Stationary Velocity and Charge Distributions of Grains in Dusty Plasmas

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Within the kinetic approach velocity and charge distributions of grains in stationary dusty plasmas are calculated and the relations between the effective temperatures of such distributions and plasma parameters are established. It is found that the effective temperature which determines the velocity grain distribution could be anomalously large due to the action of accelerating ionic bombarding force. The possibility to apply the results obtained to the explanation of the increasing grain temperature in the course of the Coulomb-crystal melting by reduction of the gas pressure is discussed.

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Recently much attention has been payed to theoretical studies of various problems of dusty plasma physics associated with grain dynamics and grain charging (formation and melting of dusty crystals, influence of charging on effective grain interaction, dust-acoustic wave excitation, effect of grain charging on fluctuations and electromagnetic wave scattering in dusty plasmas, etc.). In such studies it is convenient to treat the grain charge as a new variable (as was done for the first time in Ref. [1]). This makes it possible to statistically describe the grain charge distribution on equal footing with the spatial and velocity grain distributions. Obviously, it is very important to know what are the stationary (quasiequilibrium) grain distributions and what is the relation of these distributions to plasma parameters. In spite of the fact that statistical descriptions of dusty plasmas have been already used in many papers, as far as the authors of this letter know neither grain charge, nor velocity distributions for grains were studied within a consistent kinetic approach. Usually, the problem is avoided by neglecting the thermal dispersion of grain velocity and charge. In many cases this is a rather reasonable approximation, but it could not be valid when the properties of the grain subsystem and its dynamics are concerned.

The purpose of the present paper is to describe stationary velocity and charge distributions of grains in dusty plasmas in the case of grain charging by plasma currents and to determine the dependences of effective temperatures on plasma parameters. We study dusty plasma consisting of electrons, ions, neutral molecules and monodispersed dust particles (grains) assuming that every grain absorbs all encountered electrons and ions. Such collisions we define as charging collisions. Collisions in which plasma particles do not touch the grain surface we call Coulomb elastic collisions. Notice that the cross-sections of charging collisions are also determined by the Coulomb forces along with the geometrical size of grains.

Using the microscopic equations for dusty plasmas and the relevant BBGKY-hierarchy [2] it is possible to show that in the case of dominant influence of charging collisions the kinetic equation for the grain distribution function $f_g(X,t) \equiv f_g(r,v,q,t)$ ($q$ is the charge of the grain)
can be written as

\[
\left\{ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{q}{m_g} \mathbf{E} \cdot \nabla \right\} f_g(X,t) = - \sum_{\sigma=\epsilon,i} \int d\mathbf{v}' |\sigma_{g\sigma}(q, \mathbf{v} - \mathbf{v}')| \mathbf{v} - \mathbf{v}' f_g(X,t) \\
- \sigma_{g\sigma}(q - \epsilon_{\sigma}, \mathbf{v} - \mathbf{v}' - \delta \mathbf{v}_{\sigma}) |\mathbf{v} - \mathbf{v}' - \delta \mathbf{v}_{\sigma}| f_g(\mathbf{r}, \mathbf{v} - \delta \mathbf{v}_{\sigma}, q - \epsilon_{\sigma}, t) \right) f_{\sigma}(\mathbf{r}, \mathbf{v}', t),
\]

where \( \sigma_{g\sigma}(q, \mathbf{v}) \) is the cross-section for charging:

\[
\sigma_{g\sigma}(q, \mathbf{v}) = \pi a^2 \left( 1 - \frac{2 \epsilon_{\sigma} q}{m_{\sigma} v^2 a} \right) \theta \left( 1 - \frac{2 \epsilon_{\sigma} q}{m_{\sigma} v^2 a} \right),
\]

\( \theta(x) \) is the Heaviside step function, \( a \) is the grain radius, \( f_{\sigma}(\mathbf{r}, \mathbf{v}, t) \) is the plasma particle distribution function normalized by the particle density \( n_{\sigma} \), \( \delta \mathbf{v}_{\sigma} \equiv \left( m_{\sigma}/m_g \right) \mathbf{v}' \) is the grain velocity change due to the collision with a plasma particle, subscript \( \sigma \) labels plasma particle species, the rest of the notations is traditional. Eq. (1) could be introduced also on the basis of physical arguments as was done in Refs. [3,4]. In fact, the right-hand part of Eq. (1) describes the balance between the grains coming out from the phase volume element and those incoming to the same element due to charging collisions.

Taking into account the smallness of \( \epsilon_{\sigma} \) and \( \delta \mathbf{v}_{\sigma} \) it is possible to expand the right-hand part of Eq. (1) into a power series of these quantities. With the accuracy up to the second order Eq. (1) in the stationary isotropic and homogeneous case is reduced to

\[
\frac{\partial}{\partial t} \left[ \frac{\partial}{\partial \mathbf{v}} \left( D_{\parallel} f_g(\mathbf{v}, q) \right) + \beta \mathbf{v} f_g(\mathbf{v}, q) + \frac{\partial}{\partial q} \left( q \gamma \mathbf{v} f_g(\mathbf{v}, q) \right) \right] \\
+ \frac{\partial}{\partial q} \left( Q f(\mathbf{v}, q) \right) - I f_g(\mathbf{v}, q) = 0,
\]

where \( D_{\parallel}, \beta, Q, \gamma, I \) are the Fokker-Planck kinetic coefficients generated by charging collisions and given by

\[
D_{\parallel} \equiv \frac{1}{2} \left( \frac{m_{\sigma}}{m_g} \right)^2 \int d\mathbf{v}' \frac{(\mathbf{v} \cdot \mathbf{v}')^2}{v^2} |\mathbf{v} - \mathbf{v}'| \sigma_{g\sigma}(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{r}, \mathbf{v}') \\
\beta \equiv \beta(q, v) = - \sum_{\sigma=\epsilon,i} \frac{m_{\sigma}}{m_g} \int d\mathbf{v}' \frac{\mathbf{v} \cdot \mathbf{v}'}{v^2} |\mathbf{v} - \mathbf{v}'| \sigma_{g\sigma}(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{r}, \mathbf{v}') \\
\gamma \equiv \gamma(q, v) = \sum_{\sigma=\epsilon,i} \frac{m_{\sigma}}{m_g} \frac{\epsilon_{\sigma}}{q} \int d\mathbf{v}' \frac{\mathbf{v} \cdot \mathbf{v}'}{v^2} |\mathbf{v} - \mathbf{v}'| \sigma_{g\sigma}(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{r}, \mathbf{v}') \\
Q \equiv Q(q, v) = \sum_{\sigma=\epsilon,i} \frac{\epsilon_{\sigma}^2}{2} \int d\mathbf{v}' |\mathbf{v} - \mathbf{v}'| \sigma_{g\sigma}(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{r}, \mathbf{v}') \\
I \equiv I(q, v) = \sum_{\sigma=\epsilon,i} \epsilon_{\sigma} \int d\mathbf{v}' |\mathbf{v} - \mathbf{v}'| \sigma_{g\sigma}(q, \mathbf{v} - \mathbf{v}') f_{\sigma}(\mathbf{r}, \mathbf{v}').
\]

The quantities \( D_{\parallel}(q, v) \) and \( Q(q, v) \) characterize the grain diffusion in the velocity and charge space, respectively, \( \beta(q, v) \) and \( \gamma(q, v) \) are the friction coefficients which determine the bombardment force \( \mathbf{F}_B(q, v) = -m_g \beta(q, v) \mathbf{v} \) associated with charging collisions and the correction to this force \( \delta \mathbf{F}_B(q, v) = -m_g \gamma(q, v) \mathbf{v} \) due to the mutual influence of the charge and velocity grain distributions, \( I \) is the grain charging current. Deriving the relation for \( \beta(q, v) \) we omit the terms of higher order in \( (m_{\sigma}/m_g) \) associated with the tensor nature of the diffusion coefficient in velocity space (contribution of the transverse diffusion coefficient). With regard for
the fact that $|I(q, v)/Q(q, v)| \to \infty$ at $e_\sigma \to 0$ and $|\beta(q, v)/D_{\parallel}(q, v)| \to \infty$ at $(m_\sigma/m_g) \to 0$, it is possible to show that the asymptotical solution of Eq. (3) can be written as

$$f_g(q, v) = n_0 Z^{-1} Q^{-1}(q, v) e^{-W(q, v)/\lambda v^2} D_{\parallel}^{-1}(q, v) e^{-V(q, v) + \epsilon \delta q(v)},$$

where

$$W(q, v) = -\int_0^q dq'/ I(q', v) Q(q', v),$$

$$V(q, v) = \int_0^v dq' D_{\parallel}(q, v) \left\{ \beta(q, v') + \frac{\partial}{\partial q}(q\gamma(q, v')) - q\gamma(q, v') \right\} \frac{\partial W(q, v')}{\partial q} \frac{\partial \delta q}{\partial q}$$

+ $Q^{-1}(q, v') \frac{\partial Q(q, v')}{\partial q} \right\} + \lambda = 2\sqrt{2}\pi \beta$$

+ $Q^{-1}(q, v') \frac{\partial Q(q, v')}{\partial q} \right\} + \lambda = 2\sqrt{2}\pi \beta$

$\delta q(v) = q - q(v), \quad \beta = \frac{1}{2v} \left\{ \frac{1}{Q(q, v)} \frac{\partial Q(q, v)}{\partial v} + \frac{\partial W(q, v)}{\partial v} + \epsilon \frac{\partial q}{\partial v} \right\}.$

Eqs. (5)–(8) give the asymptotically exact solution of Eq. (3) at $(m_\sigma/m_g) \to \infty$. Further estimates require the explicit form of the kinetic coefficients. Assuming that plasma particle distributions are Maxwellian, one obtains the following stationary grain distribution with accuracy up to the zeroth order in $(q m_i/e \sigma m_g)$:

$$f_g(q, v) = n_0 Z^{-1} Q^{-1}(q, v) e^{-m_\sigma v^2/2\epsilon m_i e^2} Q^{-1}(q, v) e^{- \frac{(q-gm)}{2\epsilon m_i e^2}},$$

where

$$T_{\text{eff}}(q) = \frac{2T_i(t + z)}{t - z + \frac{(q-gm)}{q_0} z[1 + \frac{t-z}{t+z} (1 + \frac{2Z_i}{1+Z_i}(1 + t + z))]}$$

$$\tilde{T}_{\text{eff}} = \frac{2}{1+Z_i} \frac{1+t+z}{t+z} T_e,$$

and

$$D_{\parallel}(q, v) \simeq D_0 \left[ 1 + \frac{q-q_0}{q_0} \frac{z}{t+z} \right] \left( 1 + \frac{z}{t} \right)$$

$$Q(q, v) = Q_0 \left[ 1 - \frac{q-q_0}{q_0} \frac{z(t+z-Z_i)}{(t+Z_i)(1+Z_i)} \right] (t+z)(1+Z_i)$$

$$D_0 = \frac{4}{3} \sqrt{2\pi} \left( \frac{m_i}{m_g} \frac{T_i}{m_g} \right) a^2 n_i S_i$$

$$Q_0 = \sqrt{2\pi} \left( \frac{T_e}{T_i} \right) e^2 a^2 n_i S_i.$$
\( n_{0g} \) is the averaged number density of grains. Here, we use the notation

\[
z = \frac{e^2 Z_g}{aT_e}, \quad t = \frac{T_i}{Z_i T_e}, \quad S_i^2 = \frac{T_i}{m_i}, \quad Z_g = \frac{q_0}{e^i}, \quad Z_i = |\frac{e_i}{e^i}|.
\]

The quantity \( q_0 \) is the equilibrium grain charge of stationary particles satisfying the equation

\[
I(q_0, 0) = 2\sqrt{2\pi} a^2 e^i n_i S_i \left[ 1 + \frac{z}{t} - \left( \frac{m_i}{m_e} \right)^{1/2} \left( \frac{T_i}{T_e} \right)^{1/2} \frac{n_e}{Z_i n_i} e^{-z} \right] = 0.
\]  

(13)

For typical values of plasma parameters in dusty plasma experiments (\( t + z > 1 \)) and \( Z_i = 1 \) we have:

\[
\tilde{T}_{eff} \simeq T_e.
\]

In such case the thermal variation of the grain charge \(|q - q_0|^2\) is of the order of \( aT_e \) and \(|q - q_0|z \sim q_0 \sqrt{e^2/aT_e} \). This means that at weak plasma coupling defined with the grain size \((e^2/aT_e \ll 1)\) the effective temperature of the grain thermal motion \( T_{eff}(q) \equiv T_{eff} \) reduces to

\[
T_{eff} \simeq 2T_i \frac{t + z}{t - z}
\]  

(14)

and

\[
D(q, v) \simeq D_0 \left( 1 + \frac{z}{t} \right), \quad Q(q, v) \simeq Q_0 (t + z) (1 + Z_i).
\]

Thus, in such case

\[
f_g(v, q) = \frac{n_{0g}}{\sqrt{2\pi aT_{eff}}} e^{-\frac{(q - q_0)^2}{2aT_{eff}}} \left( \frac{m_g}{2\pi T_{eff}} \right)^{3/2} e^{-\frac{m_g v^2}{2T_{eff}}}.
\]  

(15)

This distribution describes the equilibrium Maxwellian velocity distribution and the Gibbs grain charge distribution with the temperatures \( T_{eff} \) and \( \tilde{T}_{eff} \) respectively. In fact, the electric energy of charge variations of the electric capacity \( a \) is equal to \((q - q_0)^2/2a \) and thus, the charge distribution described by Eq. (15) can be interpreted as an equilibrium distribution with effective temperature \( \tilde{T}_{eff} \). At \( t < 1, \ z < 1 \) the effective \( \tilde{T}_{eff} \) exceeds the electron temperature. The resulting velocity distribution is described by the effective temperature \( T_{eff} \). Even in the case of neutral grains \((z = 0)\) this temperature is equal to \( 2T_i \). The presence of the factor 2 is associated with plasma particle absorption by grains.

Charging collisions are inelastic and a part of the kinetic energy of the ions is transformed into additional kinetic energy of the grains. This is the difference between the case under consideration and conventional Brownian motion where the velocity distribution is described by the temperature of the bombarding light particles. Eq. (14) shows that the effective temperature of thermal grain motion could be anomalously high at \( z \rightarrow t \). Physically it can be explained by the decrease of the friction coefficient with increase of grain charge

\[
\beta(q, v) \simeq \frac{2}{3} \sqrt{2\pi} \left( \frac{m_\sigma}{m_g} \right) a^2 n_i S_i \left( 1 - \frac{z}{t} \right) = \beta_0 \left( 1 - \frac{z}{t} \right)
\]

The reason is that the difference between the fluxes of ions bombarding the grain surface antiparallel to the grain motion and parallel decreases with the charge increase due to the
specific properties of the ionic charging cross-section, which charge-dependent part is larger for ions moving with smaller relative velocities (i.e. in parallel direction). The condition \( z = t \) corresponds to the zero value of the friction force.

Eq. (3) and its solutions (5), (9), (15) were obtained under the assumption that the Coulomb elastic collisions could be neglected. In order to take elastic collisions into consideration Eqs. (1), (3) should be supplemented by the appropriate collision terms, for example, by the Landau, or Balescu-Lenard collision integrals. We use the Balescu-Lenard collision integral in the Fokker-Planck form which in the case under consideration (isotropic spatially homogeneous stationary distribution) can be written as

\[
\left( \frac{\partial f_g}{\partial t} \right)_C = \frac{\partial}{\partial v} \left[ \frac{\partial}{\partial v} (D_{\|C}(q,v) f_g(v,q)) + v \beta_C(q,v) f_g(q,v) \right],
\]

where \( D_{\|C}(q,v) \) and \( \beta_C(q,v) \) are the Fokker-Planck coefficients related to Coulomb elastic collisions (see, for example, [5], Chapter 8). With the accuracy up to the dominant logarithmic terms (in this approximation Eq. (16) is reduced to the Landau collision term) such coefficients can be reduced to

\[
D_{\|C}(q,v) \simeq \frac{4\sqrt{2\pi q^2}}{3m_g^2} \sum_{\sigma = e,i} n_{\sigma} \frac{e^2}{S_\sigma} \ln \Lambda_\sigma \left( 1 - \frac{v^2}{5S_\sigma^2} \right)
\]

\[
\beta_C(q,v) \simeq \frac{4\sqrt{2\pi q^2}}{3m_g} \sum_{\sigma = e,i} \frac{n_{\sigma} e^2}{S_\sigma^2 m_\sigma} \ln \Lambda_\sigma \left( 1 - \frac{v^2}{5S_\sigma^2} \right), \quad S_\sigma = \left( \frac{T_\sigma}{m_\sigma} \right)^{1/2}.
\]

In Eqs. (16), (17) we again neglect the contribution of the transverse part of the diffusion coefficient which gives a correction to \( \beta_C(q,v) \) of higher order in \((m_\sigma/m_g)\) and we disregard the grain-grain Coulomb collisions, assuming the grain density to be small \((n_g < n_i(Z_i/Z_g)^2(S_g/S_i)^{1/2}(T_g/T_i))\).

We introduced also the Coulomb logarithms \( \ln \Lambda_\sigma \) for each particle species. Usually these quantities are estimated as \( \ln \Lambda_\sigma = \ln(k_{\text{max}}/k_D) \), where \( k_D = r_D^{-1} = (\sum(4\pi e^2_n/\sigma/T_\sigma)^{1/2} \text{ and } k_{\text{max}} \text{ is the inverse distance of closest approach between colliding particles,} \)

\[
k_{\text{max}} \sim \frac{m_\sigma v^2}{|\epsilon_\sigma q|} \sim \frac{3T_\sigma}{|\epsilon_\sigma q|} = r_{L\sigma}^{-1}
\]

\((r_{L\sigma} \text{ is Landau length}). \) However, in the case of plasma particle collisions with finite-size grains this estimate could be invalid, since at \( r_{L\sigma} < a \) the Coulomb logarithm will include the contribution of collisions with particles reaching the grain surface, i.e. charging collisions.

An approximate modification of \( \Lambda_\sigma \) is achieved by treating \( \ln \Lambda_\sigma \) as a logarithmic factor appearing in the momentum transfer cross-section for Coulomb collisions. In the case of finite size grains one obtains the following logarithmic factor

\[
\ln \Lambda_\sigma = \ln \left( \sin \frac{\chi_{\text{max}}}{2} / \sin \frac{\chi_{\text{min}}}{2} \right),
\]

where \( \chi_{\text{max}} \) and \( \chi_{\text{min}} \) are the scattering angles related to the minimal and maximal impact parameters \( b_{\text{min}} \) and \( b_{\text{max}} \) by the Rutherford formula. Obviously, \( b_{\text{min}} \) should be determined from the condition that the distance of closest approach is equal to \( a \) implying

\[
b_{\text{min}} = a \sqrt{1 - \frac{2e_\sigma q}{m_\sigma v^2}} \theta(1 - \frac{2e_\sigma q}{m_\sigma v^2}).
\]
Concerning the quantity $b_{\text{max}}\sigma$, it is reasonable to put $b_{\text{max}}\sigma = r_D + a$ instead of $b_{\text{max}}\sigma = r_D$, since in the case of a finite size grain its screened potential is given by the DLVO-potential

$$\Phi(r) = \frac{q}{r}(1 + \frac{a}{r_D})^{-1} e^{-(r-a)/r_D},$$

rather than the Debye potential.

As a result we have

$$\ln \Lambda_i = \frac{1}{2} \ln \left( \frac{(r_D + a)^2 + r_{Li}^2}{(r_{Li} + a)^2} \right)$$

As is seen, at $r_{Li} \gg r_D$ the ionic Coulomb logarithm can be a small quantity in contrast to the case of ideal plasmas.

Comparing Eqs. (3) and (16) it is easy to see that in order to take elastic Coulomb collisions into account it is sufficient to make the following replacements in the obtained solutions

$$D_{\parallel}(q,v) \rightarrow \tilde{D}_{\parallel}(q,v) = D_{\parallel}(q,v) + D_{\parallel C}(q,v)$$

$$\beta(q,v) \rightarrow \tilde{\beta}(q,v) = \beta(q,v) + \beta_C(q,v).$$

In the case of weak plasma coupling ($e^2/aT_e \ll 1$)

$$\tilde{D}_{\parallel}(q,v) \simeq D_0 \left( 1 + \frac{z}{l} + \frac{z^2}{l^2} \ln \Lambda_i \right)$$

$$\tilde{\beta}(q,v) \simeq \beta_0 \left( 1 - \frac{z}{l} + 2 \frac{z^2}{l^2} \ln \Lambda_i \right),$$

Thus, the correction produced by the elastic collisions could be of the same order as that due to charging collisions. The condition for dominant influence of charging collisions is

$$\left| 1 - \frac{z}{l} \right| > 2 \frac{z^2}{l^2} \ln \Lambda_i,$$

which can be realized at small values of $z/t$, or at $z|t| > r_D^2/a^2 (r_{Li} \gg r_D^2/a).$

Rigorously speaking Eq. (16) and thus Eq. (22) are definitely valid in the case of weak coupling plasmas ($r_{Li} \ll r_D$) since this is the condition of the derivation of the Balescu-Lenard (or, Landau) collision term. However, it is possible to expect that actually the domain of validity of Eqs. (16), (22) is not too strongly restricted by such condition. This assumption is in agreement with the direct calculations of the friction coefficient (Coulomb collision frequency) in terms of the binary collision cross-sections. Beside that, as it was shown in Ref. [6], in the case of strong grain-plasma coupling the influence of the Coulomb collision is also small and the kinetic equation is reduced again to Vlasov equation. This means that fluctuation evolution equations, which solutions determine the explicit form of the Balescu-Lenard collision term, are the same as in the case of weakly coupled plasmas and thus Eq. (16) continues to be valid.
The new kinetic coefficients give the following effective temperature for thermal grain motion

\[ T_{\text{eff}} = T_i \frac{2 \left( 1 + \frac{z}{t} + \frac{z^2}{t^2} \ln \Lambda_i \right)}{1 - \frac{z}{t} + 2 \frac{z^2}{t^2} \ln \Lambda_i}, \]

i.e. elastic collisions can produce a saturation of the grain temperature. However, in the case of dominant influence of charging collision \( T_{\text{eff}} \) can be still anomalously large. This fact can be used for a qualitative explanation of the experimentally observed grain temperatures which are usually much higher than the ion temperature, \( T_g \gg T_i \) (see, for example [7,8], \( T_i \sim 0.1 \text{ eV}, T_g \sim 4 \div 40 \text{ eV} \)). Finally, we point out that the obtained results can be modified also for the case of a plasma with a neutral component. It is possible to introduce an additional collision term along with the term (16). Since the collision integral describing elastic collisions of neutrals with grains also can be represented in the Fokker-Planck form (it follows from the Boltzmann collision integral) the presence of neutrals results in new additions to \( \tilde{D}_\parallel \) and \( \tilde{\beta} \), namely

\[
\tilde{D}_\parallel(q, v) = D_0 \left( 1 + \frac{z}{t} + \frac{z^2}{t^2} \ln \Lambda_i + \frac{n_n}{n_i} \left( \frac{m_n}{m_i} \right)^{1/2} \left( \frac{T_n}{T_i} \right)^{3/2} \right),
\]

\[
\tilde{\beta}(q, v) = \beta_0 \left( 1 - \frac{z}{t} + 2 \frac{z^2}{t^2} \ln \Lambda_i + 2 \frac{n_n}{n_i} \left( \frac{m_n}{m_i} \right)^{1/2} \left( \frac{T_n}{T_i} \right)^{1/2} \right).
\]

As a result the effective temperature is modified into

\[
T_{\text{eff}} = 2T_i \frac{\left( 1 + \frac{z}{t} + \frac{z^2}{t^2} \ln \Lambda_i + \frac{n_n}{n_i} \left( \frac{m_n}{m_i} \right)^{1/2} \left( \frac{T_n}{T_i} \right)^{3/2} \right)}{\left( 1 - \frac{z}{t} + 2 \frac{z^2}{t^2} \ln \Lambda_i + 2 \frac{n_n}{n_i} \left( \frac{m_n}{m_i} \right)^{1/2} \left( \frac{T_n}{T_i} \right)^{1/2} \right)}. \tag{25}
\]

According to Eq. (25) the effective temperature increases with decreasing neutral density. The influence of neutral density changes on the effective temperature would be especially important at \( 1 - \frac{z}{t} + 2 \frac{z^2}{t^2} \ln \Lambda_i \approx 0 \). In such a case a decrease of the neutral gas pressure can produce an anomalous growth of \( T_{\text{eff}} \). That is in qualitative agreement with the experimental observation of melting of dusty crystals by reduction of the gas pressure [7,8].

The obtained results show that stationary velocity and charge grain distributions are described by effective temperatures different from those of the plasma subsystem. These effective temperatures are determined by the competitive mechanics of collisions: grain-neutral collisions and elastic Coulomb collisions result in the equalization of the effective temperature to the temperature of neutrals, or ions, respectively, while charging collisions can produce anomalous temperature growth. That could be one of the main mechanisms of grain heating.

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