Grand unification using a generalized Yang-Mills theory

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Abstract

Generalized Yang-Mills theories have a covariant derivative that employs both scalar and vector bosons. Here we show how grand unified theories of the electroweak and strong interactions can be constructed with them. In particular the SU(5) GUT can be obtained from SU(6) with SU(5)xU(1) as a maximal subgroup. The choice of maximal subgroup also determines the chiral structure of the theory. The resulting Lagrangian has only two terms, and only two irreducible representations are needed, one for fermions and another for bosons.

1 Introduction

The idea behind Generalized Yang-Mills theories\(^1\) (GYMTs) is as follows: In the usual Yang-Mills theory gauge invariance is assured through the demand that vector gauge fields \(A_\mu\) transform as \(A_\mu \rightarrow UA_\mu U^{-1} - (\partial_\mu U)U^{-1}\). In a GYMT scalar boson fields are considered to be also acceptable as gauge fields, with a transformation law similar to the one just shown. To be able to sum together scalar and vector bosons, it is necessary to express all terms of the theory, both bosonic and fermionic, in the spinorial representation. Technical details on this procedure are left for next section.
As will be seen, a gauge invariant theory is obtained, which we call a GYMT. In the usual Yang-Mills theories gauge bosons are placed in the adjoint representation; in GYMTs whatever Higgs fields are needed are placed there, too. The total number of bosons, vector plus scalar, has to equal the number of group generators, a rather stringent requirement on their numbers. The fermions are placed in another irreducible representation (irrep) of mixed chirality. The structure of the adjoint gauge field matrix is

\[
\begin{pmatrix}
\text{vector bosons} & \text{scalar bosons} \\
\text{scalar bosons} & \text{vector bosons}
\end{pmatrix},
\]

and that of the fermion field matrix is:

\[
\begin{pmatrix}
\text{left fermions} & \text{right fermions} \\
\text{right fermions} & \text{left fermions}
\end{pmatrix}.
\]

The possible dimensions of the blocks that make up the matrices are determined by the maximal subgroups of the gauge group. If we assume it to be \(SU(6)\), then there are three possible maximal subgroups, \(SU(3) \times SU(3) \times U(1)\), \(SU(4) \times SU(2) \times U(1)\), and \(SU(5) \times U(1)\). In this case, for example, the upper left blocks for both matrices would be a square of side 3, 4, and 5, respectively.

The original inspiration for our work is an old idea due to Fairlie and Ne'eman,\(^4\) of using \(SU(2/1)\) as a unification group for the Glashow-Weinberg-Salam (GWS) model, and putting the Higgs fields in the adjoint along with the vector fields. However, in spite of much effort by these and other authors, no success was achieved at the time in these matters. The belief that the gauge group had to be graded\(^1\,^2\) lead to insurmountable difficulties.

It is possible to construct a grand unified theory (GUT) of the strong and electroweak forces using a GYMT.\(^3\) The immediate benefit of this procedure is simplicity. There are only two terms in the Lagrangian, whereas in the usual GUTs there are several. As mentioned, only two irreps are needed, as compared with one or two for the fermions, at least two for the Higgs fields and one for the gauge vector fields, in the case of a usual GUT.

In high energy physics the word “unification” has a very precise meaning: one selects a large Lie group, which contains in a maximal subgroup other small Lie groups that represent the forces that are to be “unified”. Then, through the process of spontaneous symmetry breaking brought about by nonzero vacuum expectation values of Higgs fields, the smaller groups can be recovered and with them the low energy phenomenology. In the case of GYMT grand unification the same process is occurring insofar as one must choose a large Lie group so that it can contain the smaller phenomenological groups, but, besides this process, there is another kind of “unification”, brought about by putting together in one irrep vector and scalar bosons (which in turns permits to put together in the same irrep left and right fermions). To go from this second kind of “unification” to the smaller groups, it is necessary to restrict the gauge group to its maximal subgroup in order to obtain a Yang-Mills theory and be in a more familiar terrain, so to speak.
The “generalized gauge transformation” leaves invariant the Lagrangian of the theory. However, this does not mean that separate parts of the Lagrangian are necessarily kept invariant if they are transformed individually. As an example, consider the fermion matrix (2), whose chiral structure is not left invariant by the most general “generalized gauge transformation”. But the maximal subgroup does leave its chiral structure invariant. In fact, the different maximal subgroups are the possible Yang-Mills theories derivable from a particular GYMT.

A very general “generalized gauge transformation” is not going to help us recover low energy phenomenology, since it leads to expressions we are not accustomed to. The procedure is to obtain the Yang-Mills theory from the GYMT first by restraining the gauge group to a maximal subgroup; then, through the usual spontaneous symmetry breaking processes, obtain the small phenomenological groups. We shall come back to this topic in last section, in relation to the example covered in this paper, $SU(6)$, when the discussion is not so abstract.

2 GYMTs

The first thing we have to do is learn how to write covariant derivatives in the spinorial representation. Let $S$ be an element of the spinor representation of the Lorentz group, so that, if $\psi$ is a spinor, then it transforms as $\psi \rightarrow S\psi$. Then, due to the homomorphism that exists between the vector and spinor representations of the Lorentz group, we have that $A \rightarrow SAS^{-1}$ for a vector $A_\mu$ that has been contracted with the Dirac matrices. It was this homomorphism that allowed Dirac to write a spinorial equation that included the vector electromagnetic field. We have the following

**Theorem.** Let $D_\mu = \partial_\mu + B_\mu$, where $B_\mu$ is some vector field. Then:

$$
(\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) = \frac{1}{8} \text{Tr}^2 \mathcal{D}^2 - \frac{1}{2} \text{Tr} \mathcal{D}^4,
$$

where the traces are to be taken over the Dirac matrices.

We are not going to prove this theorem. Instead, we remit the reader to Ref. 1, where the proof is done in detail. Consider now a Yang-Mills theory that is invariant under a Lie group with $N$ generators. The fermion or matter sector of the non-Abelian Lagrangian has the form $\bar{\psi}i\mathcal{D}\psi$, where $D_\mu$ is a covariant derivative chosen to maintain gauge invariance, and $\psi$ is a chiral fermion multiplet. This multiplet transforms as $\psi \rightarrow U\psi$, where $U = U(x)$ is an element of the fundamental representation of the group. The covariant derivative is $D_\mu = \partial_\mu + A_\mu$, where $A_\mu = igA_\mu^a(x)T^a$ is an element of the Lie algebra and $g$ is a coupling constant. We are assuming here that the matrices $\{T^a\}$, $a = 1, \ldots, N$, constitute a representation of the group generators. Gauge invariance of the matter term is assured if

$$
A_\mu \rightarrow UA_\mu U^{-1} - (\partial_\mu U)U^{-1},
$$

or, what is the same,

$$
A \rightarrow UA^{-1} - (\partial U)U^{-1}.
$$
To construct the GYMT we generalize the transformation of the gauge field as follows: to every generator in the Lie group we choose one gauge field that can be either vector or scalar. Suppose there are \( N \) generators in the Lie group; we choose \( N \) \( V \) generators to be associated with an equal number of vector gauge fields and the other \( N \) \( S \) to be associated with an equal number of scalar gauge fields. Naturally \( N \) \( V \) + \( N \) \( S \) = \( N \). The choice has to be made so that the adjoint can be decomposed in blocks made up solely of vector or scalar fields (with the exception of the fields along the diagonal, that can be either), as shown in (1), with the block size given by the dimension of the subgroups forming the maximal subgroup. We define the generalized covariant derivative \( D \) by taking each one of the generators and multiplying it by one of its associated gauge fields and summing them together. The result is

\[
D \equiv \partial + A + \Phi
\]

where \( A = \gamma^\mu A_\mu = ig\gamma^\mu A^a_\mu T^a \), \( a = 1, \ldots, N_\text{V} \), and \( \Phi = \gamma^5 \varphi = -g\gamma^5 \varphi^b T^b \), \( b = N_\text{V} + 1, \ldots, N \).

We now define the transformation for the gauge fields to be

\[
A + \Phi \rightarrow U(A + \Phi)U^{-1} - (\partial U)U^{-1},
\]

from which one can conclude that \( D \rightarrow UDU^{-1} \). The following Lagrangian is built based on the conditions that it contain only matter fields and covariant derivatives, and possess both Lorentz and gauge invariance:

\[
\mathcal{L}_{NA} = \bar{\psi} i D \psi + \frac{1}{2g^2} \tilde{\text{Tr}} \left( \frac{1}{8} \text{Tr}^2 D^2 - \frac{1}{2} \text{Tr} D^4 \right),
\]

where the trace with the tilde is over the gauge (or Lie group) matrices and the one without it is over matrices of the spinorial representation of the Lorentz group. (For some fermion irreps a small modification is necessary. See Section 5.)

Although we have constructed this non-Abelian Lagrangian based only on the conditions just mentioned, its expansion into component fields results in expressions that are traditional in Yang-Mills theories. The expansion is done in detail in Ref. 1, and results in

\[
\mathcal{L}_{NA} = \bar{\psi} i(\partial + A) \psi - g\bar{\psi} i \gamma^\mu \varphi^b T^b \psi + \frac{1}{2g^2} \tilde{\text{Tr}} (\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^2
\]

\[
+ \frac{1}{g^2} \tilde{\text{Tr}} (\partial_\mu \varphi + [A_\mu, \varphi])^2.
\]

The reader will recognize familiar structures: the first term on the right looks like the usual matter term of a gauge theory, the second like a Yukawa term, the third like the kinetic energy of vector bosons in a Yang-Mills theory and the fourth like the gauge-invariant kinetic energy of scalar bosons in the non-Abelian adjoint representation.
3 A case study: the GWS model

The GWS model can be written\(^1\) as a GYMT using the group \(U(3)\). It is necessary to use this group and not \(SU(3)\), as we shall see. Referring to (9), notice that one can infer, from the terms \(\bar{\psi}iA\psi\) and \(\frac{1}{2g^2}\text{Tr}([A_\mu, A_\nu])^2\), that the quantum numbers of the particles of a GYMT are the same and can be calculated in the same ways as if it were a conventional Yang-Mills theory. In particular, notice that a maximal subgroup, that would only keep diagonal terms in (1), would have the same diagonal generators (and thus the same quantum numbers), as the full group.

Originally, in Ref. 1, the authors calculated the quantum numbers directly from the diagonal generators of the group, but in this paper we use, when possible, group-theoretical techniques. We follow the conventions used in Slansky’s handbook “Group theory for unified model building”\(^5\) except that we shall write hypercharges as subindices and not between parenthesis. In particular we use the relation

\[
Q = T_3 + \frac{1}{2}Y,
\]

with a positive sign for the hypercharge term as in this reference and not as we have done in previous papers, with a minus sign. The group theory results we use are for the most part listed in Slansky’s Table 58 on branching rules for maximal subgroups.

If we try to construct a GYMT of the GWS model using \(SU(3)\), the bosons are placed in the adjoint 8. This irrep has, under the maximal subgroup \(SU(3) \supset SU(2) \times U_Y(1)\), the branching rule \(8 = 1_0 + 2_3 + \bar{2}_3 + 3_0\); that is, the 8 is the direct sum of an isospin singlet with \(Y = 0\), a doublet with \(Y = 3\), and so on. The gauge field matrix \(G_3\), that appears in the covariant derivative \(D_3 = \partial + G_3\) would, omitting all numerical factors, look like

\[
G_3 = \begin{pmatrix}
\bar{B} + A_3 & A_+ & \varphi_1 \\
A_- & \bar{B} - A_3 & \varphi_2 \\
\varphi_1^* & \varphi_2^* & -2B
\end{pmatrix}.
\] (10)

Algebraically, to obtain the quantum number of a field one must take the commutator of the field with the diagonal generator associated with that quantum number. Thus the hypercharges of the gauge fields in \(G_3\) are given by the coefficients of the fields in \([\lambda_8, G_3]\), where \(\lambda_8 \propto \text{diag}(1, 1, -2)\). For the case of \(\varphi_1\) or \(\varphi_2\) one gets \(Y = 3\). Alternatively, one can obtain this result directly and immediately from the branching rule for the 8, since the Higgs fields belong in the 2_3. The problem is that the GWS Higgs field has \(Y = 1\), not \(Y = 3\).

It is not possible to construct a GYMT for the GWS using \(SU(3)\), but with \(U(3)\) it is possible to do it. For in this group there is the additional generator \(\lambda_9 \propto \text{diag}(1, 1, 1)\), and it is possible use it, along with \(\lambda_8\), to form an alternative representation with generators \(\lambda_{10} \propto \text{diag}(1, 1, -1)\) and \(\lambda_0 \propto \text{diag}(1, 1, 2)\), which can be expressed as two orthogonal linear combinations of \(\lambda_8\) and \(\lambda_9\). We define \(\lambda_0\) to be the correct hypercharge generator, and with it the value \(Y = 1\) is obtained for the hypercharge of the Higgs. With this hypercharge we have a natural choice for the
fermions of the GWS model:

\[
\psi = \begin{pmatrix}
    e^c_R \\
    \nu^c_R \\
    e^c_L
\end{pmatrix}.
\]

This choice of hypercharge gives all the particles of the GWS model with their correct quantum numbers. We do not obtain the Higgs potential energy density \( V(\varphi) \) from GYMTs.

It seems peculiar that one has to go to an unusual representation of \( U(3) \) to be able to construct the GYMT. The explanation for this peculiarity becomes evident if one considers grand unified GYMTs. The group \( SU(6) \) has the maximal subgroup \( SU(6) \supset SU(3) \times SU(3) \times U(1) \), which, in a way, seems the logical one to use since it contains two \( SU(3) \), one for the GWS model and another for chromodynamics. We actually attempted grand unification first this way\(^3\) but we were forced to use an unusual representation of the generators of \( SU(6) \) to obtain the correct quantum numbers of the particles of the Standard Model. We also overlooked, in part due to the difficult algebra, that it is necessary to use a mixed field for the hypercharge generator, as we shall explain next section. Here we shall use a different maximal subgroup, \( SU(6) \supset SU(5) \times U(1) \). With it is not necessary to go to a new representations to get the correct quantum numbers for the vector bosons, Higgs bosons, or fermions of the Standard Model, and the relation between the \( SU(5) \) GUT and \( SU(6) \) GYMT becomes wonderfully transparent.

### 4 The bosons of the \( SU(6) \supset SU(5) \times U(1) \) GYMT GUT

The GYMT based on \( SU(6) \) will use the adjoint 35 for the bosons and the 15 for the fermions. This last irrep is the antisymmetric product of two 6’s, the 6 being the fundamental representation. To obtain the particle spectrum of the theory we use the maximal subgroup \( SU(5) \times U(1) \). The branching rule for the 35 is \( 35 = 1_0 + 5_6 + \bar{5}_{-6} + 24_0 \). The 5 is the fundamental of \( SU(5) \), and the 24 is the adjoint. Schematically, the gauge fields look like this:

\[
G_6 = \begin{pmatrix}
    24 & 5 \\
    5 & 1
\end{pmatrix} = \begin{pmatrix}
    G_5 + \Omega \\
    \varphi^\dagger \\
    -5\Omega
\end{pmatrix}.
\]

Here \( G_6 \) transforms as the adjoint 24 of \( SU(5) \); \( \varphi \), the GWS Higgs field, as the 5; \( \varphi^\dagger \) as the \( \bar{5} \); and \( \Omega \) as the singlet, all according to the branching rule 35 = 1_0 + 5_6 + \bar{5}_{-6} + 24_0. These irreps of the branching of the 35 under \( SU(6) \supset SU(5) \times U(1) \) take the place of the irreps of the normal \( SU(5) \) GUT, with the difference while the GUT Higgs in \( SU(5) \) transforms as another 24 here it does it as the singlet 1_0. While in the usual GUT theory each irrep is arbitrarily chosen, in GYMTs they are basically given by the choice of the gauge group and its maximal subgroup.
The hypercharge generator for the $SU(5)$ GUT is of the form $\text{diag}(-2, -2, -2, 3, 3)$, so for our $SU(6)$ case
\begin{equation}
 Y \propto \text{diag}(-2, -2, -2, 3, 3, 0),
\end{equation}
and we assign to it the vector boson $B$, in a way similar to the way it is done in $SU(5)$ GUT. (We shall see that actually one has to assign to it a mixture of $B$ and $\Omega$, this last being a scalar we proceed to introduce.) From the structure of the maximal subgroup $SU(5) \times U(1)$ one knows that there is a generator
\begin{equation}
 Z \propto \text{diag}(1, 1, 1, 1, 1, -5).
\end{equation}
We shall give the name *ultracharge* to the quantum number associated with the $Z$ diagonal generator. It is necessary to assign a scalar boson, we call $\Omega$, to this generator, since we cannot assign a vector boson to it because in that case it should have had already been observed, being massless. There is no problem assigning a scalar field, for, as can be seen in (9), the terms that would couple scalar bosons among themselves are missing. These terms drop out from Lagrangian (8) for algebraic reasons. This scalar $\Omega$ has to be the same one that generates the large-scale GUTs masses to the leptoquark bosons, since there are no more bosons available. It should have a very large mass, since Higgs bosons are usually assumed to have masses of the order of their vacuum expectation values.

Just as in a normal GUT, phenomenology is obtained through the maximal subgroup $SU(5) \supset SU_I(2) \times SU_C(3) \times U_Y(1)$. The branching of the adjoint is
\begin{equation}
 24 = (1,1)_0 + (3,1)_0 + ( \bar{2},3)_{-5} + (2, \bar{3})_5 + (1,8)_0,
\end{equation}
where each irrep is given in the form $(m,n)_Y$, where $m$ is in $SU_I(2)$ and $n$ in $SU_C(3)$. The bosons contained in $G_5$ are the usual four vector bosons of the GWS model, the eight gluons, and the twelve leptoquark bosons. If we add to these the five complex Higgs fields $\varphi$ and the scalar singlet $\Omega$ we obtain the 35 fields that make up the GYMT adjoint.

While this arrangement seems nicely precise, there is one serious difficulty with it. As can be seen from the branching rule for the 35, the ultracharge of all the bosons is zero, so that, in particular, the leptoquarks do not acquire a very large mass. We propose a simple solution to this problem that is reminiscent of the special representation of the generators of the $U(3)$ group that was necessary to introduce to be able to express the GWS model as a GYMT. It consists in assigning to the hypercharge generator a field $\Xi$ that is the mixture of two fields that have already been introduced:
\begin{equation}
 \Xi = B + \gamma^5\Omega.
\end{equation}
There is no change in the field assigned to the ultracharge generator; it is still the $\Omega$. With this assignment the leptoquarks (and only them) couple to the $\Omega$, as can be seen in (15) from the hypercharge assignments, so they obtain a very large mass.
In next section we study the fermion sector of the model. The reader may perhaps find it unlikely that the scheme we have introduced could work with fermions, first, because the mixed field $\Xi$ would apparently imply the coupling of $\Omega$ to fermions, giving them a mass on the scale of grand unification, and, second, that, while no fermion must couple to the $\Omega$, some of them must necessarily couple to the $\varphi$ bosons to reproduce the GWS model. Actually the fermion terms appear with a serendipitous combination of chiralities that gives all the correct results.

5 The fermions of the $SU(6) \supset SU(5) \times U(1)$ GYMT GUT

The fermions in the $SU(6)$ GYMT GUT are placed in the antisymmetric 15. The branching rule for this irrep for the maximal subgroup $SU(5) \times U(1)$ is $15 = 5_{-4} + 10_2$, where the subindex is the ultracharge. In turn the branchings of these two irreps are $5 = (1, 3)_{-2} + (2, 1)_3$ and $10 = (2, 3)_{+1} + (1, 3)_{-4} + (1, 1)_6$ where, as before, the irreps are expressed in the form $(m, n)_Y$, where $m$ is in $SU_I(2)$ and $n$ in $SU_C(3)$. These two irreps also appear in normal $SU(5)$, and on the basis of the isospin, hypercharge, and color quantum numbers their particle content is $5 \to (d_1, d_2, d_3, e, \nu)_R$ and $10 \to (u_1, d_1, u_2, d_2, u_3, u^{ec}, u^{3c}, e)_L$. The quark colors have been denoted 1, 2 and 3. It is possible to place these two irreps in a single one, using larger unification groups, but then the two irreps should be $\overline{5}$ and $10$, charge conjugating the 5, in order to have the same chiralities. However, in a GYMT GUT just the opposite happens: the two irreps must necessarily have different chiralities to be included in a larger irrep, so that, denoting the chirality of the irrep by a subindex, the $5_R$ and the $10_L$ are just what we need.

The fermion irreps fit into the antisymmetric 15 in the following way:

$$\psi = \begin{pmatrix} 0 & 5_{5R} \\ 10_L & 0 \end{pmatrix}. \quad (17)$$

This field transforms as the antisymmetrized Kronecker product of two fundamentals of $SU(6)$, so that, if $U$ is an element of the fundamental representation, then under a group transformation $\psi \to U \psi U^T$ and $\bar{\psi} \to U^* \bar{\psi} U^{-1}$. Furthermore, the adjoint is the product of the fundamental times its conjugate, $G_6 \to U G_6 U^{-1}$, so that the correct invariant fermion term in this GYMT Lagrangian would need a trace over the gauge group:

$$\overline{\text{Tr}} \ (\bar{\psi} i(\partial + G_6) \psi). \quad (18)$$

It is not necessary to prove that the 5 and 10 predict the correct fermion spectrum, since they are the same irreps that are used in the usual $SU(5)$ GUT. What we do have to verify is that the mixed field $\Xi$ causes no inconsistencies with the fermion phenomenology of the Standard Model. Expanding (18) into all its components, one obtains three types of terms:
1. Type $\bar{\chi}_R \phi \theta_R$ or $\bar{\chi}_L \phi \theta_L$, where $C_\mu$ is any one of the vector bosons of the Standard Model. (The definition of $\bar{\chi}_R$ is $(\chi_R^\dagger \gamma^0)_L$, so it is a spinor with left chirality.) These terms represent the usual couplings of the Standard Model between fermions and vector bosons, including leptoquarks.

2. Type $\bar{\chi}_L \varphi \theta_R$ or $\bar{\chi}_R \varphi \theta_L$, where $\varphi$ is the Higgs that gives mass to the $W^\pm$ and $Z^0$, the carriers of the weak interaction.

3. Type $\bar{\chi}_R \Omega \theta_R$ or $\bar{\chi}_L \Omega \theta_L$, that are zero due to the opposite chiralities. As we said, it was important that these terms cancelled.

Fortunately, in each case the correct combination of chiral spinors has appeared. Any variation would have been phenomenologically unacceptable.

The couplings with the vector bosons are the same as those predicted by the $SU(5)$ GUT. There is a term present in the $SU(5)$ GUT that is missing in this $SU(6) \supset SU(5) \times U(1)$ GYMT. The $SU(5)$ gauge invariance allows two Yukawa terms that couple fermions $5_R$ and the $10_L$ with the Higgs boson $5$. The two invariants are $\bar{\Pi}_L \times 5 \times 5_R$ and $\bar{\Pi}_R \times 5 \times 10_L = 10_L \times 5 \times 10_L$. In this GYMT there is only one term that involves fermions and it is given in (18). The only $SU(6)$ gauge invariant is $\bar{15} \times 35 \times 15$, which contains only the first of the $SU(5)$ invariants. Fortunately all the fermion kinetic energy terms are present in $\bar{\Pi}_L \times 5 \times 5_R$, so what is missing is this GWS Higgs' contribution to the mass of the right quark.

6 Final comments

We have defined GYMTs, and shown that is possible to construct a satisfactory GUT with them. The group $SU(6)$ with the $SU(5) \times U(1)$ maximal subgroup gives results that are very simple to interpret. With $SU(5) \times U(1)$ one obtains all the correct quantum numbers of the particles of the Standard Model right away, without having to perform first a rotation in root space, as it is the case using other maximal subgroups. Furthermore, the relation between the $SU(5)$ GUT and the $SU(6)$ GYMT becomes transparent. To put it bluntly, the first is the block expansion, as dictated by the maximal subgroup, of the matrices of the second. Another important point is that to the $SU(6)$ hypercharge generator a mixture of two fields must be assigned to correctly obtain the Standard Model: the usual hypercharge vector and the GUT Higgs. There are still 35 boson fields for the 35 $SU(6)$ irrep.

One advantage of this formulation of the Standard Model is that it only requires two irreps, one for all the bosons and another for all the fermions. Not only does the GWS Higgs, but also the GUT Higgs, fit in the $SU(6)$'s 35. The Lagrangian of the GYMT has only two terms: the fermion kinetic energy and a term in powers of $D$.

Notice that our motivation to restrict the symmetry to $SU(5) \times U(1)$ is due to the need to obtain a theory one is familiar with, in this case the underlying $SU(5)$ Yang-Mills structure. Then there are two additional spontaneous symmetry breakings, one from $SU(5)$ to $SU(3)_C \times SU(2)_L \times U_Y(1)$ and another from $SU(3)_C \times SU(2)_L \times U_Y(1)$ to $U_{EM}(1)$,
but these are of the usual type, due to nonzero vacuum expectation values of Higgs fields.

We still do not know how to include in a natural way in the model the Higgs potential $V(\varphi, \Omega)$. What we have done is simply to assign a GWS-scale vacuum expectation value to the colorless, chargeless, component of the Higgs field $\varphi$, and a GUT-scale to the Higgs field $\Omega$, which is a colorless and chargeless singlet. Incidentally, the right neutrino cannot be included in a GYMT GUT using $SU(6)$. To include it would necessitate a larger group. As was mentioned at the end of last section, the term that couples the GWS Higgs with the up quark in the $SU(5)$ GUT seems to be missing in this GYMT, so that this quark does not get mass generation from $\varphi$. However the fermionic term that is present does contain the up quark’s kinetic energy term.

Just as in Yang-Mills theories when the covariant derivative acts on a field $X$ its gauge fields acquire certain coefficients called the “charges” of each gauge field with respect to the field $X$, in this generalization the same thing has to be done. Thus, when $D$ acts on the leptonic triplet, its gauge fields are going to be multiplied by constants $Q_V$ and $Q_S$, with the result $D\psi = (\partial + Q_V A + Q_S \Phi)\psi$. From our knowledge of the Standard Model we conclude that $Q_V = 1$ and that there are three $Q_S$, one for each generation, with rather small values.

While it is the maximal subgroup that generates the phenomenology of a GUT GYMT, there is an additional, powerful symmetry present: the gauge group itself, $SU(6)$, in this case. The implications of this symmetry seem to us to be a worthwhile subject of study for the future.

References