Inclusive decays and lifetimes of doubly charmed baryons

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Abstract

We extend the analysis of weak decays of heavy hadrons to the case of doubly charmed baryons, \( \Xi^{++}_{cc} \), \( \Xi^+_c \) and \( \Omega^+_c \). Doubly charmed baryons are modeled as a heavy-light system containing a heavy \( cc \)-diquark and a light quark. Such a model leads to preasymptotic effects in semileptonic and nonleptonic decays which are essentially proportional to the meson wave function. Very clear predictions for semileptonic branching ratios and lifetimes of doubly charmed baryons are obtained.

Talk given by H. Štefančić at the XVII Autumn School "QCD: Perturbative or Nonperturbative?", Lisbon, Portugal, 29 Sept - 4 Oct 1999.
To appear in the Proceedings
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1 Introduction

In the last decade, significant progress in weak decays of heavy hadrons has been achieved. The qualitative picture of the lifetime hierarchy predicted for singly charmed baryons and charmed mesons has been found to be in agreement with experiments.

On the other hand, although the inverse bottom-quark mass appears to be a better expansion parameter of the inclusive decay formalism, it seems that the predictions in beauty decays do not quite follow the success from the charmed sector, especially in the prediction of absolute lifetimes of beauty particles.

The following natural step towards the investigation of weak-decay dynamics is therefore the consideration of heavy baryons containing two charmed quarks. The triplet of doubly charmed baryons ($\Xi^{++}_{cc}$, $\Xi^{+}_{cc}$ and $\Omega^{+}_{cc}$) exhibits significant preasymptotic effects, already found to be important in singly charmed decays.

We discuss some special features coming from the doubly heavy nature of doubly charmed baryons and give predictions for their lifetimes and semileptonic branching ratios (BR). Some wrong prefactors in for the four-quark contributions are corrected and numerical results are reevaluated.

*Talk given by H. Štefančić at the XVII Autumn School "QCD: Perturbative or Nonperturbative?", Lisbon, Portugal, 29 Sept - 4 Oct 1999.
2 Preasymptotic effects and the wave function in doubly charmed baryon decays

The decay rate of a doubly charmed baryon is expanded through the OPE technique, similarly as for a singly charmed baryon, in the series of the product of the short-distance part (Wilson coefficient functions denoted by \( c^f_i \)) and the long-distance part (matrix elements \( \sim \langle H_{cc} | O_i | H_{cc} \rangle \)):

\[
\Gamma(H_{cc} \to f) = \frac{G_F^2 m_c^5}{192 \pi^3} |V|^2 \left\{ \frac{1}{2M_{H_{cc}}} c^f_3 \langle H_{cc} | \bar{r}c | H_{cc} \rangle \right. \\
+ c^f_5 \langle H_{cc} | g_s \sigma^{\mu\nu} G_{\mu\nu} c | H_{cc} \rangle \right\} \frac{m_c^2}{m_c^2} + \sum \left. c^f_6 \langle H_{cc} | (\bar{r} \Gamma_i q)(\bar{q} \Gamma_j c) | H_{cc} \rangle \right\} \frac{m_c^3}{m_c^3} + O(1/m_c^4). \] (1)

The matrix elements are specific for a given doubly charmed hadron. The leading \( O(m_c^5) \) term is given by the HQET expression:

\[
\langle H_{cc} | \bar{r}c | H_{cc} \rangle = 1 - \frac{1}{2} \frac{\mu^2_G(H_{cc})}{m_c^2} + \frac{1}{2} \frac{\mu^2_G(H_{cc})}{m_c^2}, \] (2)

where \( \mu^2_G \) parametrizes the matrix element of the chromomagnetic operator \( g_s \sigma^{\mu\nu} G_{\mu\nu} c \) and \( \mu^2_G \) is the matrix element of the kinetic energy operator.

The value of the kinetic energy matrix element is obtained using some phenomenological features of the meson potential 6:

\[
\mu^2_\pi = m_c v_c^2 (\frac{m_q T}{2m_c^2 + m_c^* m_q} + \frac{T}{2m_c^*}) m_c^2, \] (3)

where \( T \) is the average kinetic energy of a light quark and a heavy diquark. In all expressions in the paper, \( m \) refers to the current mass that is used in the HQET and OPE expansions, while \( m^* \) refers to the constituent mass used in model calculations.

The expression for \( \mu^2_G \) is the following

\[
\mu^2_G = \frac{2}{3} (M_{ccq}^* - M_{ccq}) m_c - (\frac{2}{9} g^2 \frac{|\phi(0)|^2}{m_c^*} + \frac{1}{3} g^2 \frac{|\phi(0)|^2}{m_c^*}). \] (4)

The first term describes the hyperfine interaction between a light quark and a heavy diquark, while the second one accounts for the hyperfine interaction of heavy quarks in a heavy diquark. \( \phi(0) \) is the wave function of the \( cc \) pair in a diquark.

The second term in the expression (1) describing the chromomagnetic interaction, receives a similar contribution from (4).
The third term in (1) describes four-quark interactions, specific for a given hadron. Four-quark operators produce the effect which is the largest of all preasymptotic effects and numerically comparable with the leading “decay” contribution. Therefore, these effects introduce the crucial difference in lifetimes and semileptonic branching ratios between various doubly charmed baryons. We state their contributions explicitly below.

In the case of semileptonic decays, the contributions of four-quark operators appear through the positive Pauli interference and can be expressed using

\[ \tilde{\Gamma}_{SL} = \frac{G_F^2}{12\pi} m_c^2 (4\sqrt{\kappa} - 1) \frac{10}{3} |\psi(0)|^2. \] (5)

For individual doubly charmed baryons, the contributions are the following:

\[ \Gamma_{4q}^{\Xi_{cc}^+} = 0, \]
\[ \Gamma_{SL}^{\Xi_{cc}^+} = s^2 \tilde{\Gamma}_{SL}, \]
\[ \Gamma_{SL}^{\Omega_{cc}^+} = c^2 \tilde{\Gamma}_{SL}. \] (6)

Expressions for the contributions of four-quark operators become more intricate in the case of nonleptonic decays owing to the more complex QCD dynamics. In general, there are three types of processes known as W-exchange, negative Pauli interference and positive Pauli interference, given by the following expressions:

\[ \Gamma_{ex} = \frac{G_F^2}{2\pi} m_c^2 \left[ c_+^2 + \frac{2}{3} (1 - \sqrt{\kappa}) (c_+^2 - c_-^2) \right] 6 |\psi(0)|^2, \]
\[ \Gamma_{int}^- = \frac{G_F^2}{2\pi} m_c^2 \left[ -\frac{1}{2} c_+ (2c_- - c_+) \right. \]
\[ \left. - \frac{1}{6} (1 - \sqrt{\kappa}) (5c_+^2 + c_-^2 - 6c_+ c_-) \right] \frac{10}{3} |\psi(0)|^2, \]
\[ \Gamma_{int}^+ = \frac{G_F^2}{2\pi} m_c^2 \left[ \frac{1}{2} c_+ (2c_- + c_+) \right. \]
\[ \left. - \frac{1}{6} (1 - \sqrt{\kappa}) (5c_+^2 + c_-^2 + 6c_+ c_-) \right] \frac{10}{3} |\psi(0)|^2. \] (7)

These effects combine to give the contributions of four-quark operators to the triplet of doubly charmed baryons

\[ \Gamma_{NL}^{\Xi_{cc}^+} = (c^4 + s^4) P_{int}(x) + c^2 s^2 (1 + \tilde{P}_{int}(x)) \Gamma_{int}^-; \]
\[ \Gamma_{NL}^{\Xi_{cc}^+} = (c^4 P_{ex}(x) + c^2 s^2) \Gamma_{ex} + (s^4 P_{int}(x) + c^2 s^2) \Gamma_{int}^-; \]
\[ \Gamma_{NL}^{\Omega_{cc}^+} = (c^4 + c^2 s^2 P_{int}(x)) \Gamma_{int}^+ + (c^2 s^2 P_{ex}(x) + s^4) \Gamma_{ex}. \] (8)
Here $s^2$ and $c^2$ stand for $\sin^2 \theta_c$ and $\cos^2 \theta_c$, respectively, and $|\psi(0)|$ denotes the baryon wave function.

In the calculation of the baryon wave function the application of the nonrelativistic quark model \(^8\) gives

$$|\psi(0)|^2 = \frac{2}{3} |\psi(0)|^2_D = \frac{2 f_D^2 M_D \kappa^{-4/9}}{12}. \quad (9)$$

In the calculations, the physical value of $f_D$ has been used (instead of the static value $F_D$), consistent with the considerations on the "mesonic" nature of doubly heavy baryons \(^4\). \(^9\). Since the calculation of this wave function relies upon the nonrelativistic quark model, the scale at which the contribution of four-quark operators is defined is lowered to the typical hadronic scale ($\mu \sim 0.5 - 1 \text{ GeV}$) by the process of hybrid renormalization, as quark models are supposed to work best at these scales.

In the calculations of inclusive decay rates all Cabibbo modes (including the suppressed ones) were taken into account. QCD corrections \(^10\), \(^11\), \(^12\) were calculated for the case of decay diagrams, while the masses of the particles in the final states were accounted for by the inclusion of appropriate mass corrections \(^13\), \(^14\).

### 3 Semileptonic inclusive rates and lifetimes - results and discussions

Numerical results are given in Table 1, together with the set of parameters used in numerical calculations. The complete list of parameters can be found in \(^4\). Numerical results show that the dependence on $\mu$ and $\Lambda_{QCD}$ is very weak in the case of $\Xi^{++}_{cc}$, while it is negligible in the case of $\Xi^{++}_{cc}$ and $\Omega^{++}_{cc}$.

The dependence of semileptonic and nonleptonic decay rates on the baryonic wave function at the origin is shown in Figure 1. In the case of $\Omega^{++}_{cc}$, the large contributions of four-quark operators make $\Gamma_{SL}(\Omega^{++}_{cc})$ and $\Gamma_{NL}(\Omega^{++}_{cc})$ strongly $|\psi(0)|^2$ dependent.

In the case of nonleptonic total decay rates, our choice of $f_D$ in calculation of $\psi(0)$ is numerically confirmed, because the choice of static value $F_D$ would cause the $\Gamma_{NL}(\Xi^{++}_{cc})$ to become negative, what is an obviously unphysical result. One can summarize numerical results in the form of hierarchies of relevant calculated quantities. The relation for semileptonic branching ratios is

$$BR_{SL}(\Xi^{++}_{cc}) \ll BR_{SL}(\Omega^{++}_{cc}) < BR_{SL}(\Xi^{++}_{cc}), \quad (10)$$
Table 1. Predictions for nonleptonic widths, semileptonic widths, semileptonic branching ratios (for one lepton species) and lifetimes of doubly charmed baryons for the values of the parameters $m_c = 1.35 \text{GeV}$, $\mu = 1 \text{GeV}$, $\Lambda_{QCD} = 300 \text{MeV}$, $f_D = 170 \text{MeV}$.

<table>
<thead>
<tr>
<th></th>
<th>$\Xi_{cc}^{++}$</th>
<th>$\Xi_{cc}^+$</th>
<th>$\Omega_{cc}^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{NL}$</td>
<td>0.655</td>
<td>4.699</td>
<td>2.394</td>
</tr>
<tr>
<td>$\Gamma_{SL}$</td>
<td>0.151</td>
<td>0.166</td>
<td>0.454</td>
</tr>
<tr>
<td>$BR_{SL}$</td>
<td>15.8</td>
<td>3.3</td>
<td>13.7</td>
</tr>
<tr>
<td>Lifetimes in ps</td>
<td>1.05</td>
<td>0.20</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Figure 1. Dependence of semileptonic and nonleptonic decay widths on the value of the wave function squared. The vertical line represents the $|\psi(0)|^2$ used in calculations, which corresponds to $f_D = 170 \text{MeV}$.

while the lifetimes satisfy the following pattern:

$$\tau(\Xi_{cc}^{++}) \sim \tau(\Omega_{cc}^+) \ll \tau(\Xi_{cc}^+) .$$

(11)

The lifetime of $\Xi_{cc}^{++}$ is strongly prolonged by the inclusion of the negative Pauli interference effects.

Finally, one can estimate the lifetime of the triply charmed baryon $\Omega_{ccc}^{++}$ using the analogous procedure applied in the preceding consideration. Since there are no light valence quarks in the structure of $\Omega_{ccc}^{++}$, four-quark operators...
give no contribution, and the lifetime can be estimated using only the leading term in the $1/m_c$ expansion. Such an approach gives

$$\tau(\Omega_{c+c}^{++}) = 0.43 \text{ ps}. \quad (12)$$

4 Conclusions

An interesting hierarchy of semileptonic branching ratios and lifetimes of doubly charmed baryons has been predicted. The large spread of results indicates the importance of preasymptotic effects. The level of agreement of these predictions with experimental results of future experiments will measure the applicability of the formalism to doubly heavy systems and will possibly shed some light on the validity of some underlying assumptions, such as quark-hadron duality.

Acknowledgements. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 00980102

References

5. B. Guberina, B. Melić, H. Stefančić, to appear in Erratum