Local Symmetries in the AdS$_7$/CFT$_6$
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Abstract

It is shown that local symmetry transformations of the maximal AdS supergravity in seven-dimensional anti de Sitter space induce those of the $N = (2,0)$ conformal supergravity on the six-dimensional boundary at infinity. Boundary values of the AdS supergravity fields form a supermultiplet of the conformal supergravity.

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1. Introduction

In the AdS/CFT correspondence [1, 2, 3] (For a review see, e.g., ref. [4].) a supergravity in \((d + 1)\)-dimensional anti de Sitter space is dual to a conformal field theory on \(d\)-dimensional boundary at infinity. Boundary values of supergravity fields play a role of sources for operators of the conformal field theory [2, 3]. It was noted in refs. [5, 6] that these boundary fields should form supermultiplets of a conformal supergravity in \(d\) dimensions. Conformal supergravities are theories which have Weyl and super Weyl symmetries in addition to local symmetries of the Poincaré supergravities.

In refs. [7, 8] we explicitly showed such a relation between AdS supergravities and conformal supergravities in the case of three-dimensional anti de Sitter space. It was shown that local symmetry transformations of the bulk AdS supergravities induce local transformations of the boundary fields, which coincide with those of two-dimensional conformal supergravities. In particular, Weyl and super Weyl transformations on the boundary are generated from general coordinate and super transformations in the bulk. Thus, the boundary fields form a supermultiplet of the conformal supergravities. A relation between local symmetries in the bulk and those on the boundary is also discussed in ref. [9].

The purpose of this paper is to study a similar relation between an AdS supergravity and a conformal supergravity in the case of seven-dimensional anti de Sitter space. It is shown that local transformations of the maximal AdS supergravity in seven dimensions [10] induce those of the \(N = (2, 0)\) conformal supergravity in six dimensions [11] on the boundary. The seven-dimensional maximal supergravity appears in a compactification of the M-theory on \(\text{AdS}_7 \times S^4\) [12], which corresponds to a configuration of \(N\) M5-branes in the large \(N\) limit [1]. A new feature in this case is the presence of third rank antisymmetric tensor fields in the seven-dimensional supergravity. They satisfy a so-called “self-duality in odd dimensions” [13] by field equations. We show that they become self-dual antisymmetric tensor fields of the six-dimensional conformal supergravity on the boundary.

As in refs. [7, 8] we partially fix the gauge for local symmetries in the bulk. Our gauge choice is a sort of axial gauge, in which the direction normal to the boundary is distinguished. We first obtain boundary behaviors of all the fields and the gauge transformation parameters after the gauge fixing by field equations and residual symmetry equations. Substituting them into the local symmetry transformations
we obtain the conformal supergravity transformations on the boundary.

2. Maximal AdS supergravity in seven dimensions

The field content of the maximal AdS supergravity in seven dimensions [10] is a vielbein $e^M_A$, Rarita-Schwinger fields $\psi^\alpha_M$, real third rank antisymmetric tensor fields $S_{MNP,I}$, SO(5)$_g$ vector fields $B_{MI}$, spin $\frac{1}{2}$ spinor fields $\lambda^i_I$ and scalar fields $\Pi^I_i$. We denote seven-dimensional world indices as $M, N, \cdots = 0, 1, \cdots, 6$ and local Lorentz indices as $A, B, \cdots = 0, 1, \cdots, 6$. The indices $I, J, \cdots = 1, \cdots, 5$ are vector indices of SO(5)$_g$, while $i, j, \cdots = 1, \cdots, 5$ and $\alpha, \beta, \cdots = 1, \cdots, 4$ are vector and spinor indices of SO(5)$_c$ respectively. SO(5)$_g$ and SO(5)$_c$ are local symmetries of the theory, which will be discussed later. The flat metric is $\eta_{AB} = \text{diag}(-1, +1, \cdots, +1)$ and the totally antisymmetric tensor $\epsilon_{M1\cdots M7}$ is chosen as $\epsilon_{0123456} = +1$. We need two kinds of gamma matrices: $8 \times 8$ matrices $\gamma^A$ for the seven-dimensional Lorentz group SO(1,6) and $4 \times 4$ matrices $\tau^i$ for SO(5)$_c$ satisfying $\{\gamma^A, \gamma^B\} = 2\eta^{AB}$, $\{\tau^i, \tau^j\} = 2\delta^{ij}$. $\gamma$'s and $\tau$'s with multiple indices are antisymmetrized products of gamma matrices with unit strength. In particular, we have $\gamma^{A_1\cdots A_7} = -e^{A_1\cdots A_7}$. The Dirac conjugate of a spinor $\psi$ is defined as $\bar{\psi} = \psi^i \gamma^i$. The spinor fields satisfy a symplectic Majorana condition $\psi^\alpha = \Omega^\alpha \beta \bar{\psi}^\beta_T$, where $C = C^T$ and $\Omega = -\Omega^T$ are charge conjugation matrices of SO(1,6) and SO(5)$_c$ satisfying

$$C^{-1}\gamma^A C = -(\gamma^A)^T, \quad \Omega^{-1}\tau^i \Omega = (\tau^i)^T. \quad (1)$$

The spinor fields $\lambda_i^\alpha$ also satisfy the SO(5)$_c$ irreducibility condition $(\tau^i)^\alpha \beta \lambda_i^\beta = 0$. The scalar fields $\Pi_i^I$ satisfy det $\Pi_i^I = 1$, i.e., $\Pi_i^I \in \text{SL}(5, \mathbb{R})$. By the local SO(5)$_c$ symmetry physical degrees of freedom of the scalar fields parametrize a coset space $\text{SL}(5, \mathbb{R})/\text{SO}(5)_c$.

The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2} e R + 4m^2 e \left(T^2 - 2T_{ij}T^{ij}\right) - \frac{1}{2} e P_{MI}P^{MI} - \frac{1}{4} e \left(F_{MN}^\text{ij} \Pi^i_I \Pi^j_J\right)^2$$

$$+ 8m^2 e \left(\Pi^{-1}_i S_{MNP,I}\right)^2 + \frac{1}{12} m e^{MNPRST} S_{MNP,I} F_{QRST}^I$$

$$- \frac{1}{2} e \bar{\psi}_M \gamma^{MNP} D_N \psi_P - \frac{1}{2} e \bar{\lambda}_I \gamma^M D_M \lambda_i + \frac{1}{2} m e T \bar{\psi}_M \gamma^M \psi_N$$

$$- \frac{1}{2} e \bar{\psi}_M \gamma^{MNP} D_N \psi_P - \frac{1}{2} e \bar{\lambda}_I \gamma^M D_M \lambda_i + \frac{1}{2} m e T \bar{\psi}_M \gamma^M \psi_N$$

$$- \frac{1}{2} e \bar{\psi}_M \gamma^{MNP} D_N \psi_P - \frac{1}{2} e \bar{\lambda}_I \gamma^M D_M \lambda_i + \frac{1}{2} m e T \bar{\psi}_M \gamma^M \psi_N$$

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$$- \frac{1}{2} e \bar{\psi}_M \gamma^{MNP} D_N \psi_P - \frac{1}{2} e \bar{\lambda}_I \gamma^M D_M \lambda_i + \frac{1}{2} m e T \bar{\psi}_M \gamma^M \psi_N$$

$$- \frac{1}{2} e \bar{\psi}_M \gamma^{MNP} D_N \psi_P - \frac{1}{2} e \bar{\lambda}_I \gamma^M D_M \lambda_i + \frac{1}{2} m e T \bar{\psi}_M \gamma^M \psi_N$$

$$- \frac{1}{2} e \bar{\psi}_M \gamma^{MNP} D_N \psi_P - \frac{1}{2} e \bar{\lambda}_I \gamma^M D_M \lambda_i + \frac{1}{2} m e T \bar{\psi}_M \gamma^M \psi_N$$
\[-\frac{1}{2}m e \left( 8T^{ij} - T \delta^{ij} \right) \lambda_i \lambda_j + 2m e T^{ij} \tilde{\lambda}_i \tau_j \gamma^M \psi_M + \frac{1}{2} e \bar{\psi}_M \gamma^N \gamma^M \tau^i \lambda^i P_{Ni j} \]
\[+ \frac{1}{16} \bar{\psi}_M \left( \gamma^{MNPQ} - 2g^{MN}g^{PQ} \right) \tau_{ij} \psi_Q F_{NP}^{IJ} \Pi^I \Pi^J \]
\[+ \frac{1}{4} e \bar{\psi}_M \gamma^{NP} \lambda_j F_{NP}^{IJ} \Pi^I \Pi^J \]
\[+ \frac{1}{32} e \bar{\lambda}_i \tau^j \tau_{kl} \gamma^{MN} \lambda_j F_{MN}^{IJ} \Pi^I \Pi^J \]
\[+ \frac{1}{2\sqrt{3}} i m e \bar{\psi}_M \left( \gamma^{MNPQR} + 6g^{MN}g^{PQR} \right) \tau^i \psi_R \Pi^{-1} \Pi^I S_{NPQ.I} \]
\[- \frac{1}{\sqrt{3}} i m e \bar{\lambda}_i \gamma^{MN} \tau^j \lambda_i \Pi^{-1} \Pi^I S_{MNPI} \]
\[+ \frac{1}{16\sqrt{3}} i m e \bar{\lambda}_i \gamma^{MN} \tau^j \lambda_i \Pi^{-1} \Pi^I S_{MNPI} \bar{\lambda}_j \Omega^M_{,[B]} - \frac{1}{64m} \Omega_3[B] + \cdots, \tag{2} \]

where we have put the gravitational constant as \(8\pi G = 1\), \(m\) is a positive constant and the dots denote four-fermi terms. The quantities in eq. (2) are defined as follows. From the scalar fields we define

\[P_{M(ij)} + Q_{M[ij]} = (\Pi^{-1})^I \left( \partial_M \Pi_{Ij} + 8m B_{MI} \Pi_{Ij} \right), \]
\[T_{ij} = (\Pi^{-1})^I (\Pi^{-1})^J \delta_{IJ}, \quad T = T_{ij} \delta^{ij}, \tag{3} \]

where \((ij)\) and \([ij]\) are symmetric and antisymmetric parts. The field strengths of the vector fields and the antisymmetric tensor fields are

\[F_{MN,I}^J = \partial_M B_{NI}^J + 8m B_{MI}^K \gamma_{NK}^J - (M \leftrightarrow N), \]
\[F_{MNPQ,I} = 4D_{[M} S_{NPQ]I}. \tag{4} \]

The SO(5)_g gauge coupling constant is \(8m\). The covariant derivative \(D_M\) contains the SO(5)_g gauge field \(B_{MI}^J\) and the composite SO(5)_c gauge field \(Q_{M[ij]}\) as well as the spin connection \(\omega_{AB}^M\), e.g.,

\[D_M \lambda_i^\alpha = \left( \partial_M + \frac{1}{4} \omega_{AB}^M \gamma_{AB} \right) \lambda_i^\alpha + Q_{M}^{ij} \lambda_j^\alpha + \frac{1}{4} Q_M^{ik}(\tau_{jk})^\alpha_\beta \lambda_i^\beta. \tag{5} \]

Finally, \(\Omega_3[B]\) and \(\Omega_5[B]\) are Chern-Simons terms satisfying in the differential form language

\[d\Omega_5[B] = 8 \text{tr}(F^4), \quad d\Omega_3[B] = 8 \text{tr}(F^2) \text{tr}(F^2). \tag{6} \]
The Lagrangian (2) is invariant under general coordinate, local Lorentz, local \( SO(5) \), local \( SO(5) \) and local super transformations up to total derivative terms. Note that there are neither Weyl nor super Weyl symmetries in the bulk of seven-dimensional spacetime. The bosonic transformations are

\[
\delta e_M^A = \xi^N \partial_N e_M^A + \partial_M \xi^N e_N^A - \lambda^A B e_M^B,
\]

\[
\delta \psi_M = \xi^N \partial_N \psi_M + \partial_M \xi^N \psi_N - \frac{1}{4} \lambda^{AB} \gamma_{AB} \psi_M - \frac{1}{4} \psi^i \tau_{ij} \psi_M,
\]

\[
\delta S_{MNP,I} = \xi^Q \partial_Q S_{MNP,I} + 3 \partial_P \xi^Q S_{MNQ,I} - \theta_I^J S_{MNP,J},
\]

\[
\delta B_M^{IJ} = \xi^N \partial_N B_M^{IJ} + \partial_M \xi^N B_N^{IJ} + D_M \theta^{IJ},
\]

\[
\delta \lambda_i = \xi^M \partial_M \lambda_i - \frac{1}{4} \lambda^{AB} \gamma_{AB} \lambda_i - \psi^j \lambda_j - \frac{1}{4} \psi^k \tau_{jk} \lambda_i,
\]

\[
\delta \Pi_i^j = \xi^M \partial_M \Pi_i^j - \theta_I^J \Pi_i^j - \psi^j \Pi_i^j,
\]

where \( \xi^M(x) \), \( \lambda^{AB}(x) \), \( \theta^{IJ}(x) \) and \( \psi^i(x) \) are transformation parameters of general coordinate, local Lorentz, \( SO(5) \) and \( SO(5) \) transformations respectively. Note that there is no antisymmetric tensor gauge symmetry for \( S_{MNP,I} \), which satisfy the “self-duality in odd dimensions” [13] by field equation.

The local supertransformations are

\[
\delta e_M^A = \frac{1}{2} \bar{\varepsilon}^A \lambda^A \psi_M,
\]

\[
\delta \psi_M = D_M \psi + \frac{1}{5} m T \gamma_M \psi - \frac{1}{40} \left( \gamma_M^{NP} - 8 \delta_M^N \gamma^P \right) \tau_{ji} \epsilon F^{IJ} \Pi_i^j \Pi_j^i + \ldots,
\]

\[
\delta S_{MNP,I} = -\sqrt{i} \left( 3 \bar{\varepsilon} \gamma_{[MN} \tau_i \psi_P] - \bar{\varepsilon} \gamma_{MP} \lambda^i \right) \Pi^{-1}_{ji} + \ldots
\]

\[
-\frac{\sqrt{3}}{32m} \bar{i} D^j \left( 2 \bar{\varepsilon} \tau_{ijk} \psi_M + \bar{\varepsilon} \gamma_{[MN} \tau_{ijk] \lambda} \right) F^{JK} \Pi^j_i \Pi^K_k,
\]

\[
-\frac{\sqrt{3}}{16m} \bar{i} D \left( 2 \bar{\varepsilon} \gamma_{NP} \lambda^i \right) \Pi^{-1}_{ji} + \ldots
\]

\[
\delta B_M^{IJ} = \left( \frac{1}{4} \bar{\varepsilon} \psi_M^A + \frac{1}{8} \bar{\varepsilon} \gamma_{M} \tau^k \psi_M^k \right) \Pi^{-1}_{ji} \Pi^{-1}_{ji} + \ldots
\]

\[
\delta \lambda_i = 2m \left( T_{ij} - \frac{1}{5} \delta_{ij} T \right) \tau^i \epsilon + \frac{1}{16} \gamma^{MN} \left( \tau_{kl} \tau_i - \frac{1}{5} \tau_i \tau_{kl} \right) \epsilon F^{IJ} \Pi^k_i \Pi^j_j + \ldots
\]

\[
+ \frac{1}{5} \bar{i} m \gamma^{MN} \left( \tau_i^j - 4 \delta^j_i \right) \epsilon \Pi^{-1}_{ij} S_{MNP,I} + \frac{1}{2} \bar{\gamma}^M \tau^i \epsilon P_{Mij} + \ldots,
\]
\[
\delta \Pi_I^j = \frac{1}{4} \Pi_I^j \left( \bar{\epsilon} \tau_j \lambda^i + \bar{\epsilon} \tau^i \lambda_j \right),
\]
where the dots denote three-fermi terms, which we ignore in the following. The transformation parameter \( \epsilon^\alpha(x) \) satisfies the symplectic Majorana condition.

### 3. Boundary behaviors of the fields

We partially fix the gauge for the local symmetries (7) and (8). The seven-dimensional AdS space is represented as a region \( r \equiv x^6 > 0 \) in \( \mathbb{R}^7 \) with coordinates \( x^M \). The boundary of the AdS space corresponds to a hyperplane \( r = 0 \) and a point \( r = \infty \). We choose the gauge fixing condition as

\[
\begin{align*}
\epsilon_r^6 &= \frac{1}{2mr}, & \epsilon_r^a &= 0, & \epsilon_r^\mu &= 0, \\
\psi_r &= 0, & B_r^{IJ} &= 0, & \Pi^T &= \Pi,
\end{align*}
\]

where \( \mu, \nu, \cdots = 0, \cdots, 5 \) and \( a, b, \cdots = 0, \cdots, 5 \) are six-dimensional world indices and local Lorentz indices respectively. The metric in this gauge has a form

\[
dx^M dx^N g_{MN} = \frac{1}{(2mr)^2} \left( dr^2 + dx^\mu dx^\nu \hat{g}_{\mu\nu} \right).
\]

The SO(6,2) invariant AdS metric corresponds to the case \( \hat{g}_{\mu\nu} = \eta_{\mu\nu} \) but we consider the general \( \hat{g}_{\mu\nu} \). We define \( \hat{\epsilon}_\mu^a \) by \( \hat{g}_{\mu\nu} = \hat{\epsilon}_\mu^a \hat{\epsilon}_\nu^b \eta_{ab} \). An SO(6,2) invariant field configuration

\[
\hat{g}_{\mu\nu} = \eta_{\mu\nu}, \quad \Pi_I^j = \delta_I^j, \quad \text{other fields} = 0.
\]

is a solution of field equations derived from the Lagrangian (2).

Let us obtain asymptotic behaviors of the fields near the boundary \( r = 0 \). The boundary conditions are chosen such that they are consistent with these boundary behaviors. We assume that the vielbein \( e_r^\mu \) behaves as \( r^{-1} \) just as in the SO(6,2) invariant case (11). Boundary behaviors of other fields are determined by their field equations. For the scalar fields \( \Pi_I^j \) we first expand them around the background (11) as

\[
\Pi_I^j = (e^\phi)_I^j = \delta_I^j \left( \delta_j^i + \phi_j^i + \frac{1}{2} \phi_j^k \phi_k^i + \mathcal{O}(\phi^3) \right),
\]
where $\phi_{ij} = \phi_{ji}$ and $\phi_i^i = 0$. Near the boundary $r = 0$ the linearized field equation for the scalar fields in the background (11) is

$$r^7 \partial_r \left( r^{-5} \partial_r \phi \right) + 8 \phi = 0.$$  

(13)

Assuming $\phi \sim r^s$ for $r \to 0$, we obtain two independent solutions: $\phi \sim r^2$ and $\phi \sim r^4$. The solution regular in the bulk is a linear combination of these two solutions and its boundary behavior is determined by a solution which becomes larger near the boundary, i.e., $\phi \sim r^2$. The same reasoning can be applied to other fields discussed below. For the vector fields $B_M$ the linearized field equation is

$$\partial_r \left( r^{-3} \partial_r B_\mu \right) = 0.$$  

(14)

We find two solutions: $B_\mu \sim r^0$ and $B_\mu \sim r^4$.

For the antisymmetric tensor fields $S_{MNP}$ the linearized field equation is

$$\frac{1}{6} \epsilon^{MNPQ_1 \cdots Q_4} F_{Q_1 \cdots Q_4} + 16 m e S^{MNP} = 0.$$  

(15)

By the $(MNP) = (r\mu\nu)$ components of this equation the components $S_{r\mu\nu}$ can be expressed by using $S_{\mu\nu\rho}$ as

$$S_{r\mu\nu} = -\frac{r}{12} \epsilon_{\mu\rho_1 \cdots \rho_4} \eta^{\rho_1 \sigma_1} \cdots \eta^{\rho_4 \sigma_4} \partial_{[\sigma_1} S_{\sigma_2 \sigma_3 \sigma_4]}.$$  

(16)

Therefore they are not independent degrees of freedom and behave as $S_{r\mu\nu} \sim r^{s+1}$ when $S_{\mu\nu\rho} \sim r^s$. The $(MNP) = (\mu\nu\rho)$ components of the field equation (15) become

$$\partial_r S_{\mu\nu\rho} - \frac{2}{r} \tilde{S}_{\mu\nu\rho} - 3 \partial_{[\mu} S_{\nu\rho]r} = 0,$$  

(17)

where the dual of $S_{\mu\nu\rho}$ is defined as

$$\tilde{S}_{\mu\nu\rho} = \frac{1}{3!} \epsilon_{\mu\nu\rho\sigma_1 \sigma_2 \sigma_3} \hat{g}^{\sigma_1 \lambda_1} \hat{g}^{\sigma_2 \lambda_2} \hat{g}^{\sigma_3 \lambda_3} S_{\lambda_1 \lambda_2 \lambda_3}.$$  

(18)

To solve this equation we introduce self-dual and anti self-dual parts of $S_{\mu\nu\rho}$ as $S^{(\pm)} = \frac{1}{2} \left( S_{\mu\nu\rho} \pm \tilde{S}_{\mu\nu\rho} \right)$. Then we find two solutions: $S^{(+)}_{\mu\nu\rho} \sim r^0$, $S^{(-)}_{\mu\nu\rho} \sim r^{-2}$ and $S^{(+)}_{\mu\nu\rho} \sim r^2$, $S^{(-)}_{\mu\nu\rho} \sim r^4$. 

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For spinor fields $\lambda$ the linearized field equation is
\[
\partial_r \lambda - \frac{3}{2r} (2 - \gamma_6) \lambda + \gamma_6 \gamma^\mu \partial_\mu \lambda = 0. \tag{19}
\]
We define the projections $\lambda_\pm = \frac{1}{2} (1 \pm \gamma_6) \lambda$. Note that $\gamma_6 = -\gamma^0 \gamma^1 \cdots \gamma^5$ is the six-dimensional chirality matrix and $\lambda_+, \lambda_-$ are Weyl spinors. We obtain two solutions: $\lambda_+ \sim r^{\frac{3}{2}}$, $\lambda_- \sim r^{\frac{5}{2}}$ for the Rarita-Schwinger field $\psi_M$ the linearized field equation is
\[
\hat{\gamma}^{\mu \nu} \left( \partial_r \psi_\nu - \frac{1}{2r} (4 - 5\gamma_6) \psi_\nu \right) - \gamma_6 \hat{\gamma}^{\mu \rho \nu} \partial_\nu \psi_\rho = 0. \tag{20}
\]
We find two solutions: $\psi_{\mu+} \sim r^{-\frac{1}{2}}$, $\psi_{\mu-} \sim r^{\frac{1}{2}}$ for the Rarita-Schwinger field $\psi_M$ the linearized field equation is
\[
\hat{\gamma}^{\mu \nu} \left( \partial_r \psi_\nu - \frac{1}{2r} (4 - 5\gamma_6) \psi_\nu \right) - \gamma_6 \hat{\gamma}^{\mu \rho \nu} \partial_\nu \psi_\rho = 0. \tag{20}
\]
Knowing these boundary behaviors of the fields we impose Dirichlet type boundary conditions as
\[
e_\mu^a \sim (2mr)^{-1} e_0^a, \quad \psi_{\mu+} \sim (2mr)^{-1} \psi_{0+}, \quad S_{\mu \nu \rho}^{(-)} \sim r^{-2} S_{0 \nu \rho}^{(-)},
\]
\[
B_\mu \sim B_{0\mu}, \quad \lambda_+ \sim (2mr)^{\frac{3}{2}} \lambda_{0+}, \quad \phi \sim (2mr)^2 \phi_0, \tag{21}
\]
where $\Phi_0 = (e_0^a, \psi_{0+}^{(--)}, S_{0 \nu \rho}^{(--)}, B_{0\mu}, \lambda_{0+}, \phi_0)$ are arbitrary functions on the boundary. Note that we imposed the boundary conditions on only half of the components for $\psi_\mu$, $S_{\mu \nu \rho}$ and $\lambda$ since their field equations are first order [14, 15]. Other components of the fields are expressed by the functions $\Phi_0$ by using the field equations. The fields $\Phi_0$ coincide with the field content of the $(2,0)$ conformal supergravity in six dimensions [11]. The precise relation between $\Phi_0$ and the fields of the conformal supergravity will be given below.

4. Local symmetries on the boundary

We now study how the fields on the boundary $\Phi_0$ transform under residual symmetry transformations after the gauge fixing. We first obtain the residual symmetries, which preserve the gauge conditions (9). By solving differential equations obtained by taking variations of the gauge conditions (9) under the transformations (7), (8)
we find parameters of the residual symmetry transformations near the boundary $r = 0$ as

$$\xi^r = -r \Lambda_0, \quad \xi^\mu = \xi_0^\mu + O(r^2),$$

$$\lambda_{ab} = \lambda_{0ab} + O(r^2), \quad \theta^{IJ} = \theta_0^{IJ} + O(r^2), \quad v^{ij} = \theta_0^{ij} + O(r^2),$$

$$\epsilon_{\pm} = (2mr)^{1\over 2} \left[ \epsilon_{0\pm} + O(r^2) \right], \quad (22)$$

where $\Lambda_0, \xi_0^\mu, \lambda_{0ab}, \theta_0^{ij}$ are arbitrary functions of $x^\mu (\mu = 0, \cdots, 5)$. Order $O(r)$ and $O(r^2)$ terms are non-local functionals of these functions and the fields $\Phi_0$.

Substituting eqs. (21), (22) into eq. (7) and taking the limit $r \to 0$ we find the bosonic transformations on the boundary as

$$\delta e_{0\mu}^a = \Lambda_0 e_{0\mu}^a + \xi_0^\nu \partial_\nu e_{0\mu}^a + \partial_\mu \xi_0^\nu e_{0\nu}^a - \lambda_0^a b e_{0\mu}^b,$$

$$\delta \psi_{0\mu+} = \frac{1}{2} \Lambda_0 \psi_{0\mu+} + \xi_0^\nu \partial_\nu \psi_{0\mu+} + \partial_\mu \xi_0^\nu \psi_{0\nu+} - \frac{1}{4} \lambda_0^{ab} \gamma_{ab} \psi_{0\mu+} - \frac{1}{2} \theta_0^{ij} \tau_{ij} \psi_{0\mu+},$$

$$\delta S_{0\mu\nu\rho,I}^{(-)} = 2 \Lambda_0 S_{0\mu\nu\rho,I}^{(-)} + \xi_0^\sigma \partial_\sigma S_{0\mu\nu\rho,I}^{(-)} + 3 \lambda_0^{ab} \gamma_{ab} S_{0\mu\nu\rho,I}^{(-)} - \theta_0^{IJ} S_{0\mu\nu\rho,I}^{(-)},$$

$$\delta B_{0\mu}^{IJ} = \xi_0^{\nu} \partial_\nu B_{0\mu}^{IJ} + \partial_\mu \xi_0^{\nu} B_{0\nu}^{IJ} + D_0 \theta_0^{IJ},$$

$$\delta \lambda_{0i+} = -\frac{3}{2} \Lambda_0 \lambda_{0i+} + \xi_0^\nu \partial_\nu \lambda_{0i+} - \frac{1}{4} \lambda_0^{ab} \gamma_{ab} \lambda_{0i+} - \theta_0^{ij} \lambda_{0j+} - \frac{1}{4} \theta_0^{jk} \tau_{jk} \lambda_{0i+},$$

$$\delta \phi_{0ij} = -2 \Lambda_0 \phi_{0ij} + \xi_0^k \partial_k \phi_{0ij} - \theta_0^{ik} \phi_{0kj} - \theta_0^{ij} \phi_{0bk}. \quad (23)$$

The transformations with the parameters $\xi_0^\mu, \Lambda_0, \lambda_0^{ab}$ and $\theta_0^{ij}$ represent general coordinate, Weyl, local Lorentz and SO(5) gauge transformations in six dimensions respectively. In particular, seven-dimensional general coordinate transformation in the direction $M = r$ became six-dimensional Weyl transformation. Weights of the Weyl transformation are determined by the powers of $r$ appearing in the boundary behaviors of the fields (21). The weights given in eq. (23) are consistent with those of the conformal supergravity [11].

On the other hand, substituting eqs. (21), (22) into eq. (8) and taking the limit
$r \to 0$ we find the fermionic transformations on the boundary as

$$
\delta \epsilon_{0 \mu}^a = \frac{1}{2} \epsilon_{0+} \gamma^a \psi_{0+},
$$

$$
\delta \psi_{0+} = D_{0 \mu} \epsilon_{0} - \frac{1}{2 \sqrt{3}} i m \tau^i \gamma_{0 \mu} \psi_{0+} S_{0 \mu \rho \sigma, i}^{(-)} + 2 m \gamma_{0 \mu} \epsilon_{0-},
$$

$$
\delta S_{0 \mu \rho \sigma, i}^{(-)} = \frac{\sqrt{3}}{12} \left( 3 \epsilon_{0+} \tau_i \gamma_{0 [\mu \nu] \psi_{0\rho]} - 3 \epsilon_{0-} \tau_i \gamma_{0 [\mu \nu] \psi_{0\rho]} - \epsilon_{0+} \gamma_{0 \mu \rho \sigma} \lambda_{0 i+} \right)
$$

$$
- \frac{\sqrt{3}}{8 m} i D_{0 \mu} \left( \epsilon_{0+} \tau_i \gamma_{0 \rho} \psi_{0 \rho} \right),
$$

$$
\delta B_{0 \mu}^{ij} = \frac{1}{4} \epsilon_{0+} \tau^i \psi_{0 \mu} - \frac{1}{4} \epsilon_{0+} \tau^i \psi_{0 \mu} + \frac{1}{8} \epsilon_{0+} \tau^k \tau^i \gamma_{0 \mu} \lambda_{0 k+},
$$

$$
\delta \lambda_{0 i+} = -2 m \tau^j \epsilon_{0+} \phi_{0 ij} + \frac{1}{16} \left( \tau_{jk} \tau_i - \frac{1}{5} \tau_i \tau_{jk} \right) \gamma_{0 \mu} \epsilon_{0+} F_{0 \mu \nu j}
$$

$$
- \frac{1}{\sqrt{3}} i m \left( \delta^i_j - \frac{1}{5} \tau_i \tau_j \right) \gamma_{0 \mu} \epsilon_{0-} S_{0 \mu \rho, j}^{(-)}
$$

$$
- \sqrt{3} i m \left( \delta^i_j - \frac{1}{5} \tau_i \tau_j \right) \gamma_{0 \mu} \epsilon_{0+} S_{0 \mu \rho, j},
$$

$$
\delta \phi_{0 ij} = \frac{1}{2} \left( \epsilon_{0+} \tau (\lambda_{0 j}) - \epsilon_{0-} \tau (\lambda_{0 i}) \right),
$$

(24)

where $S_{0 \mu \rho, i}^{(-)} = S_{0 \mu \rho, i=1}^{(-)}$, $B_{0 \mu}^{ij} = B_{0 \mu}^{i=1, j=1}$. The underlined fields are not independent fields on the boundary but can be expressed by the fields $\Phi_0$. The transformations (24) are actually equivalent to those of the conformal supergravity [11]. To see the equivalence we redefine the fields as

$$
\tilde{\epsilon}_{\mu}^a = \epsilon_{\mu}^a,
$$

$$
\tilde{\psi}_{\mu} = \psi_{0 \mu+},
$$

$$
\tilde{T}_{abc, i} = -4 \sqrt{3} i m \left( S_{0abc, i}^{(-)} + \frac{\sqrt{3}}{16 m} i \tilde{\psi}_{0 [a+} \tau_i \gamma_{0 \beta] \psi_{0 \beta+}} \right),
$$

$$
\tilde{V}_{\mu}^{ij} = -8 m B_{0 \mu}^{ij},
$$

$$
\tilde{\chi}_{i}^{\gamma} = 15 m \lambda_{0 i+},
$$

$$
\tilde{D}_{ij} = -120 m^2 \phi_{0 ij},
$$

(25)

and the transformation parameters as

$$
\tilde{\epsilon} = \epsilon_{0+},
$$

$$
\tilde{\eta} = 2 m \epsilon_{0-}.
$$

(26)

By dropping tildes on the fields to avoid unnecessary complications in equations we obtain the transformations for fermionic residual symmetry as

$$
\delta \epsilon_{\mu}^a = \frac{1}{2} \tilde{\epsilon} \gamma^a \tilde{\psi}_{\mu},
$$

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\[
\delta \psi_\mu = D_\mu \epsilon + \frac{1}{24} \tau^i \gamma^{abc} \epsilon T_{abc,i} + \gamma_\mu \eta,
\]
\[
\delta T_{abc,i} = -\frac{1}{32} \bar{\epsilon} \tau_i \gamma^d \epsilon R_{de}(Q) - \frac{1}{15} \bar{\epsilon} \gamma_{abc} \chi_i,
\]
\[
\delta \psi_\mu = -\frac{1}{15} \bar{\epsilon} \tau^k \tau^j \gamma_\mu \chi_k - \bar{\eta} \tau^j \psi_\mu,
\]
\[
\delta \chi_i = \frac{1}{4} \tau^j \epsilon D_{ij} - \frac{15}{128} \left( \tau_{jk} \tau_i - \frac{1}{5} \tau_i \tau_{jk} \right) \gamma_{\mu \nu} \epsilon R_{\mu \nu}^{jk}(V)
\]
\[
+ \frac{5}{32} \left( \delta_i^j - \frac{1}{5} \tau_i \tau^j \right) \gamma_{abc} \psi_\mu D_{\mu} T_{abc,j} + \frac{5}{8} \left( \delta_i^j - \frac{1}{5} \tau_i \tau^j \right) \gamma_{abc} \eta T_{abc,j},
\]
\[
\delta D_{ij} = \bar{\epsilon} \tau_i \gamma_{\mu} D_\mu \chi_j - 2 \bar{\eta} \tau_i \chi_j,
\]

where
\[
R'_{\mu \nu}(Q) = 2 D_{[\mu} \psi_{\nu]} + \frac{1}{12} \tau^i \gamma^{abc} \gamma_{[\mu} \psi_{\nu]} T_{abc,i},
\]
\[
\phi_\mu = -\frac{1}{16} \left( \gamma^{\mu \sigma} \gamma_\mu - \frac{3}{5} \gamma_\mu \gamma^{\sigma \rho} \right) R_{\rho \sigma}(Q),
\]
\[
R_{\mu \nu}(Q) = R'_{\mu \nu}(Q) + 2 \gamma_{[\mu} \phi_{\nu]},
\]
\[
R_{\mu \nu}^{ij}(V) = 2 \partial_{[\mu} V^{ij}_{\nu]} + 2 V^{ik}_{[\mu} V^{j]}_{\nu k},
\]
\[
D_{\mu} \chi_i = D_{\mu} \chi_i - \frac{1}{4} \tau^j \psi_\mu D_{ij} + \frac{15}{128} \left( \tau_{jk} \tau_i - \frac{1}{5} \tau_i \tau_{jk} \right) \gamma_{\mu \nu} \epsilon R_{\mu \nu}^{jk}(V)
\]
\[
- \frac{5}{32} \left( \delta_i^j - \frac{1}{5} \tau_i \tau^j \right) \gamma_{abc} \psi_\mu D_\mu T_{abc,j} - \frac{5}{8} \left( \delta_i^j - \frac{1}{5} \tau_i \tau^j \right) \gamma_{abc} \phi_\mu T_{abc,j}.
\]

To obtain eq. (27) we have used field equations derived from the Lagrangian (2) to express the underlined fields in eq. (24) as
\[
\psi_{0\mu} = \frac{1}{2m} \phi_\mu, \quad \lambda_{0i} = \frac{1}{20m^2} \gamma^\mu D_\mu \chi_i, \quad S_{0\mu \nu r,i} = \frac{1}{96 \sqrt{3} m^2} i D^\rho T_{\mu \nu \rho \mu, i}. \tag{29}
\]

The transformations (27) coincide with those in the conformal supergravity [11]. The transformations with the parameters \( \epsilon \) and \( \eta \) represent local super and super Weyl transformations in six dimensions respectively. The bosonic transformations of the fields defined in eq. (25) are easily obtained from eq. (23) and also coincide with those in the conformal supergravity.

Thus, we have shown that boundary values of the fields in the maximal AdS supergravity in seven dimensions form a supermultiplet of the \( N = (2, 0) \) conformal
supergravity in six dimensions, and that local symmetry transformations in the
bulk induce local transformations of the conformal supergravity on the boundary.
Using these results we can compute anomalies of local symmetries on the boundary
as in the AdS$_3$/CFT$_2$ case [8] and obtain Ward identities for correlation functions
of the boundary conformal field theory. Weyl anomaly in a purely gravitational
background was obtained in ref. [14]. The gauge anomaly is easily obtained from
the well-known relation between the Chern-Simons terms and chiral anomalies [16].
Super Weyl anomaly can be also obtained as in ref. [8].

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