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THE INVARIANCE OF THE VACUUM IS

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In quantum field theory, we often encounter groups of transformations generated by the space integral of the fourth component of a local vector current. We know such groups generate invariances of the system if they leave the Hamiltonian invariant, for from this we can deduce that the vacuum is an invariant state, and that the transformations of field operators induced by the group are invariances of the Wightman functions. It is the purpose of this note to point out that all this follows from an apparently much more restricted assumption: we prove that if the vacuum is invariant under such a group, then so is the Hamiltonian ¹⁾.

Proof: if the vacuum is invariant under the group, then the generator of the group must annihilate the vacuum. That is to say,

$$\int V_0(\underline{x}, t) d^3 \underline{x} |0\rangle = 0. \quad (1)$$

Let $|n\rangle$ be an arbitrary state with vanishing three-momentum. Then,

$$\langle n | \int V_0(\underline{x}, t) d^3 \underline{x} |0\rangle = 0. \quad (2)$$

But because of the way we have chosen the momentum, this is equivalent to

$$\langle n | V_0(\underline{x}) |0\rangle = 0, \quad (3)$$

which is the same as,

$$\langle n | \partial^\mu V_\mu(\underline{x}) |0\rangle = 0. \quad (4)$$

But Lorentz invariance tells us that if this is true in any one Lorentz frame, it is true in all Lorentz frames. Since any momentum eigenstate may be obtained by applying a Lorentz transformation to a state of zero three-momentum, Eq. (4) is valid with any momentum eigenstate on the left. Since these are a complete

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set,

$$\partial^\mu v_\mu(x)|0\rangle = 0. \quad (5)$$

But it is known ²⁾ that any local operator which annihilates the vacuum is in fact zero. Thus,

$$\partial^\mu v_\mu = 0. \quad (6)$$

This implies that

$$\int v_0(\underline{x}, t) d^3 \underline{x}$$

is independent of time, i.e., commutes with the Hamiltonian.

Recently, there has been some interest ³⁾ in groups of transformations generated by the space integrals of the space components of axial vector currents. The same method can be used to show that if the vacuum is invariant under such a group, then the Hamiltonian is also. Equation (6) is replaced by

$$\partial_\mu A_\nu - \partial_\nu A_\mu = 0. \quad (7)$$

Otherwise the proof is substantially the same.