SCATTERING MATRIX ANALYSIS OF THE NLC ACCELERATING STRUCTURE*
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1 INTRODUCTION

In the Next Linear Collider (NLC)[1], long trains of short, intense bunches are accelerated through the linac on their way to the collision point. One serious problem that needs to be addressed is the multi–bunch, beam break–up instability in the linac. To counteract this instability the accelerating structures are designed so that the dipole mode wakefields are detuned and weakly damped. Detuning is accomplished by gradually varying the dimensions of the 206 cavity cells in each structure, and weak damping by surrounding the structure with four manifolds, which allow the dipole modes to weakly couple out of the cells. In order to design and predict the performance of such a structure in the NLC, it is necessary to be able to calculate accurately the strength of its long–range wakefields.

NLC detuned structures were designed first using an equivalent circuit approach, for example, the double band model of Ref. [2]. With the introduction of weak damping, a more elaborate equivalent circuit approach was required[3]. Other methods that have been used for detuned structures are the open–mode, field expansion method[4], and a finite element calculation employing 206 parallel processors[5]. A scattering matrix method can also be applied to cavities that consist of a series of waveguide sections[6], such as the NLC structure, and such a method has also been applied to the problem of detuned accelerating structures[7],[8]. Among its natural strengths when applied to structure problems are the ability to model the effects of complicated 3 dimensional couplers and to investigate the behavior of higher passband modes. In this report we will employ a computer program that uses such an approach [9], to demonstrate that the scattering matrix method can yield accurate results concerning the wakefields of damped, detuned accelerating structures.

We begin by calculating the dipole mode impedance and wakefield of a slightly simplified version of a detuned (DT) structure, results which we compare with those of the double band circuit model. An important related problem is the effect of complicated 3 dimensional couplers and to investigate the behavior of higher passband modes. In this report we will employ a computer program that uses such an approach [9], to demonstrate that the scattering matrix method can yield accurate results concerning the

2 CIRCUIT MODEL COMPARISON

In this report we consider cylindrically symmetric, disk–loaded accelerating structures at X-band (with fundamental frequency $f_0 = 11.424$ GHz), but with the simplification that the irises are rectangular, and not rounded. Let us consider an earlier version of the NLC structure, a DT structure. It consists of 206 cells, with Gaussian detuning of the dipole modes, with a central synchronous frequency $f_s = 15.07$ GHz, an rms $(f_s)_{rms} = 0.025 f_s$, and a total width $\Delta f_s = 0.1 f_s$. The structure period $p = 8.75$ mm, iris thickness $p - q = 1.46$ mm, cavity radius $b \approx 11$ mm, and iris radius $a$ varying from 5.9 mm at the beginning of the structure to 4.1 mm at the end.

DT structures have been modelled using a double band circuit model computer program[2]. If $M$ is the number of cells in a structure, this program generates $2M$ coupled mode frequencies $f_m$ and kick factors $k_m$. The transverse wakefield is then given by

$$W_\perp(s) = 2 \sum_{m=1}^{2M} k_m \sin(2\pi f_m s/c) \ e^{-\pi f_m s/Qc}, \quad (1)$$

with $s$ the distance between driving and test particles, $c$ the speed of light, and $Q$ the quality factor due to the wall resistivity.

For the scattering matrix calculations we follow the approach of Ref. [6], implemented in the program described in Ref. [9]. A structure with $M$ cells is modeled by a set of $2M$ joined waveguide sections of radii $a_m$ or $b_m$, filled with a number of dipole TE and TM waveguide modes. For each section the S-matrix is obtained, and the S-matrix for the composite chain is constructed by cascading the individual S-matrices. From this the longitudinal dipole impedance for a beam offset at $r = 4$ mm is calculated, from which, using the Panofsky-Wenzel theorem, the transverse impedance $Z_\perp$ is obtained. The real part of the impedance $R_\perp$ consists of a series of infinitesimally narrow spikes, and to reduce the calculation time for our problem the peaks have been artificially widened by introducing an imaginary 1.5 MHz frequency shift, corresponding to a $Q$ of 5000 (the width of the peaks is still narrow compared to the minimum mode spacing $\sim 7$ MHz, however). The number of modes used in the cavity region is 15 TE and 15 TM modes, and in the iris region 8 TE and 8 TM modes.

To compare with the circuit model results, the impedance can be fit, for resonant frequencies and kick factors, to a sum of Lorentzian distributions. Or, the area under $R_\perp(f)$, averaged over frequency bins—the average impedance $\bar{R}_\perp$—can be compared to $\frac{1}{2} k \sin^2(d/\ell)$ of the circuit model. Results for all first and second band modes, comparing the kick factors, the density of modes, and $\bar{R}_\perp$.
for the two methods are shown in Fig. 1 (to facilitate comparison, discrete results are connected by straight lines). The agreement is very good. There is disagreement in the $k$ of the (much weaker) second band modes, but for these modes it is not expected that the double–band circuit model be accurate. With the S-matrix approach, to obtain the wakefields, the fitted mode frequencies and kick factors are summed according to Eq. 1. Finally, the envelope of the dipole wake (with $Q$ set to 6500, appropriate for copper wall losses) up to $s = 80$ m (the NLC bunch train length) for the two methods is shown in Fig. 2.

3 INTERNAL MISALIGNMENTS

If there are internal misalignments in the accelerating structure, due to e.g. fabrication errors, then even if the beam is aligned to the nominal structure axis there will be residual transverse wakefield kicks, both at frequencies near the fundamental mode (11 GHz) and the 1st band dipole modes (15 GHz). In the NLC these kicks can, independent of orbit, increase the beam projected emittance, and destroy the resolution of the structure as a beam position monitor.

Internal misalignments break cylindrical symmetry in geometry, and we study this problem with a 3–dimensional version of the S-matrix program. In the NLC what is normally used to estimate the effects of internal structure misalignments (due to the dipole modes) is a perturbative approach based on the circuit model solution[10]. The method assumes that, for small misalignments, the mode frequencies and eigenfunctions do not change significantly, and the kick factors associated with them can be obtained through an integration of fields along a path described by the misalignments. The result is linear in cell offset, and the effect of a misaligned structure can be decomposed into many single-cell misalignments.

Let us study the effect of modes near 15 GHz on a beam on–axis in a structure with a single cell misaligned. We consider a 21–cell section of the DT structure described above (cells 175–195, with $a$ varying from 4.527–4.343 mm). The structure has been shortened to save computing time, and this particular section was chosen since it contains many trapped dipole modes that do not reach, and therefore are not affected by, the boundary cells. For the simulations, 45 waveguide modes are used in the cavity region (15 each of TM and TE dipole, and TM monopole) and 15 modes in the iris region (5 modes of each type). To avoid fundamental mode standing waves, matching cells were added at the ends (with $a_{in} = 6.39$ mm, $b_{in} = 11.112$ mm, $a_{out} = 5.65$ mm, and $b_{out} = 10.815$ mm), and the resulting SWR at 11.4 GHz is below 1.02.

When one cell (of cells 187–192) is misaligned, the S-matrix program finds that only very few modes are excited significantly, and that nearly the total effect is due to just one or two modes. To obtain kick factors the impedance was fitted to a set of Lorentzians, as before. The results for six cases, showing the kick of the one or two dominant modes, are given in Fig 3. Note that here $k$ has been normalized to 1 mm offset of the shifted cell; $k$ has also been scaled to the full 206 cell structure (by multiplying the 21–cell result by 21/206). Also shown in Fig. 3, for comparison, is the results of the perturbation calculation based on the double–band circuit model, but with frequencies all shifted down by 44 MHz. The agreement is remarkably good. It validates the perturbation theory, showing that there is no missing contribution due to having broken cylindrical symmetry. Finally, note that the sum of kick factors due to one misaligned cell is almost the same for all cases, $\sim 0.3 \text{ V/pC/mm/m}$, and that this value is about the same as is found in the peak of the distribution for $k$ of Fig. 1a.
3.1 Transverse Kick at 11 GHz

When the monopole field interacts with a shifted cell it excites a dipole field that causes a transverse kick independent of the position of a witness bunch. This is true whether the field is external or excited by the beam. The dipole field does not propagate far from the shifted cell because the fundamental frequency is far below the first dipole passband. In Fig. 4 we plot the ratio of transverse to longitudinal voltage \( V_t/V_L \) excited by the beam at the fundamental mode frequency. The integral over the real part is in each case zero. Then the transverse impedance was calculated in the frequency range 11.4–11.5 GHz, and the results were inverse Fourier transformed to yield a wakefield (see Fig. 5). Note that these kicks are much weaker than those of the dipole modes near 15 GHz.

4 THE DDS STRUCTURE

The S-matrix calculation was performed for the geometry of cell 106 of DDS3, using periodic boundary conditions. DDS3 is a disk–loaded structure, surrounded by 4 manifolds, with rectangular cross–sections that are aligned radially outward, and that couple through a slot to the structure cells. The dimensions are: \( a = 4.783 \text{ mm, } b = 10.757 \text{ mm, } p–g = 1.464 \text{ mm}; \) the manifold cross–section is 5.0 mm×10.496mm, and the distance from manifold edge to structure axis is 11.921 mm. Modelled was 1/4 of the cell; in the cavity region 23 waveguide modes (including manifold modes) were used, in the iris region 28 waveguide modes. The waveguide modes were obtained using the 2-d finite element code SLANS[11]. Final results are compared in Fig. 6 with those obtained with MAFIA[12], and we see that agreement is good.

5 REFERENCES