Mass scales, supersymmetry breaking and open strings

I. Antoniadis\textsuperscript{a} and A. Sagnotti\textsuperscript{b}

\textsuperscript{a} Centre de Physique Théorique (CNRS UMR 7644)  
École Polytechnique  
F-91128 Palaiseau FRANCE

\textsuperscript{b} Dipartimento di Fisica  
Università di Roma “Tor Vergata”  
INFN, Sezione di Roma “Tor Vergata”  
Via della Ricerca Scientifica 1  
I-00133 Roma ITALY

Abstract

We review physical motivations and possible realizations of string vacua with large internal volume and/or low string scale and discuss the issue of supersymmetry breaking. In particular, we describe the key features of Scherk-Schwarz deformations in type I models and conclude by reviewing the phenomenon of “brane supersymmetry breaking”: the tadpole conditions of some type-I models require that supersymmetry be broken at the string scale on a collection of branes, while being exact, to lowest order, in the bulk and on other branes.

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1. Introduction

We have long been accustomed to accept that typical string effects, confined to very high scales, are beyond the reach of conceivable experiments. This state of affairs is directly implied by the often implicit identification of the scale $M_s = (1/l_s)$ of string excitations with the Planck scale $M_p$, and has a rather stringent motivation in weakly-coupled heterotic strings, that have been extensively studied during the last decade [1]. In this case, both gravitational and gauge effects originate from the sphere topology, and the reduction on an internal space of volume $V$ gives, in a self-explanatory notation, the four-dimensional effective action:

$$S_H = \int d^4x \frac{V}{\lambda_H^2} \left( l_H^{-8} R + l_H^{-6} F^2 + \ldots \right).$$

This expression relates the four-dimensional Planck mass $M_p$ and the four-dimensional gauge coupling $g$ at the string (unification) scale to the heterotic string coupling $\lambda_H$ and to the heterotic string scale $M_H$, according to

$$M_H = gM_p, \quad \lambda_H = g \frac{\sqrt{V}}{l_H^3}.$$  

Thus, with the gauge coupling at unification determined by the minimal supersymmetric Standard Model with the “desert” hypothesis, $g \sim 0.2$, one finds $M_H \sim 10^{18}\text{GeV}$. Moreover, perturbative string descriptions, with $\lambda_H < 1$, require internal volumes close to the string size.$^1$

There are a few motivations, however, to consider string realizations with small string scale and/or large extra dimensions [2]. These were already suggested by early studies of heterotic models although, as we have seen, they are outside the perturbative setting. Aside from the obvious phenomenological interest in meeting string effects at low energies, two main motivations have been associated over the years with large extra dimensions. The first was actually suggested by the estimated value of the unification scale $M_{GUT}$, close

$^1$Up to T-dualities, we can always refer to compactification volumes larger than the string size.
enough to the string scale to let one wonder whether the two should coincide. Taking into account string threshold corrections [3], that grow linearly with internal radii, one could try to attribute the lack of coincidence to a geometric scale a couple or orders of magnitude below $M_s$, at which new (Kaluza-Klein) physics would set in. This, however, would require generically that one of the internal radii be much larger than the others, with the result of driving the heterotic description toward strong coupling, as can be seen from eq. (1.2).

Another motivation comes from supersymmetry breaking, that has long been induced in heterotic models via a string extension of the Scherk-Schwarz mechanism [4, 5, 6, 2]. In Field Theory, the Scherk-Schwarz mechanism [4] resorts to global symmetries to allow for more general mode expansions in the internal space, and can be used, in principle, to induce supersymmetry breaking at arbitrary scales if bosons and fermions are treated differently. In String Theory, however, the breaking scale is necessarily $O(1/R)$, where $R$ is a typical geometric scale of the internal space [6], and similar restrictions are met if the breaking is induced by magnetic deformations [7]. Thus, taken at face value, these results imply that TeV-scale supersymmetry breaking requires TeV-scale extra dimensions, with corresponding, nearly accessible, Kaluza-Klein towers of excitations [2, 8].

With large extra dimensions, the effective field theory presents inevitable subtleties. Most notably, at energies close to a geometric scale $1/R$ related to a (large) extra dimension, the corresponding Kaluza-Klein tower starts contributing to renormalizations, and generically tends to drive the system to strong coupling [9], unless special conditions are met [2]. For the gauge couplings, this can be avoided if the Kaluza-Klein towers fill $N = 4$ multiplets, a condition nicely met in interesting cases of string compactifications with no $N = 2$ sectors relative to the large compact coordinate [2].

With the advent of string dualities [10], the heterotic string has lost its central role in the comparison with low-energy physics, while the problem of strong coupling can now be analyzed in a more quantitative fashion, resorting to (dual) weakly-coupled descriptions
provided by other string models. Interestingly, these dual descriptions incorporate the salient features, already envisaged from the perturbative heterotic corner, that are needed to grant a smooth behavior across the compactification scale. It is therefore instructive to explore the strong-coupling problem from the heterotic corner with different numbers of large extra dimensions. In the next Section, we thus present a brief review of the results of [11], in particular for what concerns type I models, and discuss some issues raised by this analysis. For a more detailed review, the interested reader may resort to [12]. Our main motivation here is to show how, in several of the resulting cases, the dual string models are characterized by low string scales, that at times can reach the TeV region, with additional interesting effects related to their towers of higher-spin excitations [13]. In fact, as will become clear in the following, in any string theory other than the heterotic, the simple relation (1.2) that fixes the string scale in terms of the Planck mass does not hold, and therefore the string tension becomes an arbitrary parameter [14].\(^2\) It can be anywhere below the Planck scale, even at a few TeV. The main advantage of having a string tension in the TeV region, aside from its obvious experimental interest, is that it offers an automatic solution to the gauge hierarchy problem, alternative to low-energy supersymmetry or technicolor [16]. Weakly coupled, low-scale strings can be realized introducing either extra large transverse dimensions felt only by gravitational interactions or an infinitesimal string coupling. In the former case, the quantum gravity scale is also low, while gauge interactions are confined to lower-dimensional p-branes. In the latter case, gravitational and string interactions remain suppressed by the four-dimensional Planck mass. There is one exception to this general rule, allowing for large longitudinal dimensions without a low string scale, when the Standard Model is embedded in a six-dimensional fixed-point theory described by a tensionless string [11].

The very issue of supersymmetry breaking needs to be reconsidered in low-scale string

\(^2\)It was recently realized that the heterotic string scale can also be lowered at weak coupling via small instantons [15].
models. With the string scale in the TeV region, the original motivation for low-energy supersymmetry is apparently lost, since in non-supersymmetric string vacua the bulk vacuum energy is typically determined by the string scale. While this is a very favorable state of affairs for the bulk spectrum, in type I models the cosmological constant induced on our world-brane is actually enhanced by the volume of the transverse space. As a result, it is typically larger than the string scale, and tends to destabilize the hierarchy that one tries to enforce.

In Section 3 we discuss supersymmetry breaking by compactification in type I strings [17, 18, 19]. Referring to very simple models in nine dimensions, we show how the Scherk-Schwarz mechanism allows in this case two distinct possibilities, according to whether the shifts are parallel or transverse with respect to the resulting branes. In the latter case, they are ineffective on their massless modes, that display an enhanced supersymmetry. This is the phenomenon commonly denoted “brane supersymmetry”, first noted in [17] and developed further in [18, 19]. In the last Section, referring to a six-dimensional example, we review how the consistent definition of some type I models requires that, to lowest order, supersymmetry be broken at the string scale on a collection of branes, while the bulk is unaffected [20]. This phenomenon, we believe, is particularly intriguing. For the first time, supersymmetry breaking is not an option, but is required by the very consistency of a class of string models. The vacuum energy, restricted to the brane from which supersymmetry breaking originates, is naturally protected against the destabilizing effects of gravitational radiative corrections, since to lowest order the bulk is supersymmetric. Moreover, if our world is modeled resorting to this framework, the current experimental limits on short-distance gravitational effects [21] leave open the exciting possibility of an (almost) exact supergravity a (sub)millimeter away from it.

This is a joint version of the talks presented by the authors at STRINGS ’99. Transparencies and audio are available at [22].
2. Large extra dimensions: the heterotic string and its duals

2.1. Type I strings and D-branes

In ten dimensions, the strongly coupled $SO(32)$ heterotic string is dual to the type I string\(^3\), a theory where gravity is described by unoriented closed strings, while gauge interactions are described by unoriented open strings whose ends are confined to D-branes. Therefore, in this setting some of the six internal compact dimensions are longitudinal (parallel) and some are transverse to the D-branes. In particular, if the Standard Model were localized on a $p$-brane (with $p \geq 3$), there would be $p - 3$ longitudinal and $9 - p$ transverse compact dimensions. In contrast to the heterotic string, here gauge and gravitational interactions appear at different orders of perturbation theory, and the corresponding effective action reads

\[
S_I = \int d^{10}x \frac{1}{\lambda_I^2 l_P^8} \mathcal{R} + \int d^{p+1}x \frac{1}{\lambda_I l_P^{p-3}} F^2 ,
\]

where the factor $1/\lambda_I$ in the gauge kinetic terms reflects their origin from the disk diagram.

Upon compactification to four dimensions, the Planck length and the gauge couplings are given, to leading order, by

\[
\frac{1}{l_P^2} = \frac{V_\parallel V_\perp}{\lambda_I^2 l_P^8} , \quad \frac{1}{g^2} = \frac{V_\parallel}{\lambda_I l_P^{p-3}} ,
\]

where $V_\parallel (V_\perp)$ denotes the compactification volume longitudinal (transverse) to the $p$-brane. The second relation links the weak coupling $\lambda_I < 1$ to sizes of the longitudinal space comparable to the string length ($V_\parallel \sim l_P^{p-3}$), while the transverse volume $V_\perp$ remains unrestricted. Combining eqs. (2.2) gives

\[
M_P^2 = \frac{1}{g^2 v_\parallel} M_I^{2+n} R_\perp^n , \quad \lambda_I = g^2 v_\parallel ,
\]

\(^3\)T-dualities turn this model into the type I' string, that in lower dimensions can also describe a class of M-theory compactifications.
to be compared with the heterotic relations (1.2). Here $v_\parallel \gtrsim 1$ is the longitudinal volume in string units, and we are considering an isotropic transverse space with $n = 9 - p$ compact dimensions of radius $R_\perp$.

The relations (2.3) imply that the type I/I' string scale can be made hierarchically smaller than the Planck mass at the expense of introducing extra large transverse dimensions that interact only gravitationally [16, 23]. The weakness of four-dimensional (4D) gravity $M_I/M_P$ may then be attributed to the largeness of the transverse space $R_\perp/l_I$. However, the (higher-dimensional) gravity becomes strong at the string scale, although the string coupling is weak, and indeed the first of eq.(2.3) can be understood as a consequence of the $(4 + n)$-dimensional Gauss law for gravity, with

$$G^{(4+n)}_N = g^{4l_1^{2+n}v_\parallel} (2.4)$$

Newton’s constant in $4 + n$ dimensions. Taking the type I string scale $M_I$ at 1 TeV, one finds a size for the transverse dimensions $R_\perp$ varying from $10^8$ km, to .1 mm ($10^{-3}$ eV), and down to .1 fermi (10 MeV) for $n = 1, 2, 6$ large dimensions. Aside from the $n = 1$ case, obviously excluded, all other cases are actually consistent with observations, although barely for $n = 2$ [24]. In particular, sub-millimeter transverse directions are compatible with the present constraints from short-distance gravity measurements, that have tested Newton’s law only down to the cm [21].

2.2. Type IIA strings

Upon compactification to 6 or fewer dimensions, the heterotic string admits another dual description in terms of the type IIA string compactified on a Calabi-Yau manifold. For simplicity, we restrict ourselves to compactifications on $K3 \times T^2$, yielding $N = 4$ supersymmetry, or more generally on Calabi-Yau manifolds that are $K3$ fibrations, yielding $N = 2$ supersymmetry. In contrast to heterotic and type I strings, non-abelian gauge symmetries in type IIA models arise non-perturbatively (at arbitrarily weak coupling) in
singular compactifications, where the massless gauge bosons are provided by D2-branes wrapped around (vanishing) non-trivial 2-cycles. The resulting gauge interactions are localized on $K3$, while matter multiplets arise from further singularities, and are completely localized in the 6d internal space. As a result, the gauge kinetic terms are independent of the string coupling $\lambda_{IIA}$, and the corresponding effective action is

$$S_{IIA} = \int d^{10}x \frac{1}{\lambda^2_{IIA} l_{IIA}^6} R + \int d^{6}x \frac{1}{l_{IIA}^2} F^2 ,$$

(2.5)

to be compared with (1.1) and (2.1). Upon compactification to four dimensions, for instance on a two-torus $T^2$, the gauge couplings are determined by its size, $v_{T^2}$ in string units, while the Planck mass is controlled by the 6d string coupling $\lambda_{6IIA}$:

$$\frac{1}{g^2} = v_{T^2} , \quad \frac{1}{l_P^2} = \frac{v_{T^2}}{\lambda^2_{6IIA} l_{IIA}^2} = \frac{1}{\lambda^2_{6IIA} g^2 l_{IIA}^2} .$$

(2.6)

The area of $T^2$ should therefore be of order $l_{IIA}^2$, while the string scale is now related to the Planck mass according to

$$M_{IIA} = g \lambda_{6IIA} M_P = g \lambda_{IIA} M_P \frac{l_{IIA}^2}{\sqrt{V_{K3}}} ,$$

(2.7)

with $V_{K3}$ the volume of $K3$. Thus, in contrast to the type I relation (2.3), only sensitive to the volume of the internal six-manifold, one now has the freedom to use both the string coupling and the $K3$ volume to separate the Planck mass from a string scale at, say, 1 TeV.

In particular, with a string-size internal manifold, an ultra-weak coupling $\lambda_{IIA} = 10^{-14}$ can account for the hierarchy between the electroweak and Planck scales [11]. In this setting, despite the fact that the string scale is so low, gravity remains weak up to the Planck scale, while string interactions are suppressed by the tiny string coupling, or equivalently by the 4d Planck mass. Thus, no observable effects are left for particle accelerators, aside from the production of KK excitations along the two TeV dimensions of $T^2$ felt by gauge interactions. Furthermore, the excitations of the gauge multiplets have $N = 4$ supersymmetry, even when $K3 \times T^2$ is replaced by a Calabi-Yau threefold that is a $K3$ fibration, while matter multiplets
are localized on the base (replacing the $T^2$) and have no KK excitations, like the twisted states of heterotic orbifolds.

2.3. Type IIB strings

In type IIB constructions, gauge symmetries still arise from vanishing 2-cycles of $K3$, but take the form of tensionless strings in 6 dimensions, that originate from D3-branes wrapped on them. Only after a further $T^2$ reduction to four dimensions does this theory reduce to an ordinary gauge theory, whose coupling now involves the shape (complex structure) $u_{T^2}$, rather than the volume $v_{T^2}$, of the torus. In this case one finds [11]

$$\frac{1}{g^2} = u_{T^2}, \quad \frac{1}{l_P^2} = \frac{v_{T^2}}{\lambda_{IIB}^2},$$

where, for instance, for a rectangular torus the shape is the ratio of the two radii, $u_{T^2} = R_1/R_2$. Comparing with eq. (2.6), it is clear that the situation in type IIB is the same as in type IIA, unless the size of $T^2$ is much larger than the string length. Actually, since $T^2$ is felt by gauge interactions, its size cannot be larger than the TeV$^{-1}$, and thus the type IIB string scale should be much larger than the TeV. In particular, for a rectangular torus of radii $R$ and $g^2 R$

$$M_{IIB}^2 = g\lambda_{IIB} M_P \frac{1}{R},$$

so that the lowest value for the string scale, with a string coupling of order unity and $R \sim$ TeV$^{-1}$, is $10^{11}$ GeV [11]. This, as we will see, is precisely the case that describes the heterotic string with a single TeV dimension, and is the only example of a weakly coupled theory with longitudinal dimensions larger than the string length. In the energy range between the KK scale $1/R$ and the type IIB string scale, one has an effective 6d theory without gravity at a non-trivial superconformal fixed point described by tensionless strings, corresponding to D3-branes wrapped on the vanishing 2-cycles of a singular $K3$. Since the type IIB coupling is of order unity, gravity becomes strong at the type IIB string scale, and the main experimental signatures at TeV energies are in this case Kaluza-Klein excitations, as in type IIA models with tiny string coupling.
2.4. Relation with the heterotic string

As we mentioned previously, the type I/I′ and type IIA/IIB theories provide dual descriptions of the heterotic string at strong coupling. Somewhat surprisingly, it turns out that all TeV scale string models that we have discussed can be recovered as different strongly coupled decompactification limits, with only one large scale in addition to the Planck-size heterotic tension. More precisely, let us consider the heterotic string compactified on a six-torus with \( k \) large dimensions of radius \( R \gg l_H \) and \( 6 - k \) string-size dimensions. Applying the standard duality maps [10, 11], it is simple to show that the type I′ theory with \( n \) transverse dimensions provides a weakly coupled dual description for the heterotic string only with \( k = 4, 5, 6 \) large dimensions, since otherwise the remaining T-dualities needed to obtain volumes above the resulting string scale lead to strong coupling. \( k = 4 \) is described by \( n = 2, k = 6 \) (for the \( SO(32) \) gauge group) is described by \( n = 6 \), while for \( k = 5 \) one finds a type I′ model with five large and one extra-large transverse dimensions. The case \( k = 4 \) is particularly interesting: the heterotic string with 4 large dimensions at a TeV is described by a perturbative type I′ theory with the string scale at a TeV and gauge interactions confined to D7-branes with two transverse dimensions of millimeter size, T-dual to the two string-size heterotic coordinates. On the other hand, the type II theory provides a weakly coupled description for \( k = 1, 2, 3, 4 \) and \( k = 6 \) (for \( E_8 \times E_8 \)). In particular, \( k = 1 \) is described by type IIB with string tension at intermediate energies, \( k = 2 \) is described by type IIA with tension and all compactification radii at a TeV and an infinitesimal coupling \( \lambda_{IIA} \sim l_H/R \), while for \( k = 3 \) all four (transverse) \( K3 \) directions are extra large.

3. Scherk-Schwarz deformations in type-I strings

Scherk-Schwarz deformations can be introduced in type I strings following [5, 6], but present a few interesting novelties, that may be conveniently exhibited referring to a pair of 9D models [17]. To this end, we begin by recalling that, for the type IIB string, (the
fermionic part of) the partition function can be written in the compact form

\[ T = |V_8 - S_8|^2 \]  

(3.1)

resorting to the level-one SO(8) characters

\[
O_8 = \frac{\vartheta_3^4 + \vartheta_4^4}{2\eta^4}, \quad V_8 = \frac{\vartheta_3^4 - \vartheta_4^4}{2\eta^4},
\]

\[
S_8 = \frac{\vartheta_2^4 - \vartheta_1^4}{2\eta^4}, \quad C_8 = \frac{\vartheta_2^4 + \vartheta_1^4}{2\eta^4},
\]

(3.2)

where the \( \vartheta_i \) are Jacobi theta functions and \( \eta \) is the Dedekind function. In the usual toroidal reduction, where bosons and fermions have the momentum modes

\[
p_L = \frac{m}{R} + \frac{nR}{\alpha'}, \quad p_R = \frac{m}{R} - \frac{nR}{\alpha'},
\]

(3.3)

the 9D partition function is

\[ T = |V_8 - S_8|^2 Z_{mn}, \]

(3.4)

where

\[ Z_{mn} \equiv \sum_{m,n} \frac{q^{\alpha' p_L^2/4} q^{\alpha' p_R^2/4}}{\eta \bar{\eta}}. \]

(3.5)

A simple modification results in a Scherk-Schwarz breaking of space-time supersymmetry. There are actually two inequivalent choices, described by

\[
T_1 = Z_{m,2n}(V_8 \bar{V}_8 + S_8 \bar{S}_8) + Z_{m,2n+1}(O_8 \bar{O}_8 + C_8 \bar{C}_8)
\]

\[ - Z_{m+1/2,2n}(V_8 \bar{S}_8 + S_8 \bar{V}_8) - Z_{m+1/2,2n+1}(O_8 \bar{C}_8 + C_8 \bar{O}_8) \]

(3.6)

and

\[
T_2 = Z_{2m,n}(V_8 \bar{V}_8 + S_8 \bar{S}_8) + Z_{2m+1,n}(O_8 \bar{O}_8 + C_8 \bar{C}_8)
\]

\[ - Z_{2m,n+1/2}(V_8 \bar{S}_8 + S_8 \bar{V}_8) - Z_{2m+1,n+1/2}(O_8 \bar{C}_8 + C_8 \bar{O}_8) , \]

(3.7)

that may be associated to momentum or winding shifts of the usual fermionic modes (\( V_8 \bar{S}_8 \) and \( S_8 \bar{V}_8 \)) relatively to the usual bosonic ones (\( V_8 \bar{V}_8 \) and \( S_8 \bar{S}_8 \)). The two choices are inequivalent, since T-duality along the circle interchanges type-IIB and type-IIA strings [25].
Both deformed models have tachyon instabilities at the scale of supersymmetry breaking for the low-lying modes, $O(1/R)$ for the momentum deformation of eq. (3.5) and $O(R/\alpha')$ for the winding deformation of eq. (3.6).

The open descendants [26] are essentially determined by the choice of Klein-bottle projection $\mathcal{K}$ [27], while the other amplitudes $\mathcal{A}$ and $\mathcal{M}$ reflect the propagation of closed-string modes between boundaries and crosscaps. In displaying the amplitudes of [17], we implicitly confine our attention to internal radii such that (closed-string) tachyon instabilities are absent, and choose Chan-Paton assignments that remove them from the open sectors as well. We also impose some (inessential) NS-NS tadpoles, in order to bring the resulting expressions to their simplest forms.

Starting from the model of eq. (3.5), corresponding to momentum shifts, the additional amplitudes are

\begin{align}
\mathcal{K}_1 &= \frac{1}{2} (V_8 - S_8) Z_m, \\
\mathcal{A}_1 &= \frac{n_1^2 + n_2^2}{2} (V_8 Z_m - S_8 Z_{m+1/2}) + n_1 n_2 (V_8 Z_{m+1/2} - S_8 Z_m), \\
\mathcal{M}_1 &= -\frac{n_1 + n_2}{2} (\hat{V}_8 Z_m - \hat{S}_8 Z_{m+1/2}),
\end{align}

while the tadpole conditions require that $n_1 + n_2 = 32$. Supersymmetry, broken in the whole range $R > \sqrt{\alpha'}$, is recovered asymptotically in the decompactification limit. This vacuum, first described in [28], is interesting in its own right, since it describes the type I string at finite temperature (with Wilson lines), but includes a rather conventional open spectrum, where bosonic and fermionic modes have the usual $O(1/R)$ Scherk-Schwarz splittings of field-theory models.

On the other hand, starting from the model of eq. (3.6), corresponding to winding shifts, the additional amplitudes are [17]

\begin{align}
\mathcal{K}_2 &= \frac{1}{2} (V_8 - S_8) Z_{2m} + \frac{1}{2} (O_8 - C_8) Z_{2m+1},
\end{align}
\[
\mathcal{A}_2 = \left( \frac{n_1^2 + n_2^2 + n_3^2 + n_4^2}{2} (V_8 - S_8) + (n_1 n_3 + n_2 n_4)(O_8 - C_8) \right) Z_m
+ \left( (n_1 n_2 + n_3 n_4)(V_8 - S_8) + (n_1 n_4 + n_2 n_3)(O_8 - C_8) \right) Z_{m+1/2},
\]
\[
\mathcal{M}_2 = -\frac{n_1 + n_2 + n_3 + n_4}{2} \hat{V}_8 Z_m + \frac{n_1 - n_2 - n_3 + n_4}{2} \hat{S}_8 (-1)^m Z_m,
\]

(3.9)

while the tadpole conditions now require that \(n_1 + n_2 = n_3 + n_4 = 16\). Supersymmetry is recovered in the limit of vanishing \(R\), where the whole tower of winding modes present in the vacuum-channel amplitudes collapses into additional tadpole conditions that eliminate \(n_2\) and \(n_3\). This is precisely the phenomenon of [29], spelled out very clearly by these partition functions. The resulting open sector, described by

\[
\mathcal{A}_2 = \frac{n_1^2 + n_4^2}{2} (V_8 - S_8) Z_m + n_1 n_4 (O_8 - C_8) Z_{m+1/2},
\]
\[
\mathcal{M}_2 = -\frac{n_1 + n_4}{2} \hat{V}_8 Z_m + \frac{n_1 + n_4}{2} \hat{S}_8 (-1)^m Z_m,
\]

(3.10)

has the suggestive gauge group \(SO(16) \times SO(16)\), and is rather peculiar. In the limit of small breaking \(R\), aside from the ultra-massive \((O, C)\) sector, it contains a conventional \((V, S)\) sector where supersymmetry, \textit{exact} for the massless modes, is effectively broken \textit{at the string scale} for the massive ones by the unpairing of the corresponding Chan-Paton representations. This is the phenomenon of “brane supersymmetry” that we alluded to in the Introduction [17, 18, 19], here present only for the massless modes. However, as originally suggested in [18], this setting can be generalized to allow for entire open sectors with exact supersymmetry, as in [32, 33]. The arguments of [30] can then connect, via a sequence of duality transformations, the \(SO(16) \times SO(16)\) gauge group to the two Horava-Witten walls [31] of M-theory, with the end result that this peculiar breaking can be associated to an 11D Scherk-Schwarz deformation. We are thus facing a simple perturbative description of a phenomenon whose origin is non-perturbative on the heterotic side. Several generalizations have been discussed, in six and four dimensions, with partial or total breaking of supersymmetry [17, 19, 32, 33].

After suitable T-dualities, these results can be put in a very suggestive form: while the
conventional Scherk-Schwarz breaking of $T_1$ results from shifts parallel to a brane, the M-theory breaking of $T_2$ results from shifts orthogonal to a brane, and is naturally ineffective on its massless modes.

4. Brane supersymmetry breaking

The last phenomenon that we would like to review in this talk, “brane supersymmetry breaking” [20], provides an answer to an old puzzle in the construction of open-string models where, in a number of interesting cases, the tadpole conditions have apparently no consistent solution [34]. The simplest example is provided by the six-dimensional $T^4/Z_2$ reduction where, as in [27], the Klein-bottle projection is reverted for all twisted states. In the resulting projected closed spectrum, described by

$$
\mathcal{T} = \frac{1}{2}|Q_o + Q_v|^2\Lambda + \frac{1}{2}|Q_o - Q_v|^2\frac{2\eta}{\theta^2} + \frac{1}{2}|Q_s + Q_c|^2\frac{2\eta}{\theta^4}
$$

$$
\mathcal{K} = \frac{1}{4}\left\{(Q_o + Q_v)(P + W) - 2 \times 16(Q_s + Q_c)\right\},
$$

(4.1)

the massless modes include 17 tensor multiplets and 4 hypermultiplets\(^4\). In writing eq. (4.1), where $\Lambda$ is the whole Narain lattice sum while $P$ and $W$ are its restrictions to only momenta or windings, we have resorted to the supersymmetric combinations of SO(4) characters

$$
Q_o = V_4O_4 - C_4C_4 , \quad Q_v = O_4V_4 - S_4S_4 ,
$$

$$
Q_s = O_4C_4 - S_4O_4 , \quad Q_c = V_4S_4 - C_4V_4 .
$$

(4.2)

The reversal of the Klein-bottle projection for twisted states changes the relative sign of the crossecap contributions for N and D strings or, equivalently, the relative charge of the

\(^4\)A quantized NS-NS $B_{ab}$ would lead to similar models with lower numbers tensor multiplets, that may be analyzed in a similar fashion [35].
O5 orientifold planes relative to the O9 ones. This is clearly spelled out by the terms at the origin of the lattices,
\[ \tilde{K}_0 = \frac{25}{4} \left\{ Q_o \left( \sqrt{v} \pm \frac{1}{\sqrt{v}} \right)^2 + Q_v \left( \sqrt{v} \mp \frac{1}{\sqrt{v}} \right)^2 \right\}, \tag{4.3} \]
where the upper signs refer to the standard choice, while the lower ones refer to the reverted Klein bottle of eq. (4.1). In the latter case one is forced to cancel a negative background O5 charge, and this can be achieved introducing antibranes in the vacuum configuration. The corresponding open sector [20] results from a combination of D9 branes and D\( \bar{5} \) antibranes, and involves the \( N \) and \( D \) charges and their orbifold breakings \( R_N \) and \( R_D \):
\[ A = \frac{1}{4} \left\{ (Q_o + Q_v)(N^2 P + D^2 W) + 2ND(Q'_s + Q'_c) \left( \eta \frac{\theta}{\theta_1} \right)^2 \right\} \tag{4.4} \]
\[ + \ (R_N^2 + R_D^2)(Q_o - Q_v) \left( 2\eta \frac{\theta}{\theta_2} \right)^2 + 2R_NR_D(-O_4 S_4 - C_4 O_4 + V_4 C_4 + S_4 V_4) \left( \eta \frac{\theta}{\theta_3} \right)^2 \right\} \]
\[ \mathcal{M} = -\frac{1}{4} \left\{ NP(\hat{O}_4 \hat{V}_4 + \hat{V}_4 \hat{O}_4 - \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4) - DW(\hat{O}_4 \hat{V}_4 + \hat{V}_4 \hat{O}_4 + \hat{S}_4 \hat{S}_4 + \hat{C}_4 \hat{C}_4) \right. \]
\[ -N(\hat{O}_4 \hat{V}_4 - \hat{V}_4 \hat{O}_4 - \hat{S}_4 \hat{S}_4 + \hat{C}_4 \hat{C}_4) \left( \frac{2\eta}{\theta_2} \right)^2 + D(\hat{O}_4 \hat{V}_4 - \hat{V}_4 \hat{O}_4 + \hat{S}_4 \hat{S}_4 - \hat{C}_4 \hat{C}_4) \left( \frac{2\eta}{\theta_2} \right)^2 \right\} . \]
Supersymmetry is broken on the antibranes, and indeed the amplitudes involve the new characters \( Q'_s \) and \( Q'_c \), corresponding to a chirally flipped supercharge, that may be obtained from eq. (4.2) upon the interchange of \( S_4 \) and \( C_4 \), as well as other non-supersymmetric combinations. The tadpole conditions determine the gauge group \([SO(16) \times SO(16)]_9 \times [USp(16) \times USp(16)]_5\), and the 99 spectrum is supersymmetric, with (1,0) vector multiplets for the \( SO(16) \times SO(16) \) gauge group and a hypermultiplet in the \( (16,16,1,1) \). On the other hand, the 55 spectrum is non supersymmetric and, aside from the \([USp(16) \times USp(16)]\) gauge vectors, contains quartets of scalars in the \( (1,1,16,16) \), right-handed Weyl fermions in the \( (1,1,120,1) \) and \( (1,1,1,120) \) and left-handed Weyl fermions in the \( (1,1,16,16) \). Finally, the ND sector, also non supersymmetric, comprises doublets of scalars in the \( (16,1,1,16) \) and in the \( (1,16,16,1) \), and additional (symplectic) Majorana-Weyl fermions in the \( (16,1,16,1) \) and \( (1,16,1,16) \). These fields are a peculiar feature of six-dimensional
space time, where the fundamental Weyl fermion, a pseudoreal spinor of $SU^*(4)$, can be subjected to a Majorana condition if this is supplemented by the conjugation in a pseudoreal representation. All irreducible gauge and gravitational anomalies cancel in this model, while the residual anomaly polynomial requires a generalized Green-Schwarz mechanism [36] with couplings more general than those found in supersymmetric models.

It should be appreciated that the resulting non-BPS configuration of branes and anti-branes has no tachyonic excitations, while the branes themselves experience no mutual forces. Brane configurations of this type have received some attention lately [37], and form the basis of earlier constructions of non-supersymmetric type I vacua [38] and of their tachyon-free reductions [39]. As a result, the contributions to the vacuum energy, localized on the antibranes, come solely from the Möbius amplitude. The resulting potential, determined by uncancelled D5 NS-NS tadpole, is

$$V_{\text{eff}} = c e^{-\phi_0} \frac{1}{\sqrt{v}} = c e^{-\phi_{10}} = \frac{c}{g_{\text{YM}}^2},$$

(4.5)

where $\phi_{10}$ is the 10D dilaton, that determines the Yang-Mills coupling $g_{\text{YM}}$ on the antibranes, and $c$ is a positive numerical constant. This potential (4.5) is clearly localized on the antibranes and positive, consistently with the interpretation of this mechanism as global supersymmetry breaking. One would also expect that, in the limit of vanishing D5 coupling, supersymmetry be recovered, at least from the D9 viewpoint. While not true in six dimensions, due to the peculiar chirality flip that we have described, the expectation is actually realized after compactification to four dimensions, with suitable subgroups of the antibrane gauge group realized as internal symmetries.

Several generalizations of this model have been discussed in [20]. Some include tachyon-free combinations of branes and antibranes of the same type, that extend the construction of [40]. This more general setting has the amusing feature of leading to the effective stabilization of some geometric moduli, while some of the resulting models, related to the $Z_3$ orientifold of [41], have interesting three-family spectra of potential interest for
phenomenology.

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References


[10] For a recent review, see A. Sen, hep-th/9802051.


\[22\] http://strings99.aei-potsdam.mpg.de/cgi-bin/viewit.cgi.


\[25\] For a review, see: A. Giveon, M. Porrati and E. Rabinovici, Phys. Rept. 244 (1994) 77.


