \hat{A}_{p-1} = \delta_\lambda \hat{A}_{n-p-2} = \delta_\lambda \phi = 0 . \quad (24)\]

\( \delta_\lambda \) originates from a general coordinate transformation in the \( n \)th direction. When decomposing the dual of a \( p \)-form \( \omega_p \), we make use of the relation
\[ * \omega_p = e^{-\varphi} \hat{*} \omega_{p-1} + (-)^p e^{\varphi} \hat{*} (\hat{\omega}_p - \hat{\omega}_{p-1} W) (W + dx^n) , \]
where the Hodge star \( \hat{*} \) in \( n-1 \) dimensions involves the reduced Levi-Civita tensor \( \hat{\epsilon}_{a_1 \ldots a_{n-1}} = e^{-\varphi} \varepsilon_{a_1 \ldots a_{n-1} n} \) and indices are raised with \( g^{ab} \).
There are now two ways of determining the covariant derivatives and gauge transformations of the $\hat{A}$-fields, which turn out to give the same results. One may either start from the first order formulation and solve the dimensionally reduced equations of motion for the auxiliary fields, or one may directly decompose the equations (9) and (10). Let us follow the latter approach: we define a covariant derivative $\hat{D}$ by the relations

$$DA_p = \hat{D}A_p + \hat{D}A_{p-1} dx^n, \quad DA_{n-p-1} = \hat{D}A_{n-p-1} + \hat{D}A_{n-p-2} dx^n. \quad (25)$$

The decomposition of the left-hand sides is straightforward, and by comparison of the terms with and without $dx^n$ one derives the action of $\hat{D}$ on the four gauge fields. Similarly to the gauge fields themselves, the descendants $\hat{D}A_p$ and $\hat{D}A_{n-p-1}$ transform noncovariantly under $\delta_\lambda$. It is convenient to use $\delta_\lambda$-invariant generalized field strengths instead, analogous to those appearing in Kaluza-Klein supergravity models [6],

$$\mathcal{F}_{p+1} = \hat{D}A_p - \hat{D}A_{p-1} W, \quad \mathcal{F}_{n-p} = \hat{D}A_{n-p-1} - \hat{D}A_{n-p-2} W. \quad (26)$$

Then also the connection $\hat{V}$ appears always in a $\delta_\lambda$-invariant combination, which we denote by

$$U = \hat{V} - \phi W. \quad (27)$$

Since the original $n$-dimensional fields do not appear anymore, we shall omit all hats in the following. Explicitly, one finds

$$\mathcal{F}_{p+1} = dA_p - dA_{p-1} W - gE^{-1}U [e^{-\varphi} \star dA_{n-p-2} + gi_U (dA_p - dA_{p-1} W)$$
$$+ (-)^p g e^{-2\varphi} \phi dA_{p-1}]
$$

$$+ g i_U (dA_{n-p-1} - dA_{n-p-2} W) - (-)^{n-p} g e^{-2\varphi} \phi dA_{n-p-2}],$$

and

$$DA_{p-1} = dA_{p-1} - gE^{-1}U [(-)^{n-p} e^{\varphi} \star (dA_{n-p-1} - dA_{n-p-2} W) + gi_U dA_{p-1}$$
$$- gE^{-1}\phi [(-)^p e^{-\varphi} \star dA_{n-p-2} + (-)^p g i_U (dA_p - dA_{p-1} W)$$
$$+ g e^{-2\varphi} \phi dA_{p-1}]
$$

$$DA_{n-p-2} = dA_{n-p-2} - gE^{-1}U [(-)^{(p+1)(n-1)} e^{\varphi} \star (dA_p - dA_{p-1} W) + gi_U dA_{n-p-2}$$
$$- gE^{-1}\phi [(-)^{p(n-1)} e^{-\varphi} \star dA_{p-1} - (-)^{n-p} g i_U (dA_{n-p-1} - dA_{n-p-2} W)$$
$$+ g e^{-2\varphi} \phi dA_{n-p-2}].$$

where now

$$E = 1 + g^2 U^a U_a + g^2 e^{-2\varphi} \phi^2. \quad (30)$$

The above expressions enter the gauge transformations $\delta_\epsilon$ of the forms $A_p$ etc., which are found to read

$$\delta_\epsilon A_p = g \epsilon [e^{-\varphi} \star DA_{n-p-2} + (-)^{n-p} e^{\varphi} \star \mathcal{F}_{n-p} W]
\[ \delta_t A_{p-1} = (-)^{n-p} g e^{\varphi} \ast F_{n-p} \]
\[ \delta_t A_{n-p-1} = -(-)^{(n+1)} g e^{e^{-\varphi} \ast DA_{p-1} - (-)^p e^\varphi \ast F_{p+1} W} \]
\[ \delta_t A_{n-p-2} = -(-)^{(p+1)(n-1)} g e^{e^\varphi \ast F_{p+1}} \]  
(31)

while those of the remaining fields are simply
\[ \delta_t U = de \]  \[ \delta_t \phi = \delta_t W = \delta_t \varphi = \delta_t g_{ab} = 0 \]  
(32)

We observe that \( DA_{p-1} \) and \( DA_{n-p-2} \) contain terms which are accompanied by the scalar \( \phi \) rather than the connection \( U \). One may introduce “minimal” covariant derivatives \( D \) of \( A_{p-1} \) and \( A_{n-p-2} \) by subtracting appropriate covariant terms from \( DA_{p-1} \) and \( DA_{n-p-2} \):
\[ DA_{p-1} = DA_{p-1} + (-)^pg e^{-\varphi} \phi \ast DA_{n-p-2} \]
\[ DA_{n-p-2} = DA_{n-p-2} + (-)^{p(n-1)} g e^{-\varphi} \phi \ast DA_{p-1} \]  
(33)

This results in the following simpler expressions:
\[ DA_{p-1} = dA_{p-1} - gE^{-1}U [(-)^{n-p} E e^{\varphi} \ast (dA_{n-p-1} - dA_{n-p-2} W) + g i_U dA_{p-1} - (-)^p g^2 e^{-\varphi} \phi \ast i_U dA_{n-p-2}] \]
\[ DA_{n-p-2} = dA_{n-p-2} - gE^{-1}U [(-)^{(p+1)(n-1)} E e^{\varphi} \ast (dA_{p} - dA_{p-1} W)] + g i_U dA_{n-p-2} - (-)^{p(n-1)} g^2 \phi \ast i_U dA_{p-1} \]  
(34)

where \( E \) is a function of the scalar fields only,
\[ E = 1 + g^2 e^{-2\varphi} \phi^2 \]  
(35)

Eqs. (33) can be inverted to express \( DA_{p-1} \) and \( DA_{n-p-2} \) in terms of the minimal covariant derivatives,
\[ DA_{p-1} = E^{-1} [DA_{p-1} - (-)^p g e^{-\varphi} \phi \ast DA_{n-p-2}] \]
\[ DA_{n-p-2} = E^{-1} [DA_{n-p-2} - (-)^{p(n-1)} g e^{-\varphi} \phi \ast DA_{p-1}] \]  
(36)

Finally, we obtain the gauge invariant action in \((n-1)\)-dimensional spacetime by reduction of eq. (15). Using the eqs. (26), (27) and (36), it can be written entirely in terms of \( \delta_\lambda \)-invariant as well as \( \delta_t \)-covariant expressions,
\[ S = \frac{1}{2} \int \left\{ e^{\varphi} (dU + \phi dW) \ast (dU + \phi dW) + (-)^n e^{-\varphi} d\phi \ast d\phi \\
+ (-)^{n(p+1)} e^{\varphi} (F_{p+1} \ast F_{p+1} + F_{n-p} \ast F_{n-p}) \\
+ (-)^{np} E^{-1} e^{-\varphi} (DA_{p-1} \ast DA_{p-1} + DA_{n-p-2} \ast DA_{n-p-2}) \\
- 2g E^{-1} U (\ast DA_{p-1} \ast F_{n-p} - (-)^{(p+1)} \ast DA_{n-p-2} \ast F_{p+1}) \\
- 2(-)^{p(n-1)} g^2 E^{-1} e^{-\varphi} \phi U (DA_{n-p-2} \ast F_{n-p} - (-)^n DA_{p-1} \ast F_{p+1}) \right\} \]
As compared to (15), the new feature of the action (37) is the coupling of $\phi$, $\varphi$ and $W$. Indeed, if we set these fields to zero, (37) reduces to an action of the form (15) in $n' = n - 1$ dimensions, for two pairs of exterior form gauge fields with form degrees $(p, n' - p - 1)$ and $(p - 1, n' - p)$: one gets (15) with $X = X_1 + X_2$ and $K = K_1 + K_2$, where the subscripts 1 and 2 refer to the first and second pair respectively $(X_i$ and $K_i$ match their counterparts in (13) and (14)) up to sign factors which can be removed by trivial field redefinitions $A \rightarrow \pm A)$. Clearly, one may iterate the dimensional reduction procedure to derive more involved models of the above type. Again, after setting to zero scalar and vector fields coming from $G_{\mu\nu}$ and $V$, the resulting actions reduce to the form (15) for more pairs of exterior form gauge fields.

Comments

To understand the structure of the models, it is useful to keep in mind that they are consistent deformations of the standard Einstein-Maxwell action for exterior form gauge fields, plus kinetic terms for the scalar fields in the models obtained by dimensional reduction. The non-polynomial structure of the interactions can be traced back to the properties of the global symmetry (2) that is gauged. Namely, the first order deformation $S_1$ of the $n$-dimensional Einstein-Maxwell action is given by $S_1 = (-)^{n-p} \int V \ast dA_p \ast dA_{n-p-1}$. This is nothing but the Noether coupling $VJ$, where the $(n-1)$-form $J$ is the Noether current of the global symmetry (2). The corresponding first order deformation of the gauge symmetry involves the global symmetry (2): it is just $\delta_1 = \epsilon \Delta$ (i.e., $\delta_1 A_p = \epsilon \ast dA_{n-p-1}$, $\delta_1 A_{n-p-1} = -(-)^{n(p+1)} \epsilon \ast dA_p$). It can be readily checked that $\delta_1 S_1$ does not vanish, due to the fact that $J$ is not invariant under $\Delta$. As a consequence, consistency of the interactions at second order makes it necessary to introduce second order deformations both of the action and of the gauge symmetry (this can be rigorously proved by cohomological arguments along the lines of [7]). This extends to all higher orders and leads to the non-polynomial structure of the interactions and gauge transformations.

In fact the situation is somewhat similar to the coupling of the gravitational field to matter in standard gravity. Indeed, viewed as a deformation using $G_{\mu\nu} = \eta_{\mu\nu} + m_{pl}^{-1} h_{\mu\nu}$, the first order deformation couples $h_{\mu\nu}$ to the energy momentum tensor $T^{\mu\nu}$ in flat spacetime. $T^{\mu\nu}$ is the Noether current of translations and not invariant under the generators of translations given by the spacetime derivatives. This leads to interactions that are non-polynomial in $h_{\mu\nu}$ and in the coupling constant $m_{pl}^{-1}$ which are compactly constructed by means of the familiar tensor calculus.

The non-standard coupling of the scalar fields present in the dimensionally reduced models can be understood analogously from the point of view of consistent deformations. If we specialize these models by setting $W$ and $\varphi$ to zero, the first order coupling of the scalar field $\phi$ is just $\int \phi dA_{p-1} dA_{n-p-2}$ (up to a factor), i.e., it couples $\phi$ to the topological density $dA_{p-1} dA_{n-p-2}$. Again, the latter is not invariant under the first order deformation $\delta_1$ of the gauge symmetry. This enforces further interaction terms of higher order, with higher powers of $\phi$, and eventually non-polynomial interactions.
of $\phi$. Analogous statements apply in presence of $W$ and $\varphi$, where further first order interactions are present. One may disentangle the various couplings in these interactions by introducing further coupling constants $g_1$ and $g_2$ through the rescalings $W \rightarrow g_1 W$, $\varphi \rightarrow g_2 \varphi$.

Hence, though one might have suspected that the non-polynomial interactions of the scalar fields (and of $W$) in the above models are just an artefact of the dimensional reduction, such interactions are actually to be expected in gauge theories of the type studied here, whether or not the models can be obtained by dimensional reduction. In fact, the reader may check that non-polynomial interactions of scalar fields similar to those found here are present in all supersymmetric models constructed in [2, 3, 5].

Acknowledgements

FB and UT wish to thank Joaquim Gomis for his kind hospitality at the University of Barcelona. This work was supported by the Acciones Integradas program of the Deutscher Akademischer Austauschdienst and the Ministerio de Educación y Cultura. FB was supported by the Deutsche Forschungsgemeinschaft. JS is supported by a fellowship from Comisionat per a Universitats i Recerca de la Generalitat de Catalunya.

References


