Suggestions for Improved Benchmark Scenarios
for Higgs-Boson Searches at LEP2

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Abstract

We suggest new benchmark scenarios for the Higgs-boson search at LEP2. Keeping $m_t$ and $M_{\text{SUSY}}$ fixed, we improve on the definition of the maximal mixing benchmark scenario defining precisely the values of all MSSM parameters such that the new $m_h^{\text{max}}$ benchmark scenario yields the parameters which maximize the value of $m_h$ for a given $\tan\beta$. The corresponding scenario with vanishing mixing in the scalar top sector is also considered. We propose a further benchmark scenario with a relatively large value of $|\mu|$, a moderate value of $M_{\text{SUSY}}$, and moderate mixing parameters in the scalar top sector. While the latter scenario yields $m_h$ values that in principle allow to access the complete $M_A$–$\tan\beta$-plane at LEP2, on the other hand it contains parameter regions where the Higgs-boson detection can be difficult, because of a suppression of the branching ratio of its decay into bottom quarks.

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1 Introduction and theoretical basis

Within the MSSM the masses of the \( \mathcal{CP} \)-even neutral Higgs bosons are calculable in terms of the other MSSM parameters. The mass of the lightest Higgs boson, \( m_h \), has been of particular interest as it is bounded from above according to \( m_h \leq M_Z \) at the tree level. The radiative corrections at one-loop order [1, 2] have been supplemented in the last years with the leading two-loop corrections, performed by renormalization group (RG) methods [3, 4], by renormalization group improvement of the one-loop effective potential calculation [5, 6], by two-loop effective potential calculations [7, 8], and in the Feynman-diagrammatic (FD) approach [9, 10]. These calculations predict an upper bound for \( m_h \) of about \( m_h < \sim 130 \text{ GeV} \).\(^1\)

The numerical evaluations of the neutral \( \mathcal{CP} \)-even Higgs-boson masses are implemented in two Fortran codes that are used for phenomenological studies by the LEP collaborations: the program \texttt{subhpole} corresponding to the RG calculation [5], and the program \texttt{FeynHiggs} [11], corresponding to the result of the FD calculation.

The tree-level value for \( m_h \) within the MSSM is determined by \( \tan \beta \), the \( \mathcal{CP} \)-odd Higgs-boson mass \( M_A \), and the \( Z \)-boson mass \( M_Z \). Beyond the tree-level, the main correction to \( m_h \) stems from the \( t-\tilde{t} \)-sector, and for large values of \( \tan \beta \) also from the \( b-\tilde{b} \)-sector.

In order to fix our notations, we list the conventions for the inputs from the scalar top and scalar bottom sector of the MSSM: the mass matrices in the basis of the current eigenstates \( \tilde{t}_L, \tilde{t}_R \) and \( \tilde{b}_L, \tilde{b}_R \) are given by

\[
M^2_{\tilde{t}_L} = \left( \frac{m^2_t + m^2_t + \cos 2\beta \left( \frac{1}{2} - \frac{2}{3} s^2_W \right) M^2_Z}{m_t X_t} m_t X_t \right) M^2_{\tilde{t}_R} + \frac{m^2_t + \frac{2}{3} \cos 2\beta s^2_W M^2_Z}{M^2_{\tilde{t}_L}} ,
\]

\[
M^2_{\tilde{b}_L} = \left( \frac{m^2_b + m^2_b + \cos 2\beta \left( -\frac{1}{2} + \frac{1}{3} s^2_W \right) M^2_Z}{m_b X_b} m_b X_b \right) M^2_{\tilde{b}_R} + \frac{m^2_b - \frac{1}{3} \cos 2\beta s^2_W M^2_Z}{M^2_{\tilde{b}_L}} ,
\]

where

\[
m_t X_t = m_t (A_t - \mu \cot \beta) , \quad m_b X_b = m_b (A_b - \mu \tan \beta) .
\]

Here \( A_t \) denotes the trilinear Higgs–stop coupling, \( A_b \) denotes the Higgs–sbottom coupling, and \( \mu \) is the Higgs mixing parameter.

SU(2) gauge invariance leads to the relation

\[
M_{\tilde{t}_L} = M_{\tilde{b}_L} .
\]

For the numerical evaluation, a convenient choice is

\[
M_{\tilde{t}_L} = M_{\tilde{b}_L} = M_{\tilde{t}_R} = M_{\tilde{b}_R} =: M_{\text{SUSY}} ;
\]

this has been shown to yield upper values for \( m_h \) which comprise also the case where \( M_{\tilde{t}_R} \neq M_{\tilde{t}_L} \neq M_{\tilde{b}_R} \), when \( M_{\text{SUSY}} \) is identified with the heaviest one [10]. We furthermore use the short-hand notation

\[
M^2_S := M^2_{\text{SUSY}} + m^2_t .
\]

\(^1\)This value holds for \( m_t = 175 \text{ GeV} \) and \( M_{\text{SUSY}} = 1 \text{ TeV} \). If \( m_t \) is raised by 5 GeV then the \( m_h \) limit is increased by about 5 GeV; using \( M_{\text{SUSY}} = 2 \text{ TeV} \) increases the limit by about 2 GeV.
Accordingly, the most important parameters for the corrections to \( m_h \) are \( m_t \), \( M_{\text{SUSY}} \), \( X_t \), and \( X_b \). The mass of the lightest \( \mathcal{CP} \)-even Higgs boson depends furthermore on the SU(2) gaugino mass parameter, \( M_2 \). The other gaugino mass parameter, \( M_1 \), is usually fixed via the GUT relation

\[
M_1 = \frac{5}{3} \frac{s_W^2}{c_W^2} M_2. \tag{7}
\]

At the two-loop level also the gluino mass, \( m_{\tilde{g}} \), enters the prediction for \( m_h \). In \textit{FeynHiggs} the gluino mass can be specified as a free input parameter. The effect of varying \( m_{\tilde{g}} \) on \( m_h \) is up to \( \pm 2 \text{ GeV} \) for large mixing in the \( \tilde{t} \)-sector and below \( \pm 0.5 \text{ GeV} \) for vanishing mixing [10]. Within \textit{subhpole}, the gluino mass was fixed to \( m_{\tilde{g}} = M_{\text{SUSY}} \). Compared to the maximal values of \( m_h \) (obtained for \( m_{\tilde{g}} \approx 0.8 M_{\text{SUSY}} \)) this leads to a reduction of the Higgs-boson mass by up to 0.5 GeV. Within the new version, \textit{subhpole2}, arbitrary values of the gluino mass will be allowed as input.

It should be noted in this context that the FD result has been obtained in the on-shell (OS) renormalization scheme, whereas the RG result has been calculated using the \( \overline{\text{MS}} \) scheme. Owing to the different schemes used in the FD and the RG approach for the renormalization in the scalar top sector, the parameters \( X_t \) and \( M_{\text{SUSY}} \) are also scheme-dependent in the two approaches. This difference between the corresponding parameters has to be taken into account when comparing the results of the two approaches. In a simple approximation the relation between the parameters in the different schemes is given by [12]

\[
M_S^{2,\overline{\text{MS}}} = M_S^{2,\text{OS}} - \frac{8}{3} \frac{\alpha_s}{\pi} M_S^2, \tag{8}
\]

\[
X_t^{\overline{\text{MS}}} = X_t^{\text{OS}} + \frac{\alpha_s}{3\pi} M_S \left( 8 + 4 \frac{X_t}{M_S} - 3 \frac{X_t}{M_S} \ln \left( \frac{m_t^2}{M_S^2} \right) \right), \tag{9}
\]

where in the terms proportional to \( \alpha_s \) it is not necessary to distinguish between \( \overline{\text{MS}} \) and on-shell quantities, since the difference is of higher order. The \( \overline{\text{MS}} \) top-quark mass, \( m_t^{\overline{\text{MS}}}(m_t) \equiv \overline{m}_t \), is related to the top-quark pole mass, \( m_t^{\text{OS}} \equiv m_t \), in \( \mathcal{O}(\alpha_s) \) by

\[
\overline{m}_t = \frac{m_t}{1 + \frac{4}{3\pi} \alpha_s(m_t)}. \tag{10}
\]

While the resulting shift in the parameter \( M_{\text{SUSY}} \) turns out to be relatively small in general, sizable differences can occur between the numerical values of \( X_t \) in the two schemes, see Refs. [10,12]. For this reason we specify below different values for \( X_t \) within the two approaches.

The results of the RG and the FD calculation have been compared in detail in Refs. [13,12]. While the results agree in the logarithmic terms at the two-loop level [12], the FD result (program \textit{FeynHiggs}) contains further genuine two-loop corrections that are not present in the RG calculation. These corrections lead to an increase in the maximal values for \( m_h \) by up to 4 GeV. Within the one-loop effective potential computation, for large values of \( M_A \) and \( M_S \), \( m_{\tilde{g}} = M_{\text{SUSY}} \), and not too large mixing parameters in the scalar top sector, the bulk of these corrections is taken into account by incorporating the proper one-loop relation between the running top quark mass at the scale \( m_t \) and the one at the scale \( M_S \) when computing the finite threshold corrections to the Higgs quartic coupling at the scale \( M_S \) [12].
The proper relation between $m_t$ and $m_t(M_S)$ will be used in the program based on the RG improved one-loop effective potential calculation ($\text{subhpole2}$) available for public use in the near future.

2 The benchmark scenarios

By combining the theoretical result for the upper bound on $m_h$ as a function of $\tan \beta$ within the MSSM with the informations from the direct search for the lightest Higgs boson, it is possible to derive constraints on $\tan \beta$. Since the predicted value of $m_h$ depends sensitively on the precise numerical value of $m_t$, it has become customary to discuss the constraints on $\tan \beta$ within a so-called maximal mixing “benchmark” scenario [14]. In this scenario, $m_t$ was kept fixed at the value $m_t = 175$ GeV, a large value of $M_{\text{SUSY}}$ was chosen, $M_{\text{SUSY}} = 1$ TeV, and the mixing parameter in the stop sector was fixed in order to maximize the stop-induced radiative corrections to the lightest $\mathcal{CP}$-even Higgs-boson mass as a function of $\tan \beta$, $m_h(\tan \beta)$. For a recent analysis within this framework, see e.g. Ref. [15].

In this note we shall define an improved version of the maximal mixing benchmark scenario that keeps many of the features of the previous one, but maximizes also the chargino and neutralino contributions by taking small values of the $|\mu|$ and $M_2$ parameters, while yielding chargino masses which are beyond the reach of LEP2. This scenario maximizes the Higgs-boson mass as a function of $\tan \beta$ for fixed $m_t$ and $M_{\text{SUSY}}$ ($m_h^{\text{max}}$ scenario), and should therefore be useful in order to derive conservative bounds on $\tan \beta$. The $m_h^{\text{max}}$ scenario defined here is close to the one recently proposed in Ref. [16], where it was analyzed how the previous benchmark scenario [15] should be modified in order to incorporate the maximal values of $m_h(\tan \beta)$. An analysis of the experimental lower bound on $\tan \beta$, studying its dependence on the $\tilde{t}$-mass eigenvalues and the mixing angle was performed in Ref. [17]. The values of $\mu$ and $M_2$ in Ref. [17] were similar to those proposed here for the $m_h^{\text{max}}$ scenario.

In the following we will consider the $m_h^{\text{max}}$ scenario as well as the corresponding scenario with vanishing mixing in the scalar top sector. We furthermore suggest a third scenario, in which a relatively large value of $|\mu|$ is adopted, leading to interesting phenomenological consequences.

In all benchmark scenarios we fix the top-quark mass to its experimental central value,

$$m_t = m_t^{\text{exp}} \ (= 174.3 \text{ GeV}),$$

where we have indicated the current value for completeness. It should be kept in mind that internally the codes $\text{subhpole}$ and $\text{FeynHiggs}$ make use of the running top-quark mass, $\overline{m}_t$. In comparing results of different codes it is essential that not only the input value for the top-quark pole mass is the same, but also the relation employed for deriving the running top-quark mass. In $\text{subhpole}$ and $\text{FeynHiggs}$ $\overline{m}_t$ is calculated from $m_t$ according to eq. (10), taking into account corrections up to $\mathcal{O}(\alpha_s)$.

Although the soft SUSY breaking parameter $M_{\text{SUSY}}$ is renormalization-scheme-dependent, the numerical effect of the scheme dependence is rather small in general. We have checked that for the scenarios below the numerical difference of the corresponding values of the parameter $M_{\text{SUSY}}$ in the two schemes lies within about 4%. We therefore do not distinguish
between the parameters in the two schemes and define

\[ M_{\text{SUSY}}^{\overline{\text{MS}}} \approx M_{\text{SUSY}}^{\overline{\text{OS}}} =: M_{\text{SUSY}}. \] (12)

2.1 The \( m_{h}^{\text{max}} \) scenario

In this benchmark scenario the parameters are chosen such that the maximum possible Higgs-boson mass as a function of \( \tan \beta \) is obtained (for fixed \( M_{\text{SUSY}} \), \( m_t \) given by its experimental central value, and \( M_A \) set to its maximal value in this scenario, \( M_A = 1 \) TeV). The parameters are:

\[
\begin{align*}
M_{\text{SUSY}} &= 1 \text{ TeV} \\
\mu &= -200 \text{ GeV} \\
M_2 &= 200 \text{ GeV} \\
m_{\tilde{g}} &= 0.8 M_{\text{SUSY}} \\
M_A &\leq 1000 \text{ GeV} \\
X_t^{\overline{\text{OS}}} &= 2 M_{\text{SUSY}} \quad (\text{FD calculation}) \\
X_t^{\overline{\text{MS}}} &= \sqrt{6} M_{\text{SUSY}} \quad (\text{RG calculation}) \\
A_b &= A_t.
\end{align*}
\] (13)

The values for \( X_t \) in the FD calculation (\textit{FeynHiggs}) and in the RG calculation (\textit{subhpole}) specify the mixing in the scalar top sector in both approaches in such a way that \( m_h \) becomes maximal. The values of \( \mu \) and \( M_2 \) are close to their experimental lower bounds. Slightly higher Higgs-boson masses are obtained for smaller \( |\mu| \) and smaller \( M_2 \). The sign of \( \mu \) has only a small effect in this scenario.

One should take into account that the maximal value of the lightest \( CP \)-even Higgs-boson mass would increase with respect to the \( m_{h}^{\text{max}} \) benchmark scenario if, for instance, the 1\( \sigma \) upper bound on the experimental value of the top-quark mass were considered, or if the third generation squark masses were larger than the ones chosen in the benchmark scenario.

2.2 The no-mixing scenario

This benchmark scenario is the same as the \( m_{h}^{\text{max}} \) scenario, but with vanishing mixing in the \( \tilde{t} \)-sector. The parameters are:

\[
\begin{align*}
M_{\text{SUSY}} &= 1 \text{ TeV} \\
\mu &= -200 \text{ GeV} \\
M_2 &= 200 \text{ GeV} \\
m_{\tilde{g}} &= 0.8 M_{\text{SUSY}} \\
M_A &\leq 1000 \text{ GeV} \\
X_t^{\overline{\text{OS}}} &= 0 \quad (\text{FD calculation}) \\
X_t^{\overline{\text{MS}}} &= 0 \quad (\text{RG calculation}) \\
A_b &= A_t,
\end{align*}
\] (14)
where we have neglected the difference between $X_t^{OS}$ and $X_t^{MS}$, which is of minor importance in this scenario.

The difference of the $m_h$ values in the $m_h^{\text{max}}$ and the no-mixing scenario is purely an effect of the mixing in the scalar top sector. For a common $M_{\text{SUSY}}$ and low values of $|\mu|$ and $M_2$, as assumed above, restrictions on the mixing parameters in the $t$-sector as a function of $\tan \beta$ can be derived by demanding the Higgs-boson mass to be above the experimental limit. This is due to the fact that for low values of $\tan \beta$ experimentally acceptable values of $m_h$ can only be achieved for non-vanishing mixing parameters in the scalar top sector.

2.3 The large $\mu$ scenario

This benchmark scenario is characterized by a relatively large value of $|\mu|$ (compared to $M_{\text{SUSY}}$). We furthermore adopt a relatively small value of $M_{\text{SUSY}}$ and moderate mixing in the scalar top sector. The parameters are:

$$
\begin{align*}
M_{\text{SUSY}} & = 400 \text{ GeV} \\
\mu & = 1 \text{ TeV} \\
M_2 & = 400 \text{ GeV} \\
m_{\tilde \phi} & = 200 \text{ GeV} \\
M_A & \leq 400 \text{ GeV} \\
X_t^{OS} & = -300 \text{ GeV} \quad \text{(FD calculation)} \\
X_t^{MS} & = -300 \text{ GeV} \quad \text{(RG calculation)} \\
A_b & = A_t \\
m_b & \equiv m_b(m_t) = 3 \text{ GeV} \quad \text{(FD calculation)}.
\end{align*}
$$

Here we have neglected the difference between $X_t^{OS}$ and $X_t^{MS}$. This will slightly affect the comparison between the FD and the RG result, but will be of minor relevance for the general features of the large $\mu$ scenario which are discussed in the following. The value of the bottom mass, $m_b = 3$ GeV, specified for the FD calculation is chosen in order to absorb higher-order QCD corrections that are important to keep the effects of large mixing in the scalar bottom sector, which occur for large $\mu$ and $\tan \beta$, under control. In subhpole this is already taken into account internally.

As a consequence of the relatively low values of $M_{\text{SUSY}}$ and the mixing parameter in the $t$-sector chosen in this scenario, considerably lower Higgs-boson masses are obtained compared to the $m_h^{\text{max}}$ scenario. Therefore, in the large $\mu$ scenario defined here LEP2 has the potential of covering the whole $m_h-\tan \beta$-plane. It should furthermore be noted that for large values of $\tan \beta$ in this scenario radiative corrections from the scalar bottom sector become important, which, for instance, lead to a decrease of the predicted value for $m_b$ for moderate or large values of the CP-odd Higgs mass $M_A \gtrsim 150$ GeV.

On the other hand, this scenario also gives rise to regions in the MSSM parameter space where the Higgs-boson detection might be difficult, since there exist “pathological” points for which either $BR(h \to b\bar{b}) \to 0$ or $BR(H \to b\bar{b}) \to 0$ [18, 19]. Although the relevant Higgs-boson mass will in principle be within the kinematically accessible region, the non-
standard decay signatures may lead to difficulties in actually detecting the particle. For a recent analysis in this context see Refs. [19,20].

The condition, whether corrections in the Higgs sector lead to a vanishing effective coupling $h \bar{b}b$ (and consequently to $BR(h \rightarrow b \bar{b}) \rightarrow 0$ or $BR(H \rightarrow b \bar{b}) \rightarrow 0$), depends in particular on the signs and magnitudes of $\mu A_t$ and $\mu A_b$ and also on the value of $|A_t|$ [19]. Changing the sign of $X_t$ in eq. (15) leads to a scenario with similar $m_h$ values, where the radiative corrections in the Higgs sector do not give rise to “pathological” points in the $m_h-\tan \beta$-plane with vanishing branching ratio of the $CP$-even Higgs-boson decays into bottom quarks.

For large $\mu$, $\tan \beta$, and $m_{\tilde{g}}$ large SUSY-QCD corrections to the $h \bar{b}b$ vertex are possible that could make a perturbative calculation questionable [21–23, 19, 20]. Even for the relatively low value of $m_{\tilde{g}} = 200$ GeV chosen in the present scenario, very large vertex corrections from gluino exchange appear in the large $\tan \beta$ region, which can also lead to a suppression of the branching ratio into bottom quarks. It should furthermore be noted that not only the SUSY-QCD vertex corrections but also the genuine electroweak vertex contributions can become relevant. In order to obtain reliable predictions in this region of parameters, the inclusion of leading higher-order contributions is important. A proper treatment of the vertex corrections in the region of large values of $\tan \beta$ will be incorporated in the versions of the programs FeynHiggs2.0 and subhpole2 available for public use in the near future.

The same kind of SUSY-QCD corrections affects the value of the electroweak precision parameter $\Delta \rho$, which, for the values of the parameters chosen in this scenario, can exceed the experimentally allowed values for extra SUSY contributions, $\Delta \rho^{SUSY} \lesssim 10^{-3}$, for very large values of $\tan \beta$. The value of $\Delta \rho$, based on a two-loop calculation [24], is given as an output of FeynHiggs as a consistency check of the calculation. A thorough treatment of the higher order SUSY-QCD corrections is also important in this case.

Besides the suppression of the main decay channel, a problem for detecting the MSSM Higgs bosons can of course also arise from a suppression of the kinematically favored production cross section, i.e. $e^+e^- \rightarrow hZ$ or $e^+e^- \rightarrow hA$ [25]. For instance, at LEP2 this behaviour occurs in the $m_h^{\max}$ scenario for relatively large values of $\tan \beta$ and values of $M_A$ such that the lightest $CP$-even Higgs-boson mass is just above the kinematical threshold of the $hA$ channel. In this region of parameters, the lightest $CP$-even Higgs boson is within the kinematical reach of the $hA$ channel, but its coupling to the $Z$ boson is suppressed. For the large $\mu$ scenario, instead, we find that at least one of the production channels $e^+e^- \rightarrow hZ, HZ, hA$ should always be open within the kinematical reach with a sufficiently high rate. The reason for this can qualitatively be understood from the fact that the cross sections for the above production channels are approximately proportional to $\sin^2(\beta - \alpha)$, $\cos^2(\beta - \alpha)$, $\cos^2(\beta - \alpha)$, respectively, and from the relation [20]

$$m_h^2 \sin^2(\beta - \alpha) + m_H^2 \cos^2(\beta - \alpha) = m_h^2|_{M_A^2 \gg M_Z^2}.$$  

In the above, the quantities on the left-hand side are given as functions of arbitrary values of $M_A$ and the other MSSM parameters, while the right-hand side is the square of the lightest $CP$-even Higgs-boson mass for $M_A^2 \gg M_Z^2$ and the same values of the other MSSM parameters, i.e. the upper bound on $m_h$ for this set of parameters. In the large $\mu$ scenario, the upper bound on $m_h$ is about 107 GeV, which is within the kinematical reach of LEP2,
and is only obtained for relatively large values of $\tan \beta$. Therefore, the suppression of the $hZ$ or $HZ$ production cross section by very small values of one of these mixing angles implies that the complementary cross section will be of the order of the Standard Model one and that the corresponding Higgs boson is within the LEP2 kinematical reach.

3 Conclusions

We have suggested three benchmark scenarios for the Higgs-boson search at LEP2, which improve and extend the previous benchmark definitions used in the literature. The $m_h^{\text{max}}$ scenario yields the theoretical upper bound of $m_h$ in the MSSM as a function of $\tan \beta$ for fixed $m_t$ and $M_{\text{SUSY}}$. It thus allows to derive conservative constraints on $\tan \beta$ from the Higgs-boson search under the assumption that $m_t$ is given by its experimental central value and $M_{\text{SUSY}} = 1$ TeV. In the no-mixing scenario the mixing in the scalar top sector is chosen to be zero, while the other parameters are the same as in the $m_h^{\text{max}}$ scenario. Comparing the two scenarios allows to investigate the effects of mixing in the scalar top sector. As a new benchmark scenario, we propose a scenario which is characterized by a relatively large value of $|\mu|$. Moderate values are chosen for $M_{\text{SUSY}}$ and the mixing parameter in the $t$-sector. The values of the Higgs-boson masses obtained in this scenario are such that in principle a complete coverage of the $m_h$–$\tan \beta$-plane would be possible at LEP2. However, the scenario contains parameter regions in which the $BR(h \rightarrow b\bar{b})$ or $BR(H \rightarrow b\bar{b})$ is suppressed and which therefore will be difficult to access, although the corresponding Higgs-boson mass would be within the kinematical reach. Thus, other decay modes of the Higgs boson beyond the $b\bar{b}$ channel should carefully be considered in this region of parameters.

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