On the precision of calculations of Initial State Radiation in the LEP Z line-shape fits

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Abstract

Calculation of Initial State Radiation (ISR) in the fitting program MIZA and in the Standard Model program BHM is improved within the analysis of the Z line-shape measurements at LEP. The uncertainty on the determination of the Z line-shape parameters coming from the precision of ISR calculations is 2×10^{-4} on σ_0 and 0.1 MeV on m_Z and Γ_Z .

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High precision and high statistics LEP measurements performed at c.m.s. energies close to the Z mass provide the most stringent tests of the Standard Model. In the near future LEP experiments will publish results of the final analysis of the Z line-shape. The expected precision of the combined results of four LEP experiments is about 2 MeV on the Z mass m_Z and width Γ_Z , and 0.1% on the σ_0 measurement [1]. The σ_0 uncertainty includes improved precision of the theoretical calculations of the small angle Bhabha scattering of 0.061% [2, 3].

Large theoretical corrections are needed to extract the line-shape parameters m_Z , Γ_Z and σ_0 from the experimentally measured cross-sections. The largest correction, about 30% on the Z peak, comes from Initial State Radiation (ISR). The precision of the ISR calculation is discussed in this paper separately for purely photonic and fermion pair production.

Two line-shape fitting programs are used by LEP experiments: ZFITTER [4] (cross-checked with TOPAZ0 [5]), and MIZA [6]. The precision of ISR is studied here in the c.m.s. energy region between 88 and 94 GeV using the fitting program MIZA and the Standard Model program BHM [7] used by the ALEPH Collaboration.

Two methods are used to calculate photonic ISR in these programs:

- KF: $\mathcal{O}(\dots \alpha^2 L^2, \alpha^2 L, \alpha^2 L^0)$ calculations [8] including the exponentiation of Kuraev–Fadin [9]. In order to improve KF precision we have added $\mathcal{O}(\alpha^3 L^3)$ [10] terms to these programs. Note that $\mathcal{O}(\alpha^3 L^3)$ corrections are also given in Ref. [11].
- YFS[12]/JSW[13, 10] calculations using YFS inclusive exponentiation. Implementation of explicit YFS calculations in MIZA and BHM is part of this work. Note that YFS was already used in the ALEPH publication of the data up to 1992 [14] by applying numerical correction factors to the cross-sections prior to entering them into the fit [15].

Comparison of the e^+e^- cross-section calculation σ_{tot} using different ISR implementations in BHM/MIZA is given in Fig. 1 with the following notations:

KF2: $\mathcal{O}(\dots \alpha^2 L^2, \alpha^2 L, \alpha^2 L^0)_{KF}$

KF3: $\mathcal{O}(\dots \alpha^2 L^2, \alpha^2 L, \alpha^2 L^0, \alpha^3 L^3)_{KF}$

YFS2=YFS/JSW2: $\mathcal{O}(\dots \alpha^2 L^2, \alpha^2 L)_{YFS}$

YFS2_{SL2}=YFS/JSW2–SL2: $\mathcal{O}(\dots \alpha^2 L^2)_{YFS}$ (explicit $\alpha^2 L$ terms removed)

YFS3=YFS/JSW3: $\mathcal{O}(\dots \alpha^2 L^2, \alpha^2 L, \alpha^3 L^3)_{YFS}$

where (...) stands for (Born, $\alpha^1 L^1, \alpha^1 L^0$).

The size of the $\mathcal{O}(\alpha^3 L^3)$ terms in the calculations using KF exponentiation is shown by the distribution KF3–KF2. The size of these terms in the calculations using YFS/JSW exponentiation is given by the distributions YFS3–YFS2 and shows much faster convergence of the higher-order terms compared with the KF method. The distribution YFS2_{SL2}–YFS2 gives the size of the sub-leading (SL) term of second order needed for the calculation of uncertainties.

There is a large difference between calculations using the KF and YFS/JSW method (distribution KF2–YFS3) before the introduction of $\mathcal{O}(\alpha^3 L^3)$ terms into KF calculations.

The difference of σ_{tot} between the most complete calculations using KF and YFS/JSW exponentiations is not bigger than 1×10^{-4} in the whole LEP I c.m.s. energy region after introducing these terms as shown by the distribution KF3–YFS3.

Figure 1: Comparison of different ISR implementations in BHM/MIZA

What is the physical and technical precision of the YFS/JSW ISR calculations in the BHM and MIZA?

The main sources of physical precision were already studied in Ref. [16, 17]. The missing leading-log (LL) terms were estimated there to be not bigger than the YFS/JSW $\mathcal{O}(\alpha^3 L^3)$ contribution, and the missing SL contributions to be not bigger than the YFS/JSW $\mathcal{O}(\alpha^2 L)$ contribution. Note that YFS/JSW $\mathcal{O}(\alpha^3 L^3)$ and YFS/JSW $\mathcal{O}(\alpha^2 L)$ contributions *do not* denote the complete corrections of a given order but only residual terms not ac-

counted for by the YFS exponentiation of the lower order terms. This study gave uncertainties on σ_0 of 2×10^{-4} and below 0.1 MeV on the Z mass and width.

The missing terms have a negligible contribution on the uncertainty: non-log $\mathcal{O}(\alpha^2 L^0)$ terms are small compared with the $\mathcal{O}(\alpha^2 L)$ contribution; LL $\mathcal{O}(\alpha^4 L^4)$ and LL $\mathcal{O}(\alpha^5 L^5)$ are calculated [18] giving, for example, $\mathcal{O}(\alpha^4 L^4)/\mathcal{O}(\alpha^3 L^3) \sim 1/20$.

The ISR uncertainties quoted above are pessimistic. The conventional approach is to scale the above-mentioned QED uncertainties by the leading contributions and to just reduce them. We use here a modest reduction factor of 2 and obtain the final uncertainty on σ_0 of 1×10^{-4} and largely below 0.1 MeV on the Z mass and width in the photonic ISR calculations.

The technical precision of the implementation is studied by switching off electroweak corrections in BHM, leaving only Born $\times$ ISR. The cross-sections of BHM and of the original implementation by the authors [10] agrees to 7 digits for the c.m.s energies studied in the CERN report [17].

Fermion pair production is introduced to BHM/MIZA according to Ref. [19]. The physical precision was studied in Ref. [19], giving a lepton pair correction uncertainty of 0.2×10^{-4} which is negligible. Quark pair corrections are taken from Ref. [20] and were scaled down to include the effect of ISR. The whole scaling factor correction of 1.8×10^{-4} is (conservatively) taken there to be the uncertainty on the σ_0 determination. The uncertainty on the m_Z and Γ_Z is caused by the variation of quark pair correction uncertainty with the c.m.s. energy in the LEP I region. The whole correction is flat below and on the Z peak and then decreases above the peak [19]. Using Table 1 of Ref. [16] one obtains the uncertainty on m_Z and Γ_Z of 0.1 MeV.

Treating the photonic and quark pair uncertainties as independent, we obtain the total uncertainty of 2×10^{-4} on σ_0 and 0.1 MeV on m_Z and Γ_Z coming from the precision of the calculation of ISR.

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