I. INTRODUCTION

Since the advent of inflationary models, the roles played by scalar fields at different epochs of the cosmic history have been extensively investigated. Such fields have been invoked in a variety of disparate scenarios with rather different goals. Some known examples are: (i) the inflaton, the field that drives inflation [1], (ii) the axion, a cold dark matter candidate [2] and (iii) the dilaton, the field appearing in the low energy string action [3] which addresses the same issues of inflation and may provide a solution to the singularity problem [4]. More recently, inspired by the existing observational data and theoretical speculations, some authors have also suggested scalar fields (sometimes called “quintessence”) as the sough non-baryonic dark matter [5]. These “remnant” fields, might have important consequences on the formation of the large scale structure, as well as be responsible by the present day accelerated phase [6], as indicated by the latest type Ia Supernovae observations [7].

Despite their generalized use in the cosmological framework, the physical situations in which the scalar fields are commonly considered are rather particular. For example, in the new inflation case there is

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not a fundamental justification for “turning off” all the possible couplings during the slow rollover phase and doing the opposite just at the onset of the thermalisation phase of reheating (as well as afterwards, if some potential energy is still available). This can be achieved only by considering very strict initial conditions, which weakened the viability of such a scenario. But this is not an isolated case. Neglecting possible thermal couplings between scalar fields and the other constituents of the universe is a general feature assumed in nearly all scalar field models presented in the literature.

One important exception is the explosive reheating period required to succeed any version of isentropic inflation. Along this process, as the field coherently oscillates about the minimum of the potential, its energy is drained to the matter and radiation components. Either in its earlier [8] or in its modern version based on parametric resonance (sometimes called preheating) [9,10], the reheating mechanism is a relatively fast process, and virtually all the entropy of the present universe may be generated in this way. This is certainly not the most general case. In principle, a permanent or temporary coupling of the scalar field $\phi$ with other fields might also lead to dissipative processes producing entropy at different eras of the cosmic evolution. It is expected that progresses in non-equilibrium statistics of quantum fields will provide the necessary theoretical framework for discussing dissipation in more general cases (see for example [11] and references therein). Another possibility is the so-called “instant preheating” [12]. In this process, the inflaton decays continuously into another scalar field as it rolls down the potential. This second field is very short-lived and rapidly decays into fermions thus furnishing a sustained entropy generation, including for quintessence-like models.

Although a justification from first principles for dissipative effects has not been firmly achieved, such effects should not be ruled out only on readiness basis. Much work can be done in phenomenological grounds as, for instance, by applying nonequilibrium thermodynamic techniques to the problem or even studying particular models with dissipation. An interesting example of the latter case is the warm inflationary picture recently proposed [13]. Like in new inflation, a phase transition driving the universe to an inflationary period dominated by the scalar field potential is assumed. However, a standard phenomenological friction-like term $\Gamma \dot{\phi}^2$ is inserted into the scalar field equation of motion to represent a continuous energy transfer from $\phi$ to the radiation field. This persistent thermal contact during inflation is so finely adjusted that the scalar field evolves all the time in a damped regime generating an isothermal expansion. As a consequence, the subsequent reheating mechanism is not needed and thermal fluctuations produce the primordial spectrum of density perturbations [14,15](see also reference [16]).

Warm inflation was originally formulated in a phenomenological setting, but some attempts of a fundamental justification has also been presented [17]. Furthermore, a dynamical systems analysis [19] showed that a smooth transition from inflationary to a radiation phase is attained for many values of the friction parameter, thereby showing that the warm scenario may be a workable variant to inflation. As it ap-
pears, its unique negative aspect is closely related to a possible thermodynamic fine-tunning, because an isothermal evolution of the radiative component is assumed from the very beginning in some versions of warm inflation (for comments on this issue, see [17]). In other words, the thermal coupling acting during inflation is so powerful and finely adjusted that the scalar field decays ensuring a constant temperature even considering the exponential expansion of the universe.

In brief, the aim of this paper is to relax this hypothesis. However, instead of proposing another particular inflationary model, we discuss how the differences between the isentropic and the isothermal inflationary scenarios can be depicted in a convenient parameter space. As we shall see, these models are only two extreme cases of an infinite two-parametric family. Hopefully, this unified view may indicate ways to a consistent phenomenological treatments of these models based on the methods of nonequilibrium thermodynamics. We also discuss how the standard slow roll conditions are modified due to the scalar field decay.

II. SCALAR FIELD WITH DISSIPATION

We will limit our analysis to homogeneous and isotropic universes, described by the flat Friedmann-Robertson-Walker (FRW) line element

\[ ds^2 = dt^2 - a^2(t) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \]

where \( a(t) \) is the scale factor (in our units \( \hbar = c = 1 \)). The source of this spacetime is a mixture of a real and minimally coupled scalar field interchanging energy with a perfect fluid representing all the other fields. The Lagrangian density for the scalar field is

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) + \mathcal{L}_{\text{int}}, \]

where the interaction is implied by the term \( \mathcal{L}_{\text{int}} \) and \( V(\phi) \) is the scalar field potential. This field has the stress-energy tensor given by

\[ T_{\phi}^{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \mathcal{L}_{\phi} g^{\mu\nu}. \]

The other component of the mixture is a simple fluid with energy-stress tensor

\[ T_{\text{m}}^{\mu\nu} = \rho u^\mu u^\nu - p g^{\mu\nu}, \]

where energy density and pressure are given respectively by \( \rho \) and \( p \). The total energy stress tensor of the system \( T^{\mu\nu} = T_{\phi}^{\mu\nu} + T_{\text{m}}^{\mu\nu} \) obeys Einstein’s field equations, from which we obtain the equations of motion

\[ 3H^2 = \frac{8\pi}{m_{\text{pl}}^2} \left( \frac{\dot{\phi}^2}{2} + V(\phi) + \rho \right), \]
\[ 3H^2 + 2\dot{H} = -\frac{8\pi}{m_{\text{pl}}^2} \left( \frac{\dot{\phi}^2}{2} - V(\phi) + p \right). \]  

(6)

where a dot means time derivative, \( H = \dot{a}/a \) is the Hubble parameter, \( m_{\text{pl}}^2 = 1/G \) is the Planck mass, and we have used that the scalar field energy density and pressure are, respectively

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \]

(7)

\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \]

(8)

Now, assuming that the perfect fluid complies with the “gamma law” equation of state,

\[ p = (\gamma - 1)\rho, \]

(9)

the energy conservation law for this interacting selfgravitating mixture can be cast in the form

\[ \dot{\rho} + 3\gamma H \rho = 3 \Gamma \dot{\phi}^2, \]

(12)

where the dissipative coefficient \( \Gamma \) is the decay width of the scalar field and the factor 3 has been introduced for mathematical convenience. For models endowed with an isentropic inflationary stage, we know that \( \rho \approx 0 \), at the onset of the coherent oscillations, that is, when the field \( \phi \) oscillates about the minimum of its potential \( V(\phi) \). In these cases, the term \( 3 \Gamma \dot{\phi}^2 \) is generally inefficient for describing the first stages of the reheating process [9,10] and is not acting during exponential inflation. However, in the presence of a nonnegligible thermal component, a friction term can be justified under special conditions during a de Sitter regime [11]. Applications of this theory lead to complicated but promising warm inflationary models, with an unreasonably large quantity of light fields coupled to the inflaton [17], although some of these models might be physically interpreted in terms of string theory [18].

In what follows, it will be assumed the validity of this friction term as representing the thermal contact between \( \phi \) and the other fields for any epoch. We also consider the “thermal decay width” \( \Gamma \) as a generic function of the temperature, or equivalently, of the cosmic time.

Let us now define the following dimensionless parameters:
\[ x \equiv \frac{\dot{\phi}^2}{\dot{\phi}^2 + \gamma \rho}, \]  

and

\[ \alpha \equiv \frac{3H\dot{\phi}^2}{3H\dot{\phi}^2 + |\dot{\rho}|}. \]  

The adoption of these parameters is somewhat natural and may be understood as follows. The warm picture departs quantitatively from new inflation because during the inflationary stage the energy density of the material component is not negligible. More precisely, it is not negligible only in comparison to the scalar field kinetic term since the potential \( V(\phi) \) must be strongly dominant in order to generate exponential expansion. This means that one needs to compare the kinetic term \( \dot{\phi}^2 = \rho_\phi + p_\phi \) with a quantity involving the energy of the material component written in a convenient form. For the mixture, the (total) equivalent quantity is \( \rho_t + p_t = \dot{\phi}^2 + \gamma \rho \). In this way, recalling the energy conservation law, \( x \) is defined by the ratio between two “inertial mass” terms which redshift away due to the universe expansion. Additionally, it is not enough to compare the values of \( \dot{\phi}^2 \) and \( \gamma \rho \) since we also need to quantify how they evolve in the course of the expansion. This explains the introduction of the parameter \( \alpha \) involving \( 3H\dot{\phi}^2 \) and \( \dot{\rho} \). The presence of \( |\dot{\rho}| \) in this parameter is also reasonable. It comes into play because we are adopting the original isothermal scenario proposed by Berera as a limiting case. Indeed, it seems to be an extreme theoretical situation where the cooling rate of the radiation due to expansion is fully compensated by the transference of particles from the scalar field to the ordinary constituents. It is implicit in the definition of \( \alpha \) that \( \dot{\rho} \leq 0 \), since a reheating phase is unnecessary in warm inflation. However we should point out that a positive \( \dot{\rho} \) is possible and has been investigated \([15]\).

The convenience of these parameters is apparent since by construction they are dimensionless and constrained on the intervals \( 0 \leq x \leq 1 \) and \( 0 \leq \alpha \leq 1 \). In particular, for the isothermal inflation \([13,14]\) we have \( \alpha = 1 \) and \( x \to 0 \), because \( \dot{\rho} = 0 \) and \( \gamma \rho \gg \dot{\phi}^2 \). For isentropic inflation one has \( x = 1 \) and \( \alpha = 1 \) since \( \gamma \rho \ll \dot{\phi}^2 \) and \( \dot{\rho} \ll 3H\dot{\phi}^2 \) (the radiation becomes exponentially negligible). Similarly, noninteracting quintessence-like models lie at some intermediary value of \( \alpha = x \) between 0 and 1. When both parameters are equal to zero, we have the standard model (with a possible cosmological constant). Therefore, the most common solutions, with or without thermal couplings, can be portrayed in this bidimensional parameter space (see Fig. 1).
FIG. 1. Bidimensional parameter space \((\alpha, x)\) depicting thermal couplings for the most common scalar field models with an arbitrary potential \(V(\phi)\). The shadowed overside triangle is forbidden for an expanding universe. The entire diagonal line corresponds to the absence of thermal couplings between \(\phi\) and the other fields. The special points \((0,0)\) and \((1,1)\), represent the standard FRW model plus a cosmological constant and the new inflation, respectively. The point WI \((0,1)\) is the isothermal warm inflationary scenario as originally proposed by Berera [13]. The surprising feature is the large area available in this parameter space where each point represent a possible inflationary evolution.

III. INFLATION, DISSIPATION AND SLOW ROLL CONDITIONS

A fundamental ingredient of many inflationary variants is the period of “slow roll” evolution of the scalar field during which the inflaton field evolves so slowly that its kinetic term remains always much smaller than \(V(\phi)\). A full description of the two-component mixture requires the introduction of a dynamical parameter which is function of the total energy density of the mixture. In order to discuss slow roll dynamical conditions, a suitable choice of such a parameter is \(\gamma_{\text{eff}}\), derived from manipulations on Eqs. (5) and (6)

\[\gamma_{\text{eff}} = - \frac{2\dot{H}}{3H^2} = \frac{\dot{\phi}^2 + \gamma\rho}{\rho_t}.\]  

(15)

Slow roll conditions are imposed to assure (nearly) de Sitter solutions for an amount of time, say, \(\Delta t_f\), which must be long enough to solve the problems of the hot big bang. By inspection of Eqs. (5), (6) and (15) one obtains that the condition for having a de Sitter universe is

\[3H^2 \gg 2|\dot{H}| \leftrightarrow \gamma_{\text{eff}} \approx 0 \leftrightarrow V(\phi) \gg \rho + \frac{\dot{\phi}^2}{2}.\]  

(16)

Additionally, a sufficient amount of inflation requires the above relations to be valid along the interval \(\Delta t_f\). This usually implies constraints on the slope and curvature of \(V(\phi)\). However, as we shall see,
the coupling between $\phi$ and the $\gamma$-fluid during inflation may relax these constraints for a large range of intermediary situations between the isentropic and warm scenarios. In our case, these conditions are represented by (see also [13])

$$H^2 \approx \frac{8\pi}{3m_{\text{pl}}^2} V(\phi), \quad (17)$$

$$3(H + \Gamma) \dot{\phi} \approx -V'(\phi). \quad (18)$$

Notice that the discussion that follows is independent of constraints on the signal of $\rho$, so that we are not limited to the definition of $\alpha$. The approximated expressions (17) and (18) give us the following constraints on the shape of $V(\phi)$

$$\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \frac{m_{\text{pl}}^2}{16\pi} \approx \left(1 + \frac{\Gamma}{H}\right)^2 \frac{\dot{\phi}^2}{2V(\phi)} \quad (19)$$

and

$$\frac{V''(\phi)}{V(\phi)} \frac{m_{\text{pl}}^2}{24\pi} \approx -\left(1 + \frac{\Gamma}{H}\right) \frac{\dot{\phi}^2 + \gamma \rho}{2V(\phi)} - \frac{1}{3H} \frac{d}{dt} \left(\frac{\Gamma}{H}\right). \quad (20)$$

We recall that the standard slow roll conditions appearing in isentropic inflation are recovered for $\Gamma = 0$ (decoupled mixture). They imply that inflation is possible only if the potential is extremely flat, by the last inequality in (16). However, in the limit ($\Gamma \gg H$), one may see that the first and the second derivative of the potential are not necessarily small, in order to guarantee the continued accelerated expansion. Thus, the extremely flat potential of usual inflation assuring the slow down of the scalar field by an enough amount of time, may be replaced by a large friction-like term with no extreme behavior of the derivatives of $V(\phi)$. The field does not accelerate, thereby $\dot{\phi}^2$ becoming comparable to $V(\phi)$, because the friction may be really large and not because the potential is unusually flat. This explains why this class of extended scenarios may provide a solution for the fine tuning problems plaguing the new inflationary picture.

IV. A TOY MODEL

The standard procedure in scalar field cosmological model building is to assume a specific potential $V(\phi)$, motivated or not from particle physics. This potential is used to solve the $\phi$ equation of motion (11). Subsequently, if there is a nonegligible energy density stored in the other fields, the solution of $\phi$ is inserted into (12), which is solved for the energy density $\rho$. As widely known, even for a given coupling, many solutions are possible just changing the potential $V(\phi)$. To fix terminology, this method for generating solutions will be called dynamic approach.
On the other hand, the parameter space \((\alpha, x)\) can also be used as a guide in the search for new models. To show that a different (thermodynamic) route is also possible we first rewrite (12) in terms of \(x\) and \(\alpha\). We have

\[
\frac{\Gamma}{H} = \frac{1}{x} - \frac{1}{\alpha}.
\]  

(21)

It is worth noticing that (21) does not include the potential since it is representative only of the possible couplings, but not of the dynamics. In what follows we obtain a simple model example based on equations (13), (14) and (21). In principle, a more quantitative analysis will clarify the interesting features contained in the parameter space \((\alpha, x)\). In order to be as generic as possible, we also include the possibility of a decaying \(\phi\) during any era.

Since \(x\) and \(\alpha\) do not depend on \(V(\phi)\), one may show that equation (12) for the material medium can be rewritten as

\[
\dot{\rho} = -3\gamma H\rho (1 - \delta),
\]  

(22)

where for short we have introduced the quantity

\[
\delta \equiv \frac{\alpha - x}{\alpha(1 - x)}.
\]  

(23)

It is apparent that the simplest toy model is the one where the parameters \(\alpha\) and \(x\) are constants. In this case, the solution of (22) is

\[
\rho(a) = \rho_I \left( \frac{a}{a_I} \right)^{-3\gamma(1-\delta)},
\]  

(24)

where the integration constants, \(\rho_I\) and \(a_I\), are “initial” conditions just at the onset of the inflationary stage. It should be noticed that if \(\delta = 1\), or equivalently, if \(\alpha = 1\), the energy density \(\rho\) remains constant (\(\rho = \rho_I\)). In particular, if \(x \to 0\), that is, if \(\rho \gg \dot{\phi}^2\), we recover the isothermal inflationary scenario proposed by Berera (see also Fig.1). This show more clearly why models with values of \(0 < x < 1\) may also evolve isothermally during inflation (exponential or power-law). The other extreme situation is obtained if \(\delta = 0\), that is, if \(\alpha = x\), with the energy density scaling as \(\rho \sim a^{-3\gamma}\). This behavior is typical for an adiabatic expansion (very weak coupling), and in Fig. 1, it corresponds to the adiabatic line. Note that even in these circumstances, where the parameters are constants, one may expect that an intermediary situation \((0 < \delta < 1)\) be physically more probable.

Under the above conditions, the energy density of the the scalar field may also be readily obtained. First we rewrite (11) as

\[
\dot{\rho}_\phi = -3H\dot{\phi}^2 \left( 1 + \frac{\Gamma}{H} \right).
\]  

(25)

Now, since \(x = \text{const}\) we see from (13) that \(\dot{\phi}^2 = \frac{1}{1-x^2} \gamma \rho\), and using (21) and (24), this equation can be integrated in terms of the scale factor. The result is
\[ \rho_\phi = \rho_{\phi I} + \rho_I \left( \delta + x \frac{I}{1 - x} \right) \left[ \left( \frac{a}{a_I} \right)^{3(1 - \delta)} - 1 \right] \], \quad (26) 

where the constant \( \rho_{\phi I} \) is the scalar field energy density when \( a = a_I \).

In the adiabatic limit (\( \delta = 0 \)) the above expression reduces to

\[ \rho_\phi = \rho_{\phi I} + \rho_I \frac{x}{1 - x} \left[ \left( \frac{a}{a_I} \right)^{3\gamma} - 1 \right], \quad (27) \]

The isothermal case (\( \delta = 1 \)) is also readily obtained using that \( \lim_{q \to 1} \frac{f - q^{-1}}{1-q} = \ln f \). One finds

\[ \rho_\phi = \rho_{\phi I} - 3 \gamma \frac{1 - x}{1 - \delta} \rho_I \ln \left( \frac{a}{a_I} \right) \], \quad (28) \]

If \( \delta \neq 1 \), using (24) and (26), we have that the total energy density may be written as

\[ \rho_t = \rho_{\phi I} - B \rho_I + B \rho \]

where \( B = \frac{x(1 + \frac{\delta}{x})}{(1 - x)(1 - \delta)} \) is a constant.

The time dependence of the scale factor may also be obtained from (13) and (15), which give \( \gamma_{\text{eff}} = \frac{\rho}{(1 - \delta)\rho_I} \). We see that the possible evolution laws depend critically on the initial conditions. In particular, if \( B \sim \frac{\rho_{\phi I}}{\rho_I} \) we have \( \gamma_{\text{eff}} \sim \frac{3\gamma}{(1 - \delta)\rho_I} \). Thus, if \( \rho_{\phi I} \gg \rho_I \) we have exponential inflation, and if \( \rho_\phi \) dominates only moderately, the scale factor will evolve as a power law inflation with a coupled thermal component.

As one may check, in this case the potential \( V(\phi) \) scales as \( e^{-\lambda \phi} \), where \( \lambda(\delta) \) is a positive parameter. Note still that if the first two terms in (29) do not cancel each other, we may have a “remnant” cosmological constant for large values of the cosmological time.

As we have seen, this simple model is representative of two different ways of extending the original warm inflation. One is keeping the isothermal condition (\( \alpha = \delta = 1 \)) for generic values of \( x \). Another approach is to consider nonvanishing parameters \( \alpha \) and \( x \) (\( \alpha \neq x \)), which allows for warm power law inflationary models. Hopefully, in a more general framework, where the pair of parameters \( (\alpha, x) \) are time dependent functions, a consistent unified picture containing the new and warm inflation, as well as all the intermediary situations may be obtained.

**V. CONCLUSIONS**

Our analysis might be useful as an heuristic tool for building models with scalar fields coupled to a material medium. In principle, it can be applied to warm inflation-like models (with or without phase transitions involved), reheating (even for the old scenario) and quintessence-like models. As a rule, it could be tried as a first step, before attempting different potentials for the scalar fields. As we have shown, this happens because depending on the two parameters, the conditions for distinct dynamics of
the scale factor (including inflationary regimes) are relatively independent of the shape of the potential, since a large dissipative term may provide a natural slow rolling for the field. This may unconstrain the conditions on the flatness of the potential in such a way that even exotic models like the oscillating inflation of Damour and Mukhanov [20] may provide the necessary number of e-foldings and enough post-inflationary radiation temperature. The toy model presented in the last section somewhat suggest that any inflationary interpolating solution between the isentropic and isothermal limits can be represented in the bidimensional parameter space ($\alpha, x$). Particular examples will be discussed elsewhere [21].

As it appears, a more comprehensive phenomenological treatment of this matter should necessarily include thermodynamical constraints thus requiring the methods and techniques from nonequilibrium thermodynamics. As shown recently [22], a phenomenological coupling term explicitly dependent on the created particles (and not only on the scalar field) should be a natural outcome of these methods when the decay products thermalize with the heat bath. Such an approach may have interesting consequences on the old reheating and warm inflation models.

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