The Equation of State of Deconfined Matter at Finite Chemical Potential in a Quasiparticle Description

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Abstract

A quasiparticle description of the thermodynamics of deconfined matter, reproducing both the perturbative limit and nonperturbative lattice QCD data at finite temperature, is generalized to finite chemical potential. By a flow equation resulting from Maxwell’s relation, the equation of state is extended from zero to non-zero quark densities. The impact of the massive strange flavor is considered and implications for cold, charge-neutral deconfined matter in $\beta$-equilibrium in compact stars are given.

Key Words: deconfinement, equation of state, quark stars

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I. INTRODUCTION

The equation of state (EoS) represents an important interrelation of state variables describing matter in local thermodynamical equilibrium. All microscopic characteristics are integrated out and only the macroscopic response to changes of the state variables as, e.g., the temperature $T$ and the chemical potential $\mu$ are retained. Via the Gibbs equation, $e = Ts + \mu n - p$, the energy density $e$ is related to the entropy and particle densities,
\[ s = \frac{\partial p}{\partial T} \] and \[ n = \frac{\partial p}{\partial \mu} \], respectively, and the pressure \( p \). The thermodynamical potential \( p(T, \mu) \) thus provides all information needed to evaluate, e.g., sequences of stellar equilibrium configurations by means of the Tolman-Oppenheimer-Volkov equation, or the evolution of the universe via Friedman’s equations, or the dynamics of heavy-ion collisions within the framework of relativistic Euler equations. In the examples mentioned not only excited hadron matter is of relevance, rather, at sufficiently high density or temperature, a plasma state of deconfined quarks and gluons is the central issue.

Within the quantum field theoretical standard model, the interaction of quarks and gluons is described by Quantum Chromodynamics (QCD). The challenge, therefore, consists in the derivation of the EoS of deconfined matter directly from QCD. In first attempts [1] the EoS of cold quark matter was derived perturbatively up to order \( \mathcal{O}(g^4) \) in the coupling \( g \). Finite temperatures have been considered, where the perturbative expansion is nowadays extended up to the order \( \mathcal{O}(g^5) \) [2]. However, in the physically relevant region, due to the large coupling, perturbative methods seem basically to fail and, consequently, nonperturbative evaluations are needed. Present lattice QCD calculations can accomplish this task, and indeed the EoS of the pure gluon plasma [3,4] and of systems containing two or four light dynamical quark flavors [5–7] are known reliably at finite temperature, and simulations with physical quark masses will possibly be available in the future.

As a matter of fact, the current QCD lattice calculations are yet restricted to zero chemical potential \( \mu \). Attempts to extend the lattice calculations systematically to non-zero chemical potential are under way [8], but focus currently more on conceptional questions than on precise predictions as in the case of \( \mu = 0 \). However, a detailed understanding of the EoS at non-vanishing net quark density is required for a variety of physical questions as, e.g., the structure of quark cores in massive neutron stars, the baryon contrast prior to cosmic confinement, or the evolution of the baryon charge in the mid-rapidity region of central heavy-ion collisions.

Being aware of the urgent need for this EoS, we here suggest an approach, based on a quasiparticle description of quarks and gluons, to map available lattice data from \( \mu = 0 \)
to finite values of \( \mu \) and to small temperatures. Confidence in the model is gained by its quantitative agreement with the latest lattice data, which are analyzed in Sec. II, and by the smooth interpolation to the asymptotic limit. Discussed as well is the connection to the thermodynamic hard-thermal-loop resummation proposed recently [9–11], which provides a more rigorous motivation of our phenomenological approach. In Sec. III, we formulate the extension of the model to finite chemical potential, which does not require any further assumption. Although our approach is formulated in terms of gluon and quark quasiparticles and, hence, designed for ‘conventional’ deconfined matter, there are formal indications of the existence of other QCD phases. In Sec. IV the quark-gluon plasma is considered with the massive strangeness degree of freedom appropriately taken into account. Despite of two free model parameters (which can be fixed when lattice data for the physical (2+1) flavor system will be available), our approach makes some definite statements about the EoS of the quark-gluon plasma. This is shown in a detailed study of the thermodynamics of \( \beta \)-stable deconfined matter, which allows us to draw conclusions about the bulk properties of pure quark stars. Finally, a summary and an outlook are given in Sec. V.

II. THE QUASIPARTICLE MODEL

The concept of describing an interacting system in terms of a system of quasiparticles which, as appropriate ground states, reflect relevant aspects of the interaction by effective properties, is successfully used in many fields of physics. For deconfined QCD plasmas, the development of the quasiparticle idea, starting with [12–14], ran parallel to the improvement of the lattice QCD calculations. These nonperturbative numerical simulations provide the testing ground for the quasiparticle concept which is a phenomenological approach. At the same time, by identifying the quasiparticles with the asymptotically relevant excitations [15], these models yield a physically intuitive picture of the strongly interacting system. Referring to [16] for a recent survey of quasiparticle descriptions of finite-temperature lattice data, we focus in this work on the generalization of the approach to non-zero chemical potential.
A. Formulation of the model

We consider an $SU(N_c)$ plasma of gluons, with $N_c = 3$ for QCD, and $N_f$ quark flavors in thermodynamic equilibrium. Within the approach outlined below, the interacting plasma is described as a system of massive quasiparticles, a picture arising asymptotically from the in-medium properties of the constituents of the plasma. For thermal momenta $k \sim T$, the relevant modes, transversal gluons and quark-particle excitations, propagate predominantly on-shell with dispersion relations $\omega_i^2(k) \approx m_i^2 + k^2$ and

$$m_i^2 = m_{0i}^2 + \Pi_i^*, \quad (1)$$

while the plasmon (longitudinal gluon) and plasmino (quark-hole) states are essentially unpopulated [17]. Neglecting sub-leading effects, the medium contributions $\Pi_i^*$ are the leading order on-shell selfenergies. Depending on the coupling $G^2$, the temperature and chemical potential as well as the rest mass $m_{0i}$ (the latter vanishing for gluons), the $\Pi_i^*$ are given by the asymptotic values of the gauge independent hard-thermal/density-loop selfenergies [17],

$$\Pi_q^* = 2 \omega_{q0} (m_0 + \omega_{q0}), \quad \omega_{q0}^2 = \frac{N_c^2 - 1}{16N_c} \left[ T^2 + \frac{\mu_q^2}{\pi^2} \right] G^2,$$

$$\Pi_g^* = \frac{1}{6} \left[ \left( N_c + \frac{1}{2} N_f \right) T^2 + \frac{N_c}{2\pi^2} \sum_q \mu_q^2 \right] G^2. \quad (2)$$

We emphasize that the contributions $\Pi_i^*$ are generated dynamically by the interaction within the medium, so the quantities $m_i$ in (1) can be considered only as effective masses, with no ambiguities arising for the gauge boson quasiparticles.

Generalizing the approach of Ref. [18] to a finite chemical potential $\mu$ controlling a conserved particle number, the pressure of the system can be decomposed into the contributions of the quasiparticles and their mean field interaction $B$,

$$p(T, \mu; m_{0i}^2) = \sum_i p_i(T, \mu_i(\mu); m_i^2) - B(\Pi_i^*), \quad (3)$$

where $p_i = \pm d_i T \int d^3k/(2\pi)^3 \ln(1 \pm \exp\{- (\omega_i - \mu_i)/T\})$ is the pressure of an ideal gas of bosons or fermions with degeneracy $d_i$ and the state dependent effective masses (1,2). By the
general stationarity of the thermodynamic potential $\Omega = -pV$ under functional variation of the selfenergies [19], which in the present approach simplifies to $\partial p/\partial \Pi_j = 0$, $B$ is related to the quasiparticle masses,

$$\frac{\partial B}{\partial \Pi_j} = \frac{\partial p_j(T, \mu_j; m_j^2)}{\partial m_j^2}. \quad (4)$$

We remark that the stationarity of the quasiparticle pressure leads, by the Feynman-Hellmann relation, to a chiral condensate $\langle \bar{\psi} \psi \rangle \sim m_{0q}$, so the restoration of chiral symmetry of massless quark flavors in the deconfined phase is inherent in the approach. Furthermore, the stationarity implies that the entropy and the particle densities are given by the sum of the quasiparticle contributions,

$$s_i = \frac{\partial p_i(T, \mu_i; m_i^2)}{\partial T} \bigg|_{m_i^2}, \quad n_i = \frac{\partial p_i(T, \mu_i; m_i^2)}{\partial \mu_i} \bigg|_{m_i^2}. \quad (5)$$

This quasiparticle approach reproduces the leading-order perturbative results for arbitrary values of $N_c$ and $N_f$. Moreover, however, it represents a thermodynamically consistent effective resummation of the leading-order thermal contributions [20]. As a nonperturbative approach, hence, it can be expected to be more appropriate in the large-coupling regime than perturbative predictions which, as truncated power expansions, fail to be reliable there. More specifically, the effective quasiparticle description will be an appropriate framework as long as the spectral properties of the relevant excitations do not differ qualitatively from their asymptotic form. Since propagating constituents in a plasma are, on general grounds, expected to possess a spectral representation with the strength predominately accumulated in a quasiparticle peak, our approach may be anticipated to be reasonable even close to the confinement transition. In a way, this expectation has some support from the hot $\phi^4$ theory where the selfconsistent resummation of superdaisy graphs leads to a similar ‘strict-quasiparticle’ picture [21] as introduced above for QCD. By resumming the propagator beyond the leading-loop order, then, the quasiparticle properties were shown to be pronounced even for larger values of the coupling [22]. In a more direct way, the quasiparticle approach can be tested by comparison to the lattice data available for several QCD-model systems at finite temperature.
B. Test of the model

To obtain the EoS as a function of the temperature, $G^2(T)$ has to be specified in Eq. (2). The quantity $G^2$ is to be considered as an effective coupling since it parameterizes all deviations of the exact spectral function from our ‘strict-quasiparticle’ ansatz. On the other hand, to make contact to perturbation theory, it has to approach the running coupling in the asymptotic limit of large temperatures. In the form

$$G^2(T, \mu = 0) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left( \frac{T + T_s}{T_c/\lambda} \right)^2},$$

the shape of the exact spectral function is encoded in the single number $T_s/T_c$, whereas $T_c/\lambda$ is related to the QCD scale $\Lambda_{QCD}$. While the physically intuitive ansatz (6) turns out to be applicable to all systems considered in the following, other parameterizations are conceivable as well [16].

As a first example, the quasiparticle model is applied to the case of the pure SU(3) gauge plasma. Recently, new lattice results calculated with a renormalization-improved action have been published [4], which are similar but by some 3 ··· 4% larger than the data [3] obtained with a standard action. Consequently, the parameters fitting both data sets, as shown in Tab. 1, differ notably only in the number $d_g$ of the gluon quasiparticle degrees of freedom. The quantity $d_g$ is introduced here as fit parameter to account, on one hand, for possible finite size effects of the lattice data and, on the other hand, for residual sub-leading effects otherwise neglected in the quasiparticle ansatz. As expected, for both data sets the values of $d_g$ are close to $2(N_c^2 - 1) = 16$. As shown in Fig. 1 for the entropy, which is the quantity to adjust the parameters according to Eq. (5), the quasiparticle model reproduces the lattice results quantitatively. In fact, the deviations are smaller than the numerical uncertainty of the data.

The next case analyzed is the system with two light quark flavors, with an estimate for the continuum extrapolation of the pressure given in [7]. To calculate the quasiparticle pressure (3) for fixed values of the parameters, the interaction pressure $B$ is required as a
function of the temperature. It can be obtained by the relation (4), with an integration constant \( B_0 = B(T_0) \) adjusted to the lattice data, e. g., at the smallest temperature. The resulting agreement of the data and the model, with the parameters given in Tab. 1, is shown in Fig. 2. Note that the quark quasiparticle degeneracy is held fixed to \( d_q = \frac{4N_cN_f}{2(N_f^2-1)} d_g \), as in the next example.

The available lattice data [6] for the system with \( N_f = 4 \) light flavors were obtained by the extrapolation to the chiral limit of lattice simulations of quarks with a ratio of mass to temperature of 0.2 and 0.4. Being compatible with each other, both data sets are reproduced by a single set of model parameters as shown in Fig. 3. So far, however, the data are not yet continuum extrapolated. Accordingly, the quasiparticle degeneracies are considerably larger than in the previous cases (cf. Tab. 1). Thus it is suggesting to interpret, to a large amount, the fit value of \( d_g/16 \) to be due to finite size effects which, in the free limit, are indeed found to have the same order of magnitude. Concerning the function \( B(T) \) we mention that it turns out positive in the interval \( 1 \leq T/T_c \leq 2 \), and negative and small at larger values of \( T \) [23].

We conclude this section with two remarks. As anticipated, the quasiparticle approach (being tested with available lattice data) is an appropriate description even close to the confinement transition. This provides some confidence in the applicability, in the nonperturbative regime, of the phenomenological quasiparticle resummation with the effects of finite spectral widths parameterized in an effective coupling. The quasiparticle approach is also supported by the more formal development of the thermodynamical resummation of hard-thermal loops, proposed in [9] to improve the bare perturbation theory. As shown in [10], the entropy of the plasma is dominated by the asymptotic behavior of those excitations which we start with as the relevant quasiparticles. Moreover, a direct hard-thermal-loop resummation of the pressure within the Luttinger-Ward formalism results in an expression
similar to Eq. (3), with $B$ corresponding in part to the so-called $\Phi$ functional\(^1\) \cite{11}. While this resummation, being more rigorous from a theoretical point of view and containing no fit parameters, reproduces the lattice data already qualitatively, we focus here on a phenomenological description, adjustable quantitatively to data. Nevertheless, the hard-thermal-loop resummation approaches yield further confidence to the quasiparticle model, also with regard to the extension to finite chemical potential as outlined subsequently.

**III. EXTRAPOLATING TO FINITE CHEMICAL POTENTIAL**

Encouraged by the successful quasiparticle description of the $\mu = 0$ lattice data, the model is now applied to finite chemical potential. This extension requires no further assumptions and allows in a straightforward way to map the EoS at finite temperature and $\mu = 0$ into the $\mu T$ plane. This continuous mapping relying on quark and gluon quasiparticles, however, cannot provide information about other possible phases with a different (quasiparticle) structure, so the following consideration apply only to ‘conventional’ deconfined matter.

In general, the pressure is a potential of the state variables $T$ and $\mu$. As a direct consequence thereof, the Maxwell relation implies for the quasiparticle model

$$\sum_i \left[ \frac{\partial n_i}{\partial m_i^2} \frac{\partial \Pi^*}{\partial T} - \frac{\partial s_i}{\partial m_i^2} \frac{\partial \Pi^*}{\partial \mu} \right] = 0,$$

which, formally, is the integrability condition for the function $B$ defined by Eq. (4). With $\Pi^*$ depending on $G^2$, Eq. (7) represents a flow equation for the effective coupling, following directly from principles of thermodynamics. This flow equation is a quasilinear partial differential equation of the form

\(^1\)Following these parallels further, we note that the hard-thermal-loop resummed entropy is independent of $\Phi$, as the quasiparticle entropy Eq. (5) is independent of $B$. This feature, related to the underlying one-loop structure of the selfenergies in both cases, makes the entropy an interesting quantity, whereas the pressure contains the full thermodynamical information.
\[ a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b, \]  

with the coefficients \( a_{T,\mu} \) and \( b \) depending on \( T, \mu \) and \( G^2 \). It is instructive to consider first the asymptotic limit, where \( G^2 \rightarrow g^2 \ll 1 \) and Eq. (8) reduces to the homogeneous form

\[
\pi^2 \left( cT^2 + \frac{\mu^2}{\pi^2} \right) \frac{1}{\mu} \frac{\partial g^2}{\partial \mu} - \left( T^2 + \frac{\mu^2}{\pi^2} \right) \frac{1}{T} \frac{\partial g^2}{\partial T} = 0,
\]

with \( c = (4N_c + 5N_f)/(9N_f) \). This equation has solutions \( g^2 = \text{const} \) along the characteristics given by \( cT^4 + 2T^2(\mu/\pi)^2 + (\mu/\pi)^4 = \text{const} \). As a result of this elliptic flow, the renormalization scale \( T_c/\lambda \) of \( g^2(T,0) \) determines the scale \( T_c \pi c^{1/4}/\lambda \) of the \( \mu \) dependent running coupling \( g^2(0,\mu) \).

The flow of the effective coupling is elliptic also in the nonperturbative regime, thus mapping the \( \mu = 0 \) axis, where \( G^2 \) can be determined from lattice data, into the \( \mu T \) plane. As a representative example, the nonperturbative flow of the coupling of the \( N_f = 4 \) system is shown in Fig. 4. As to be expected, the characteristics emanating at \( \mu = 0 \) in the deconfined phase do not attach a certain region, at small values of \( T \) and \( \mu \), of the phase diagram which is obviously related to the confinement phase. It is noted that at \( \mu \lesssim 3T_c \) and small temperatures, indicated by intersecting characteristics, the solution of the flow equation is not unique. This ostensible ambiguity is, however, of no physical consequence, i.e., the approach is intrinsically consistent, since already outside that region the quasiparticle pressure (calculated along the characteristics) would become negative. This region of absolute thermodynamical instability, on the other hand, may be considered as an indication of a transition to a different phase, in fact already at some positive pressure. Whether this is the confined phase or, maybe already at such small chemical potential, a color-superconducting state (cf. [24] and further references therein) remains, of course, a speculation within the present phenomenological approach. In any case, the pressure (see Fig. 5) increases much slower with \( \mu \) than with \( T \), which makes the transition to another deconfined phase likely in that region. This behavior is mainly attributed to the fact that the effective coupling (see Fig. 6), as a function of increasing chemical potential, approaches the asymptotic limit more slowly than in the direction of increasing temperature.
To conclude this example, we remark that the proposed extension of the lattice EoS to finite chemical potential, though model-dependent in using a quasiparticle picture to derive the flow equation (8), is based on \textit{ab initio} calculable data. Since the model works perfectly at $\mu = 0$ and, on the other hand, the convergence problem of perturbation theory and the concept to overcome it by resummation are not specific to $\mu = 0$, we expect our extension of the data as reliable, at least semi-quantitatively. Similar flow equations can also be derived in more rigorous approaches, and on general grounds they are expected to be also of elliptic type. In this sense, the EoS at $\mu = 0$ ‘knows’ about $\mu \neq 0$. Nevertheless, as already mentioned, such continuous extensions do not access other possible QCD-phases with a different ground state; the actual EoS of strongly interacting matter is rather determined by thermodynamical stability between ‘competing’ phases.

\section*{IV. APPLICATION OF THE MODEL}

\subsection*{A. Strangeness in matter}

Although lattice simulations of the physically interesting case of two light and one medium-mass flavor are in progress [25], reliable data of the thermodynamics of QCD with physical quark current masses, i.e. with $m_0s \sim 150$ MeV for the strange flavor, are still lacking to fix the quasiparticle parameters at $\mu = 0$. Nevertheless, even when freely varying the parameters over large ranges, our model allows some predictions since the parameter dependence of the quasiparticle pressure $p^{qp}$ is weak due to the stationarity property Eq. (4). The parameters are restricted to match $p^{qp}$ to the hadronic pressure $p^{had}$ at the confinement temperature which we assume to be $T_c = 150$ MeV. The uncertainty of $p^{had}(T_c, 0) = 3.1 \cdot 10^8$ MeV$^4$, as estimated by a hadron resonance gas model, can be absorbed into the integration constant $B_0 = B(T_c, 0)$. This parameter, being related phenomenologically to the bag constant, is varied in the range $120 \text{MeV} \leq B_0^{1/4} \leq 180 \text{MeV}$. (Larger values of $B_0$ yield an increased ratio $e/T^4$ close to $T_c$, opposed to what is expected from available lattice data.)
For the second independent parameter in $p^{qp}$, the range $3 \leq \lambda \leq 11$ is considered reasonably large, as suggested by the parameters of the analyzed lattice data in Tab. 1. The EoS for a representative choice of the parameters is provided in tabular form in [26] for the plasma with vanishing net-strangeness, i.e. with the quark chemical potentials $\mu_s = 0$ and $\mu_u = \mu_d = \mu$. With the effective coupling decreasing only logarithmically from $\alpha = G^2/(4\pi) \sim 1$ at small values of the temperature and the chemical potential, the medium contributions (2) to the quasiparticle masses are large, of the order of $T$ or $\mu$. Hence, the quasiparticles are considerably under-saturated in phase space compared to the asymptotic limit, while the current mass of strange quarks turns out a less important suppression factor.

**B. Deconfined $\beta$-stable strange matter**

With the aim to study implications for quark matter stars, we consider in the following a plasma of gluons, quarks and leptons in $\beta$-equilibrium maintained by the reactions $d, s \leftrightarrow u + l + \bar{\nu}_l$, which imply the relations $\mu_d = \mu_s = \mu_u + \mu_l \equiv \mu$ among the chemical potentials. The lepton chemical potential $\mu_l$ is determined as a function of $T$ and $\mu$ by the requirement of electrical charge neutrality. Without noticeable change of the result, the lepton pressure $p^l$ can be estimated by either the pressure of a free electron gas or, doubling the degrees of freedom, a gas of free electrons and muons. The pressure $p = p^{qp} + p^l$ and the resulting energy density at $\mu = 0$ are shown in the left panels of Fig. 7 for several values of the parameters $\lambda$ and $B_0$. Already at temperatures slightly above $T_c$, the scaled energy density reaches a saturation-like behavior at about 90% of the asymptotic limit. This qualitative feature, known from the lattice simulations [3–7], is to a large extent insensitive to the specific choice of the free parameters, while $B_0$ has a distinct impact on the latent heat. The resulting EoS at vanishing temperature is displayed in the right panels of Fig. 7. In this regime, the asymptotic values are approached more slowly due to the less rapid decrease of the effective coupling with increasing values of $\mu$, similar to the $N_f = 4$ plasma. For various sets of parameters, the EoS is available in tabular form [26].
C. Pure quark stars

By the Tolman-Oppenheimer-Volkov (TOV) equation (cf. Eq. (2.212) of Ref. [27]), sequences of hydrostatic equilibrium configurations of stars can be calculated, given the relation \( e(p) \) of the star matter. Since the bulk properties of the star are less dominated by the outermost shells, we consider the case of pure quark stars as a reasonable approximation for compact stars with a large quark matter core. For energy densities up to several times the nuclear saturation density and at temperatures less than some 10 MeV, the dependence \( e(p) \) of \( \beta \)-stable quark matter, as estimated numerically by our quasiparticle model, can be parameterized by \( e = 4\tilde{B} + \tilde{\alpha}p \). While the naive bag model EoS had \( \tilde{\alpha} = 3 \), our EoS, with the considered choices of the model parameters, yields \( 3.1 \leq \tilde{\alpha} \leq 4.5 \), indicating the nontrivial nature of the interaction. For parameters \( \lambda \geq 5 \), values of \( \tilde{B}^{1/4} \geq 200 \text{MeV} \) are found, while only the extreme choice of \( \lambda = 3 \) allows \( \tilde{B}^{1/4} \) as small as 180 MeV. However, the value of \( \tilde{B} \), i.e., the energy density at small pressure, has a strong impact on the star’s mass and radius, obtained by integrating the TOV equation.

In Fig. 8, the mass as function the radius of pure quark stars is displayed for various values of the parameter \( \tilde{B} \) and \( \tilde{\alpha} \), while in Fig. 9 the maximum mass and the corresponding radius are shown as function of \( \tilde{B}^{1/4} \). For \( \tilde{B}^{1/4} > 200 \text{MeV} \), as turned out to be the most likely range according to our analysis, the quark stars have masses less than one solar mass \( M_\odot \). Compact quark stars with a larger mass can only exist for values of \( \tilde{B}^{1/4} \leq 180...200 \text{MeV} \), depending only marginally on \( \tilde{\alpha} \). We hence conclude that compact stars with masses \( M \sim M_\odot \) and radii \( R \sim 10 \text{km} \) are unlikely to be composed mainly of deconfined matter in \( \beta \)-equilibrium, irrespective of the uncertainty of the model parameters of our approach.

V. SUMMARY AND OUTLOOK

In summary we have generalized a thermodynamic quasiparticle description of deconfined matter to finite chemical potential \( \mu \) not yet accessible by present lattice calculations. Of
central importance to the model is the effective coupling \( G^2(T, \mu) \) which can be obtained at \( \mu = 0 \) from available lattice data, proving at the same time the applicability of the quasiparticle picture even close to the confinement transition. At \( \mu \neq 0 \), \( G^2 \) is determined by a flow equation following directly from the Maxwell relation of thermodynamics. As a result of the elliptic flow, the basic features of the EoS at \( \mu = 0 \), namely the nonperturbative behavior near confinement and the asymptotic behavior, are mapped into the \( \mu T \) plane as exemplified by the \( N_f = 4 \) flavor system studied on the lattice at \( \mu = 0 \).

An important consequence of the quasiparticle approach is the relation of the ‘critical’ values of temperature and chemical potential. For deconfinement matter with physical quark masses, this fact leads to the implication that pure quark stars are less massive and, hence, more distinct in the bulk properties to neutron stars than estimated by other approaches with an ad hoc choice of small values of the bag constant.

Further applications of the quasiparticle model, in connection with realistic thermodynamical descriptions of other QCD phases, is desirable. In particular, in the cold and dense regime where a color-superconducting state is expected to be important, the quasiparticle EoS can provide one part of the information about the phase boundary. With regard to the on-going heavy-ion program at the relativistic heavy-ion collider RHIC at Brookhaven National Laboratory, we finally would like to point out the applicability of our quasiparticle EoS as an input to hydro-relativistic simulations. For a realistic description of the (near-to) equilibrium states of a heavy-ion collision, detailed knowledge about the EoS at non-zero baryon densities is required. Since the results of the quasiparticle model differ considerably from that of simpler approaches as the bag model (e.g., in the reduced latent heat and the saturation-like behavior of the energy density at about 90% of the free limit), dynamical effects of a realistic EoS may even lead to observable signals of the experimental formation of the quark-gluon plasma.

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### TABLE I. Quasiparticle-model parameters adjusted to the lattice data. For the systems containing quarks, the quark degeneracy is fixed to $d_q = \frac{4N_c N_f}{2(N_c^2 - 1)} d_g$. 

<table>
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<th>$\lambda$</th>
<th>$T_s/T_c$</th>
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<td>-0.80</td>
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FIG. 1. The comparison of the quasiparticle entropy density with the lattice results, calculated with a standard Wilson action [3,4] and a renormalization-improved action [4], of the pure gluon plasma.

FIG. 2. The pressure in our model compared to the estimate of the continuum extrapolation of the lattice data [7] for the light 2-flavor case.
FIG. 3. The comparison of the energy density in our model with the chiral extrapolation of the lattice data [6] of the 4-flavor case.

FIG. 4. The characteristics of the coupling flow equation (8) for the QCD plasma with $N_f = 4$ light flavors for which $G^2(T, \mu = 0)$ is obtained from the lattice data [6]. At leading order, the characteristics are curves of constant coupling strength. The region of intersecting characteristics is of no physical relevance since the pressure becomes negative in the region below the dash-dotted line (see text).
FIG. 5. The pressure of the $N_f = 4$ plasma in the chiral limit as a function of $\mu$ and $T$, scaled by free limit.

FIG. 6. The effective coupling $\alpha = G^2/(4\pi)$ of the $N_f = 4$ plasma.
FIG. 7. Total pressure and energy density (lower and upper bands, respectively) of the charge-neutral quark-gluon plasma in $\beta$-equilibrium, scaled by the values of the free limit. The panels on the left (right) show the EoS at $\mu = 0$ ($T = 0$), for values of the model parameter $3 \leq \lambda \leq 11$ (lower and upper line, respectively, of the shaded area), and $B_0^{1/4} = 120$ MeV (top) and $B_0^{1/4} = 180$ MeV (bottom). Non-unique values of the energy only occur in the unphysical region, where $p < 0$. 
FIG. 8. The dependence of the mass of pure quark stars on the radius for several values of the parameters $\tilde{\alpha}$ and $\tilde{B}^{1/4}$.

FIG. 9. The dependence of the maximum quark star mass (upper band and right scale) and the corresponding radius (lower band and left scale) on the parameter $\tilde{B}^{1/4}$. The upper (lower) boundary of the hatched bands are for $\tilde{\alpha} = 3$ (4.5).