Particle acceleration with the axial electric field of a TEM₁₀ mode laser beam

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Abstract

Due to their finite spot size, optical beams have small axial field components. For a Gaussian TEM₁₀ mode, the axial electric field has a maximum on the optical axis while the transverse field vanishes there. The possible use of this axial field for the acceleration of highly relativistic particles has been studied.

The ratio of the axial to the maximum transverse electric field is inverse proportional to the spot size radius; for a spot size radius of $w_0 \approx 100 \lambda$, it is in the order of $4 \cdot 10^{-3}$. But the transverse field components seen by the highly relativistic particles are significantly reduced by the Lorentz transformation, thus the axial field components become important.

Finite spot size and axial field are inevitably connected to a phase velocity along the axis which is by approximately a factor of $(1 - \pi/(kw_0)^2)^{-1}$ above c . Consequently synchronism between wave and particle beam cannot be sustained for more than half a RF period. But due to the relativistic Doppler shift, particles can travel approximately the confocal length, kw_0^2 , before the field reverses sign. The particle energy increase is proportional to the square root of the laser power, and lies in the order of 10 MeV for laser powers of 10^{12} W.¹

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1. Introduction

The high fields which exist in the focus region of high power lasers cannot be used directly for acceleration of unbound particles in vacuum. For a single homogeneous plane wave, the particle will always lag in phase if it is not highly relativistic *and* moving in parallel with the wave. But in this latter case, the fields are transverse to its direction of propagation, and thus no energy exchange occurs. Extending these thoughts to the superposition of plane waves, we cite A. M. Sessler [1]:

The field pattern produced by *any* array of optical elements, provided one is not near a surface (more than a wavelength away) and not in a medium, is simply a superposition of plane waves. It is not very difficult to generalize the considerations made for a single plane wave and conclude that for a relativistic particle, which moves with essentially constant speed and in a straight line, there is no net acceleration.

To obtain synchronism, the phase velocity of the wave has to be slowed down to the particle velocity, i.e. the component of the wave vector $\vec{k}$ in the direction of particle motion $k_{\parallel}$, has to be greater than the free space wave number, $k_{\parallel} > k$. Due to $k^2 = k_{\parallel}^2 + k_{\perp}^2$, this is possible with an imaginary transverse component of $\vec{k}$, $k_{\perp} = j\alpha$. These evanescent waves decay perpendicular to their direction of propagation. They exist in the immediate vicinity of dielectric or grating structures. For highly relativistic particles, this imaginary $k_{\perp}$ has to be only small, but still the necessary distance between these surfaces and the particle beam may become inconveniently narrow.

In this paper, we will consider a laser beam propagating in z -direction, acting on a single highly relativistic charged particle propagating in parallel to it. Even though it is not guided near a surface or dielectric, such a beam has some similarity to evanescent waves: it decays exponentially in the transverse direction, it has axial fields, and there exist even straight lines parallel to the axis where $v_p < c$. What is the effect of the small axial electric field that exists in the laser beam on the particle?

2. The field of a Gaussian beam wave

The transverse electric field of a TEM_p mode of a Gaussian beam wave with $\cos k\rho$ -dependence and linear polarization in x -direction is given in cylindrical coordinates (ρ , φ , and z) by [2]

$$\frac{E_x}{E_0} = \frac{w_0}{w} \left(\sqrt{2} \frac{\rho}{w} \right)^l L_p^{(l)} \left(2 \left(\frac{\rho}{w} \right)^2 \right) e^{-\left(\frac{\rho}{w} \right)^2} \cos k\rho e^{-j\Phi}$$

$$\Phi = kz - (2p + l + 1) \arctan \frac{2z}{L} + \frac{2z}{L} \left(\frac{\rho}{w} \right)^2,$$

where w_0 denotes the beam waist radius and

$$w(z) = w_0 \sqrt{1 + \left(\frac{2z}{L}\right)^2}$$

is the beam shape with the "confocal length" $L = kw_0^2$, the meaning of L being the length of a confocal optical resonator supporting this beam. $L_p^{(l)}$ denotes the generalized Laguerre polynomial [3], and $j^2 = -1$. l and p are the azimuthal and radial order of the mode, respectively. The approximation is valid for infinite apertures and large spot sizes, $\lambda/w_0 \ll 1$.

In Fig. 1, the transverse electric field of the TEM_{10} mode, polarized in x-direction and with $\cos lp$ -dependence, is plotted. The transverse coordinates are normalized to w ; the length of the arrows is proportional to the electric field.

Fig. 1: Transverse electric field of a TEM_{10} mode

Due to the transverse variation of the transverse fields, an axial field must exist, i.e. there has to be an axial field near the center of Fig. 1 for the electric field lines to be closed. This follows directly from Maxwell's equations

$$\nabla \times Z_0 \vec{B} = j k \vec{E},$$

$$-\nabla \times \vec{E} = j k Z_0 \vec{B},$$

where $k = \omega/c$ and $Z_0 = 377 \Omega$. If we write $\nabla = \nabla_{\perp} + \vec{u}_z \frac{\partial}{\partial z}$ and assume again a linearly polarized wave, $E_x = Z_0 B_y$, we have

$$jk \vec{E} = -\vec{u}_x \frac{\partial}{\partial z} Z_0 B_y + \nabla_{\perp} \times \vec{u}_y Z_0 B_y,$$

$$jk Z_0 \vec{B} = -\vec{u}_y \frac{\partial}{\partial z} E_x - \nabla_{\perp} \times \vec{u}_x E_x.$$

The last term in each of these two equations gives rise to axial field components. The TEM_{10} Gaussian modes are thus not exactly "transverse electromagnetic"; the name "quasi TEM mode" is more appropriate. The axial fields can in good approximation be computed from the transverse variations of the transverse fields, as long as the ratio λ/w_0 is small. The axial fields are of first order in λ/w_0 while correction terms for the transverse fields in the above equations are of second order.

The normalized transverse and axial electric fields of the TEM_{10} mode are plotted in Fig. 2 for $y=0$ (Note the different scales!). While the transverse field has a zero in the center, the axial field has a significant peak there. E_0 is calculated from the laser beam power P by the formula

$$E_0 = \sqrt{\frac{2 Z_0 P}{\pi w_0^2}}.$$

The ratio of the maximum axial electric field to the maximum transverse field is $\sqrt{2} \exp(0.5)/(kw)$.

Fig. 2: Normalized transverse (solid line) and axial electric fields (dashed line) of the TEM_{10} mode

3. Lorentz transformation of the fields

The difference in the scales of the two curves in Fig. 2, $k\omega$, is usually a large number. For a beam waist radius of some hundred λ , e.g., it is in the order of 10^3 . The axial fields are by this factor smaller than the transverse fields. But they become important for highly relativistic particles due to the Lorentz transformation of the fields.

Given the fields in the laboratory frame (unprimed quantities) and the particle velocity βc and energy $\gamma = (1 - \beta^2)^{-1/2}$, we get in the rest frame of the particle (primed quantities) [4]:

$$\begin{aligned} E'_x &= \gamma (E_x - \beta Z_0 B_y), & B'_x &= \gamma (B_x + \beta E_y / Z_0), \\ E'_y &= \gamma (E_y + \beta Z_0 B_x), & B'_y &= \gamma (B_y - \beta E_x / Z_0), \\ E'_z &= E_z, & B'_z &= B_z. \end{aligned}$$

In our case of a linearly polarized wave travelling with the particle, this becomes

$$\begin{aligned} E'_x &\approx \frac{1}{2\gamma} E_x \\ E'_z &= E_z \end{aligned}$$

for the electric field. For 1 GeV electrons, e.g., the transverse field will be decreased by a factor of about 4000. Thus the axial field plays in fact a dominant role.

4. Phase relations

Two effects lead to non-synchronism between particle and wave. First there is the deviation of the particle velocity v from c . Let us define

$$X \equiv 1 - \frac{v}{c}.$$

For a highly relativistic particle, we obtain

$$X \approx \frac{1}{2\gamma^2}.$$

For a 1 GeV electron, e.g., X would be in the order of 10^{-7} .

The second effect is the deviation of the phase velocity v_ϕ of the optical beam from c , for which we define

$$Y \equiv \frac{v_\phi}{c} - 1.$$

v_ϕ is calculated from $\Phi - \omega t = \text{const}$ along the particle trajectory. Along the axis we obtain

$$Y = \frac{2(2p+l+1)}{(kw)^2 - 2(2p+l+1)} \approx \frac{2(2p+l+1)}{(kw)^2}.$$

The approximation is valid for small λ/w_0 . For a TEM_{10} mode with $w_0/\lambda = 100$, Y is approximately $5 \cdot 10^{-6}$.

Because both X and Y are generally positive quantities, the particle will lag in phase behind the wave, and there will be in fact no "net" acceleration, in full agreement with the statement by Sessler cited above. But on how long a distance could the particle be kept in phase? In the rest frame of the particle, the laser frequency would appear Doppler downshifted by the factor

$$\frac{\omega'}{\omega} = \frac{X+Y}{\sqrt{2X}}.$$

The nominator describes the non-relativistic Doppler shift, while the denominator accounts for the change of the proper time. After half a period, the particle would slip out of phase, this is in the particle frame

$$t' = \frac{1}{2} \frac{2\pi}{\omega'}.$$

Back in the laboratory frame, this time is

$$t = \frac{1}{\sqrt{2X}} t' = \frac{T}{2} \frac{1}{X+Y},$$

where T is the original RF period. For the TEM_{10} mode, it turns out that a highly relativistic particle ($X \ll Y$) can travel just the confocal length L during this time, independent of the waist radius.

5. Achievable energy gain

The maximum energy the particle could gain on its path is given by eV_{acc} , where e is the charge of the particle and the total accelerating voltage is given by the integral

$$V_{acc} = \int_{-z}^z E_z e^{j\frac{\omega}{v}\xi} d\xi.$$

V_{acc} is plotted versus z in Fig. 3 for the TEM_{10} mode and a highly relativistic particle moving along the axis. The maximum occurs always at $z = L/2$, as stated above. The maximum value is $\sqrt{Z_0 P/\pi} = E_0 w_0/\sqrt{2}$. For a given laser power, it is independent of the parameters of the optical beam. For 1 TW, e.g., and a spotsize of $3 \cdot 10^{-6} \text{ m}^2$ ($w_0 = 100\lambda$, $\lambda = 10 \mu\text{m}$), E_0 would be 15.5 GV/m. The maximum value of E_z would be 25 MV/m, and the maximum of V_{acc} would be 11 MV.

Fig. 3: The total accelerating voltage V_{acc} for a highly relativistic particle moving along the axis in a TEM_{10} -mode versus z .

For other particle trajectories and other modes, one would have slightly different formulae, but the result never seems to exceed significantly the "single-cycle acceleration", $\sqrt{Z_0 P/\pi}$ which is small even for very high laser powers.

In [5], similar results were obtained for the single-cycle electron acceleration for electrons passing a plane wave in an oblique angle.

The proportionality between the acceleration and the square root of the laser power (to the laser amplitude) is a remarkable difference between the acceleration scheme considered here and the one used in "far field accelerators" [6] where the acceleration is directly proportional to the laser power. Below a certain threshold power (which might be very high), the effect of the axial fields is greater than of the transverse fields and must not be neglected.

6. Refocussing

The low energy gain achieved by this scheme could possibly be increased by using a periodic structure to refocus the optical beam. The scheme sketched in Fig. 4 is not meant to be a technical solution, but just to illustrate the idea of refocussing; it might as well be some mirror arrangement. The phase shift in the lenses would have to be such that the particle, moving along the axis, is in the accelerating phase in each cell. In its periodicity, this structure resembles again a conventional linac. Keeping in mind that there are two types of "near field", one in which evanescent waves exist (which normally extends only the order of λ from the structure), and a second one where the field might already be described by a superposition of homogeneous plane waves (characterized by a dependence of the radiation characteristic on the distance), the proposed scheme clearly belongs to the group of "near field accelerators".

Fig. 4: Refocussing of the beam by a set of lenses

7. Conclusions

- For a highly relativistic particle travelling along with a laser beam, the transverse fields Lorentz transform down by a factor $(2\gamma)^{-1}$, thus making the small axial fields important.
- Using these axial fields in a TEM_{10} mode, the maximum achievable energy gain of a highly relativistic particle travelling on the optical axis is $e\sqrt{Z_0 P/\pi}$. This value is obtained on the "confocal length" L and is independent of the spot size w_0 . For $P = 1$ TW and highly relativistic electrons, this would be approximately 11 MeV.
- The effect is of first order in $\sqrt{P}$, and thus has to be considered when second order effects are investigated, for example in "far field accelerators".
- For the time being the axial field components of laser beams do not seem to lead to competitive particle acceleration for practically available laser powers.

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References

1. A. M. Sessler *The quest for ultrahigh energies* Am. J. Phys., Vol. **54**, No. 6, 505 (1986)
2. H. Kogelnik, T. Li *Laser beams and resonators* Proceedings of the IEEE, Vol. **54**, No. 10, 1312(1966)
3. M. Abramowitz, I. A. Stegun *Handbook of Mathematical Functions* New York: Dover Publications (1970)
4. J. D. Jackson *Classical Electrodynamics* New York: Wiley (1975)
5. M. J. Feldman, R. Y. Chiao *Single-cycle electron acceleration in focused laser fields* Physical Review A, Vol. **4**, No. 1, 352 (1971)
6. J. D. Lawson *Laser Accelerators: Where do we stand?* CAS – ECFA – INFN Workshop, Frascati, (1984)

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