ABSTRACT

In recent years, the evidence for the existence of an ultra-compact concentration of dark mass associated with the radio source Sgr A* in the Galactic Center has become very strong. However, an unambiguous proof that this object is indeed a black hole is still lacking. A defining characteristic of a black hole is the event horizon. To a distant observer, the event horizon casts a relatively large “shadow” with an apparent diameter of ~10 gravitational radii due to bending of light by the black hole, nearly independent of the black hole spin or orientation. The predicted size (~30μarcseconds) of this shadow for Sgr A* approaches the resolution of current radio-interferometers. If the black hole is maximally spinning and viewed edge-on, then the shadow will be offset by ~8 parsecs from the center of mass, and will be slightly flattened on one side. Taking into account scatter-broadening of the image in the interstellar medium and the finite achievable telescope resolution, we show that the shadow of Sgr A* may be observable with very long-baseline interferometry at sub-millimeter wavelengths, assuming that the accretion flow is optically thin in this region of the spectrum. Hence, there exists a realistic expectation of imaging the event horizon of a black hole within the next few years.

Subject headings: black hole physics — relativity — Galaxy: center — galaxies: active — submillimeter — techniques: interferometric

1. INTRODUCTION

High resolution spectroscopy (especially with the Hubble Space Telescope) of galactic nuclei has produced an abundance of evidence for compact dark mass concentrations of up to $10^8 M_\odot/pc^3$, whose nature is strongly suspected to be indicative of supermassive black holes (Kormendy & Richstone 1995). Even better evidence exists for the galaxy NGC 4258 and the Milky Way, for which long-baseline interferometry (VLBI) reveal the presence of jets-like outflow (Falcke, Mannheim, & Biermann 1993, Falcke et al. 1998). The presence of compact radio source Sgr A* lies at the dynamical origin (Menten et al. 1997, Ghez et al. 1998). The nature of Sgr A* is still unclear, since its structure is completely washed out by strong interstellar scattering at cm-wavelengths (Lo et al. 1998). It is only at millimeter-wavelengths that we may begin to see some internal structure (Bower & Backer 1998, Krichbaum et al. 1998). Though the dark mass concentration could in principle be distributed in the form of exotic objects on a scale slightly larger than the size of Sgr A*—but with difficulties accounting for its radiation characteristics (Melia & Coker 1999)—it is expected to be associated with Sgr A* itself, since the latter, unlike the surrounding stars, has a tightly restricted proper motion indicating that it is very heavy (Reid et al. 1999, Genzel et al. 1997).

The key spectral features of Sgr A* are a slightly inverted cm-wavelength spectrum, an apparent excess (or bump) at sub-millimeter (sub-mm) wavelengths, and a steep cut-off towards the infrared (Falcke et al. 1998, Serabyn et al. 1997). The radio emission is circularly polarized but undetected in linear polarization (Bower et al. 1999a, Bower et al. 1999b). Proposed models for the radio emission range from quasi-spherical inflows (Melia 1992, Melia 1994, Narayan, Yi, & Mahadevan 1995) to a jet-like outflow (Falcke, Mannheim, & Biermann 1993, Falcke & Biermann 1999).

The sub-mm bump is particularly interesting since this should be the signature of a very compact synchrotron emitting region with a size of a few Schwarzschild radii (Falcke 1996, Falcke et al. 1998). The presence of compact radio emission in Sgr A* at a wavelength as short as 1.4 mm has been confirmed recently by a first VLBI detection at this wavelength (Krichbaum et al. 1998). This detection is exciting for several reasons. First, it lies in a region of the spectrum where the intrinsic source size should become apparent over scatter-broadening by the intervening screen (Melia, Jokipii, & Narayan 1992). Second, this component is sufficiently bright to be detected with VLBI techniques at even shorter wavelengths, and third, Sgr A* is sufficiently close that the size scale where general relativistic effects are significant could be resolved with VLBI at sub-mm wavelengths. In addition, at sub-mm wavelengths, the various models predict that the synchrotron emission is not self-absorbed, allowing a view into...
the region near the horizon. The horizon has a size of 
\((1 + \sqrt{1 - a^2})R_g\), where \(R_g \equiv GM/c^2\), \(M\) is the mass of the black hole, \(G\) is Newton's constant, \(c\) the speed of light, \(a_* \equiv Jc/(GM^2)\) is the dimensionless spin of the black hole in the range 0 to 1, and \(J\) is the angular momentum of the black hole.

Bardeen (1973) described the idealized appearance of a black hole in front of a planar emitting source, showing that it literally would appear as a ‘black hole’. At that time such a calculation was of mere theoretical interest and limited to just calculating the envelope of the apparent black hole. To test whether there is a realistic chance of seeing this ‘black hole’ in Sgr A* (Falcke et al. 1998), we here report the first calculations obtained with our general relativistic (GR) ray-tracing code that allows us to simulate observed images of Sgr A* for various combinations of black hole spin, inclination angle, and morphology of the emission region directly surrounding the black hole and not just for a background source. A more detailed description of our calculations is in preparation (Agol, Falcke, & Melia 1999).

2. THE APPEARANCE OF A BLACK HOLE

We determine the appearance of the emitting region around a black hole under the condition that it is optically thin. For Sgr A* this might be the case for the sub-mm bump (Falcke et al. 1998) indicated by the turnover in overall specific intensity, \(I_\nu\), observed at infinity is an integral of the emissivity, \(j_\nu\), times the differential path length along geodesics (Jaroszynski & Kurpiewski 1997). In line with the qualitative discussion of this paper, we assume that \(j_\nu\) is independent of frequency, and that it is either spatially uniform, or scales as \(r^{-2}\). These two cases cover a large range of conditions expected under several reasonable scenarios, be it a quasi-spherical infall, a rotating thick disk, or the base of an outflow.

The calculation of the photon trajectories and the intensity integrated along the line-of-sight is based on the standard formalism (Thorne 1981, Viergutz 1993, Rauch & Blandford 1994, Jaroszynski & Kurpiewski 1997). Our calculations take into account all the well-known relativistic effects, e.g., frame dragging, gravitational redshift, light bending, and Doppler boosting. The code is valid for all possible spins of the black hole and for any arbitrary velocity field of the emission region.

For a planar emitting source behind a black hole, a closed curve on the sky plane divides a region where geodesics intersect the horizon from a region whose geodesics miss the horizon (Bardeen 1973). This curve, which we refer to as the “apparent boundary” of the black hole, is a circle of radius \(\sqrt{27}R_g\) in the Schwarzschild case \((a_*=0)\), but has a more flattened shape of similar size for a Kerr black hole, slightly dependent on inclination. The size of the apparent boundary is much larger than the event horizon due to strong bending of light by the black hole. When the emission occurs in an optically thin region surrounding the black hole, the case of interest here, the apparent boundary has the same exact shape since the properties of the geodesics are independent of where the sources are located. However, photons on geodesics located within the apparent boundary that can still escape to the observer experience strong gravitational redshift and a shorter total path length, leading to a smaller integrated emissivity, while photons just outside the apparent boundary can orbit the black hole near the circular photon radius several times, adding to the observed intensity (Jaroszynski & Kurpiewski 1997). This produces a marked deficit of the observed intensity inside the apparent boundary, which we refer to as the “shadow” of the black hole.

We here consider a compact, optically-thin emitting region surrounding a black hole with spin parameter \(a_*=0\) (i.e., a Schwarzschild black hole) and a maximally spinning Kerr hole with \(a_*=0.998\). In the set of simulations shown here, we take the viewing angle \(i\) to be \(45^\circ\) with respect to the spin axis (when it is present), and we consider two distributions of gas velocity \(v\). The first has the plasma in free-fall, i.e., \(v^r = -\sqrt{2(r/a^2 + r^2)}\Delta/a\) and \(\Omega = 2\pi/r/A\), where \(v^r\) is the Boyer-Lindquist radial velocity, \(\Omega\) is the orbital frequency, \(\Delta \equiv r^2 - 2r + a^2\), and \(A \equiv (r^2 + a^2)^2 - a^2\Delta \sin^2 \theta\). (We have set \(G = M = c = 1\) in this paragraph.) The second has the plasma orbiting in rigidly rotating shells with the equatorial Keplerian frequency \(\Omega = 1/(r^{3/2} + a)\) for \(r > r_{\text{ms}}\) with \(v^\theta = 0\), and infalling with constant angular momentum inside \(r < r_{\text{ms}}\) (Cunningham 1975), with \(v^\phi = 0\) for all \(r\).

In order to display concrete examples of how realistic our proposed measurements of these effects with VLBI will be, we have simulated the expected images for the massive black hole candidate Sgr A* at the Galactic Center. For its measured mass (Eckart & Genzel 1996, Ghez et al. 1998) \(M = 2.6 \times 10^8 \, M_\odot\), the scale size for this object is the gravitational radius \(R_g = 3.9 \times 10^{-3}\) cm, which is half of the Schwarzschild radius \(R_s = 2GM/c^2\).

To simulate an observed image we have to take two additional effects into account: interstellar scattering and the finite telescope resolution achievable from the ground. Scattered-broadening at the Galactic Center is incorporated by smoothing the image with an elliptical Gaussian with a FWHM of \(21.2 \mu\text{arcsec} \times (\lambda/1.3\,\text{mm})^2\) along the major axis and 12.8 \(\mu\text{arcsec} \times (\lambda/1.3\,\text{mm})^2\) along the minor axis (Lo et al. 1998). The position angle of this ellipse is arbitrary since we do not know yet the spin axis of the black hole on the sky and we have assumed PA = 90° for the major axis. The telescope resolution— in an idealized form—is then added by convolving the smoothed image with a spherical Gaussian point-spread function of FWHM 33.5 \(\mu\text{arcsec} \times (\lambda/1.3\,\text{mm})^{-1} (l/8000\,\text{km})^{-1}\) —the possible resolution of a global interferometer with 8000 km baselines (Krichbaum 1996). In reality the exact point-spread-function will of course depend on the number and placement of the participating telescopes.

In Figure 1, we show the resulting image of Sgr A* for a maximally rotating black hole viewed at an angle of \(i = 45^\circ\), for a compact region in free fall, with an emissivity \(j_\nu = \nu^0 r^{-2}\). We first show the original, unsmoothed image of the emission region as calculated with the GR code in panel (a), and then present the simulated ‘observed’ images at 0.6 and 1.3 mm wavelengths in panels (b) and (c), respectively. The two distinct features that are evident in Figure 1a are (1) the clear depression in \(I_\nu\)—the shadow—produced near the black hole, which in this
Fig. 1.—An image of an optically thin emission region surrounding a black hole with the characteristics of Sgr A* at the Galactic Center. The black hole is here either maximally rotating ($a_*=0.998$, Figs. 1a–c) or non-rotating ($a_*=0$, Figs. 1d–f). The emitting gas is assumed to be in free fall with an emissivity $\propto r^{-2}$ (top) or on Keplerian shells (bottom) with a uniform emissivity (viewing angle $i=45^\circ$). Figs. 1a&d show the GR ray-tracing calculations, Figs. 1b&e are the images seen by an idealized VLBI array at 0.6 mm wavelength taking interstellar scattering into account, and Figs. 1c&f are those for a wavelength of 1.3 mm. The intensity variations along the $x$-axis (solid green curve) and the $y$-axis (dashed purple curve) are overlayed. The vertical axes show the intensity of the curves in arbitrary units and the horizontal axes show the distance from the black hole in units of $R_g$ which for Sgr A* is $3.9 \times 10^{11}$ cm $\sim 3$ $\mu$arcseconds. Particular example represents a modulation of up to 90% in intensity from peak to trough, and (2) the size of the shadow, which here is $9.2R_g$ in diameter. This represents a projected size of 27 $\mu$arcseconds, which is already within a factor of two of the current VLBI resolution (Krichbaum et al. 1995). The shadow is a generic feature of various other models we have looked at, including those with outflows, cylindrical emissivity, and various inclinations or spins.

To illustrate the expected image for another extreme case, we show in Figure 1d the analogue to Figure 1a for the case with $a_*=0$ (i.e., no rotation), an emitting plasma orbiting in Keplerian shells (as described above), and a uniform $j_{\nu}$ for $r<25R_g$. Even though these conditions are distinctly different compared to those of Figure 1a, the black hole shadow is still clearly evident, here representing a modulation in $I_{\nu}$ in the range of 50–75% from peak to trough (Fig. 1d), and with a diameter of roughly 10.4 $R_g$. In this case, the emission is asymmetric due to the strong Doppler shifts associated with the emission by a rapidly moving plasma along the line-of-sight (with velocity $v_\phi$).

The important conclusion is that the diameter of the shadow—in marked contrast to the event horizon—is fairly independent of the black hole spin and is always of order $10R_g$. Indeed, this is consistent with the observed 0.8 mm size limit $>4R_g$ of Sgr A* from a lack of scintillation (Gwinn et al. 1991). The presence of a rotating hole viewed edge-on will lead to a shifting of the apparent boundary (by as much as $2.5R_g$, or 8 $\mu$arcseconds) with respect to the center of mass, or the centroid of the outer emission region.

Interestingly, the scattering size of Sgr A* and the resolution of global VLBI arrays become comparable to the size of the shadow at a wavelength of about 1.3 mm. As one can see from Figures 1c&f the shadow is still almost completely washed out for VLBI observations at 1.3 mm, while it is very apparent at a factor two shorter wavelength (Figures 1b&e). In fact, already at 0.8 mm (not shown here) the shadow can be easily seen. Under certain conditions, i.e., a very homogeneous emission region, the shadow would be visible even at 1.3 mm (Fig. 1f).

3. HOW REALISTIC IS SUCH AN EXPERIMENT?

The arguments for the feasibility of such an experiment are rather compelling. First of all, the mass of Sgr A* is very well known within 20%, the main uncertainty being the exact distance to the Galactic Center. Since, as we have shown, the unknown spin of the suspected black hole contributes only another 10% uncertainty, we can conservatively predict the angular diameter of the shadow in Sgr A* from the GR calculations alone to be $\sim 30 \pm 7$ $\mu$arcseconds independent of wavelength. As seen in Fig. 1, the finite telescope resolution and the scatter broadening will make the detectability of the shadow a function of wavelength and emissivity; however, the size of the shadow will remain of similar order and under no circumstances can become smaller.

The technical methods to achieve such a resolution at wavelengths shortwards of 1.3 mm are currently being de-
veloped and a first detection of Sgr A* at 1.4 mm with VLBI has already been reported. The challenge will be to push this technology even further towards 0.8 or even 0.6 mm VLBI. Over the next decade many more telescopes are expected to operate at these wavelengths. Depending on how short a wavelength is required, the projected time scale for developing the necessary VLBI techniques may be about ten years. A fundamental problem preventing such an experiment is not now apparent, but in light of our results, planning of the new sub-mm-telescopes should include sufficient provisions for VLBI experiments.

A potential problem with our model may occur if $j_0$ has an inner cutoff which is larger than that of the horizon, making the shadow larger than predicted due to a decrease in emissivity rather than to GR effects. However, first of all, the truncation of accretion disk emission at the marginal stable orbit $r_{\text{ms}}$ is somewhat arbitrary (Cunningham 1975) and, secondly, if it exists such a cutoff would likely be frequency dependent, while there will be a frequency-independent minimum radius due to the general relativistic effects we have described. Another problem could be the unknown morphology of the emission region. Anisotropy, strong velocity fields, and density inhomogeneities would make an identification of the shadow in an observed image more difficult. However, inhomogeneities are unlikely to be a major issue, since the time scale for rotation around the black hole in the Galactic Center is only a few hundred seconds and hence much less than the typical duration of a VLBI observation. The strong shear near the black hole would tend to smooth out any inhomogeneities very quickly. Indeed, sub-mm variability studies on such short time scales (Gwinn et al. 1991) have yielded negative results. The same argument applies to emission models which are offset from the black hole, e.g., are one-sided. Since the shadow of the black hole has a very well defined shape it would under any conditions appear as a distinct feature, given that the dynamic range of the map is large enough (i.e., $\nu > 10^{01}$, considering a range of emission models, Agol, Falcke, & Melia 1999).

Finally, synchrotron self-absorption could pose a problem. So far the available sub-mm spectra show a flattening of the spectrum around 1.3-0.6 mm indicating a turnover towards an optically thin spectrum. Given the current observational uncertainties one could in principle construct simple models where the flow does not become optically thin until 0.2 mm. Improved simultaneous measurements at sub-mm wavelengths are therefore highly desirable to exactly measure the spectral turnover since the experiment we propose here will only work for an optically thin flow. At hundreds of microns the atmosphere becomes optically thick, making much more expensive space-based observations necessary. At X-ray wavelengths, the accretion flow will be optically thin to electron scattering, so there may be a better chance of detecting the shadow with future space-based X-ray interferometry as proposed in the MAXIM experiment.

### 4. SUMMARY

The importance of the proposed imaging of Sgr A* at sub-mm wavelengths with VLBI cannot be overemphasized. The bump in the spectrum of Sgr A* strongly suggests the presence of a compact component whose proximity to the event horizon is predicted to result in a shadow of measurable dimensions in the intensity map. To our knowledge, such a feature is unique and Sgr A* seems to have all the right parameters to make it observable. The observation of this shadow would confirm the widely held belief that most of the dark mass concentration in the nuclei of galaxies such as ours is contained within a black hole, and it would be the first direct evidence of the existence of an event horizon. A non-detection with sufficiently developed techniques, on the other hand, might pose a major problem for the standard black hole paradigm. Because of this fundamental importance, the experiment we propose here should be a major motivation for intensifying the current development of sub-mm astronomy in general and mm- and sub-mm VLBI in particular.

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