Nuclear symmetry energy in presence of hyperons in the nonrelativistic Thomas-Fermi approximation

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Abstract

We generalise the finite range momentum and density dependent Seyler-Blanchard nucleon-nucleon effective interaction to the case of interaction between two baryons. This effective interaction is then used to describe dense hadronic matter relevant to neutron stars in the nonrelativistic Thomas-Fermi approach. We investigate the behaviour of nuclear symmetry energy in dense nuclear and hyperon matter relevant to neutron stars. It is found that the nuclear symmetry energy always increases with density in hyperon matter unlike the situation in nuclear matter. This rising characteristic of the symmetry energy in presence of hyperons may have significant implications on the mass-radius relationship and the cooling properties of neutron stars. We have also noted that with the appearance of hyperons, the equation of state calculated in this model remains causal at high density.
The study of matter far off from normal nuclear matter density is of interest in understanding various properties of neutron stars. The matter density in the core of neutron stars could exceed up to a few times normal nuclear matter density. Our knowledge about dense matter is very much constrained by a single density point in the whole density plane i.e. normal nuclear matter density or the saturation density. The empirical values of various properties of symmetric nuclear matter i.e. binding energy, bulk symmetry energy, compressibility are only known at this density. All models are fitted to those properties at the saturation density and then extrapolated to high density regime.

The symmetry energy is an essential input in understanding gross properties of neutron stars. The bulk symmetry energy is defined as the difference between the energy per particle for pure neutron matter and that of symmetric nuclear matter at normal nuclear matter density. The empirical value of the bulk symmetry energy lies in the range 30-40 MeV. Nuclear symmetry energy controls Fermi momenta of baryons, particle fractions and the equation of state of dense matter. Since a dense system like neutron stars is an infinite one, the volume and symmetry energy terms in the Bethe-Weizsäcker mass formula contribute to the total energy of the system. As a consequence, the energy of the system is lowered when the system is more symmetric i.e. its symmetry energy is less.

Though various nonrelativistic as well as relativistic models are fitted to the symmetry energy at the saturation density, there is no consensus among the models about the behaviour of nuclear symmetry energy far off from normal nuclear matter density. It was earlier noted by many authors [1–3] that the symmetry energy, in nonrelativistic models, initially increased and afterwards it either decreased with density or saturated to a value. It was attributed to the role of the tensor interaction [1–4] in isospin singlet (T=0) nucleon pairs. At low density, the attraction due to the tensor force dominates over the short range repulsion in T=0 nucleon pairs. As a consequence, symmetric nuclear matter is more attractive than pure neutron matter and the symmetry energy increases with density initially. At high density, the tensor interaction in T=0 channel vanishes [4] and the short range repulsion in isospin singlet nuclear pairs wins over that of isospin triplet pairs. As a result,
nuclear symmetry energy falls at high density regime leading to energetically favourable
pure neutron matter and the disappearance of protons. It was shown by Engvik et al. [5]
that the symmetry energy increased with density in the lowest order Brueckner calculations
using modern nucleon-nucleon (NN) potentials. Such a behaviour of the symmetry energy
was also reported in another Brueckner calculation using realistic nucleon-nucleon poten-
tials [6]. On the other hand, Akmal et al. [7] found that the symmetry increased at lower
densities and then decreased at high density in variational chain summation (VCS) method
using one such modern NN potential i.e. A18. The difference between those calculations
may be stemmed from the neglect of higher order terms in Brueckner calculations. Akmal
et al. also observed that the proton fraction calculated in the VCS approach using A18 plus
three nucleon interaction, increased with density. However, they noted that the too strong
repulsion in the three nucleon force resulted in overestimation in the proton fraction or the
symmetry energy.

In relativistic mean field (RMF) models [3,8], the symmetry energy always increases with
density. Here, the mean $\rho$-meson field is responsible for the interaction part of the symmetry
energy and it increases with density. However, two main features – the tensor force and
different repulsive strengths in isospin singlet and isospin triplet nucleon pairs, are absent
in RMF calculations.

In various nonrelativistic models, the fall of the symmetry energy occurs beyond a few
times normal nuclear matter density. On the other hand, the formation of hyperons is a
possibility at about 2-3 times normal nuclear matter density. Therefore, it may be a serious
flaw to consider a dense matter system consisting only of nucleons at high density. Also,
nonrelativistic models consisting only of nucleons violate causality at high density. This
problem might be rectified with the appearance of hyperons. Hyperons are created at the
cost of nucleons’ energy. With the formation of hyperons, Fermi momenta (velocities) of
nucleons will be reduced. On the other hand, hyperons being heavier than nucleons will
have smaller Fermi velocities. In this situation, all baryons may be treated as nonrelativistic
particles in a dense system. Strange hadron systems were studied extensively in RMF
models [8–11]. Recently, there have been some calculations on strange hadronic matter in
the nonrelativistic Brueckner approximation using baryon-baryon potentials [6,12] and also
using phenomenologically constructed energy density functional [13].

In this letter, we investigate the density dependence of nuclear symmetry energy in
the nonrelativistic Thomas-Fermi approximation using a momentum and density depen-
dent finite range Seyler-Blanchard effective interaction. The momentum dependent Seyler-
Blanchard nucleon-nucleon effective interaction was extensively applied to the determination
of the parameters of mass formula by Myers and Swiatecki [14]. However, the energy de-
pendence of the single particle potential was too strong because of the strong momentum
dependence in the effective interaction. Later, the momentum dependent Seyler-Blanchard
effective interaction was modified to include a two-body density dependent term which sim-
ulated three body effects and the energy dependence of the single particle potential was
exploited to delineate the momentum and density dependence of the effective interaction
[15–17]. This modified Seyler-Blanchard (SBM) interaction was used in the description of
heavy ion collisions [15], dense matter properties [16,17] and neutron stars [18]. Here, we
generalise the SBM interaction to the case of baryon-baryon interaction with the inclusion
of hyperons in addition to nucleons. Later, we exploit this momentum and density depen-
dent finite range baryon-baryon effective interaction to calculate nuclear symmetry energy
in nuclear and hyperon matter relevant to neutron stars.

The interaction between two baryons with separation $r$ and relative momentum $p$ is given
by

$$V_{\text{eff}}(r, \rho, p) = -C_{B_i B_j} \rho \left[1 - \frac{p^2}{b^2} - d^2 (\rho_1 + \rho_2)^n \right] e^{-r/a},$$  \hspace{1cm} (1)$$

where $a$ is the range parameter and $b$ defines the strength of repulsion in the momentum
space; $d$ and $n$ are two parameters determining the strength of the density dependence; $\rho_1$
and $\rho_2$ are total baryon densities at the sites of two interacting baryons. We have all the
baryons of SU(3) octet and leptons ($e^-, \mu^-$) in our calculation. The constituents of matter
in neutron stars are highly degenerate and the chemical potentials of baryons and leptons
are much larger than the temperature of the system. Therefore, our calculation is confined to zero temperature case. The single particle potential for baryon $B_1$ is defined as,

$$V_{B_1}(p_1, \rho) = V^0_{B_1} + p_1^2 V^1_{B_1} + V^2$$

$$= \frac{2}{(2\pi)^3} \int d\vec{p}_2 d\vec{r}_2 V_{eff} \left[C_{B_1 B_1} \Theta(p_{F_{B_1}} - p_2) + \sum_{B_2 \neq B_1} C_{B_1 B_2} \Theta(p_{F_{B_2}} - p_2) + V^2\right], \quad (2)$$

where, $V^0_{B_1}, V^1_{B_1}$ are the momentum independent and dependent parts of the single particle potential, respectively and $V^2$, the rearrangement contribution [16] arising out of the density dependence of the two-body effective interaction, is given by

$$V^2 = \frac{1}{2} \int d\vec{r} \frac{\partial v_2}{\partial \rho} \sum_{B_1} \rho_{B_1} \left[C_{B_1 B_1} \rho_{B_1} + \sum_{B_2 \neq B_1} C_{B_1 B_2} \rho_{B_2}\right], \quad (3)$$

with $v_2 = d^2(2\rho)^n e^{-r/a} r/a$. Here, the total baryon density is denoted by $\rho$ and the summations over $B_1$ and $B_2$ go over all the species of SU(3) baryon octet. The density for baryon $B$ is denoted by $\rho_B$ and Fermi momentum by $P_{F_B}$. The effective mass is defined as,

$$m^*_B = \left[\frac{1}{m_B} + 2V^1_B\right]^{-1}. \quad (4)$$

We have from equations (1), (2) and (3)

$$V^0_{B_1} = 4\pi a^3 (d^2(2\rho)^n - 1) [C_{B_1 B_1} \rho_{B_1} + \sum_{B_2 \neq B_1} C_{B_1 B_2} \rho_{B_2}]$$

$$+ \frac{4a^3}{\pi b^2} [C_{B_1 B_1} \frac{p_{F_{B_1}}^5}{5} + \sum_{B_2 \neq B_1} C_{B_1 B_2} \frac{p_{F_{B_2}}^5}{5}], \quad (5)$$

$$V^1_{B_1} = \frac{4\pi a^3}{b^2} [C_{B_1 B_1} \rho_{B_1} + \sum_{B_2 \neq B_1} C_{B_1 B_2} \rho_{B_2}], \quad (6)$$

$$V^2 = 4\pi a^3 d^2 n(2\rho)^{n-1} \sum_{B_1} \rho_{B_1} [C_{B_1 B_1} \rho_{B_1} + \sum_{B_2 \neq B_1} C_{B_1 B_2} \rho_{B_2}] \quad (7)$$

The chemical potential of baryon $B$ is given by,

$$\mu_B = \frac{P_{F_B}^2}{2m^*_B} + V^0_B + V^2. \quad (8)$$

The symmetry energy is an essential ingredient in understanding dense matter. As our calculations are concerned with a system having density far off from normal nuclear matter
density, it is very much necessary to know the behaviour of nuclear symmetry energy at high density. The energy per nucleon in asymmetric matter may be written as [19,20],

\[ E(\rho, \beta) = E(\rho, \beta = 0) + \beta^2 E_{\text{sym}}(\rho), \]  

(9)

where \( \beta = \frac{(\rho_n - \rho_p)}{\rho} \) is the asymmetry parameter and \( \rho_n \) and \( \rho_p \) are neutron and proton densities, respectively; \( E(\rho, \beta = 0) \) and \( E_{\text{sym}}(\rho) \) are energy per nucleon in symmetric matter and nuclear symmetry energy, respectively. It can be shown that the symmetry energy is related to neutron and proton chemical potentials [20]. Neutron and proton chemical potentials are defined respectively as \( \mu_n = \frac{\partial \epsilon}{\partial \rho_n} \) and \( \mu_p = \frac{\partial \epsilon}{\partial \rho_p} \), where \( \epsilon = \rho E(\rho, \beta) \) is the energy density. The expression of nuclear symmetry energy follows from equation (9) and the definitions of the chemical potentials as,

\[ \mu_n - \mu_p = 4\beta E_{\text{sym}}(\rho). \]  

(10)

Putting the expression for chemical potentials (equation (8)) along with equations (5)-(7) in equation (10), we obtain

\[ 4\beta E_{\text{sym}}(\rho) = E_{\text{kin}} + V_s, \]  

(11)

where the kinetic and the interaction parts of the symmetry energy are respectively given by,

\[ E_{\text{kin}} = \left[ \frac{P_{F_n}^2}{2m_n^*} - \frac{P_{F_p}^2}{2m_p^*} \right], \]  

(12)

and

\[ V_s = [4\pi a^3(d^2(2\rho)^n - 1)(\rho_n - \rho_p) + 4a^3 \frac{\rho_n^5 - \rho_p^5}{5\pi b}(p_{F_n}^5 - p_{F_p}^5)](C_{nn} - C_{np}). \]  

(13)

The five parameters of nucleon-nucleon interaction in equation (1) - two strength parameters \( C_{BBS} \) (one for \( pp \) or \( nn \) interaction and the other for \( np \) or \( pn \) interaction), \( a, b \) and \( d \) are determined for a fixed value of \( n = 1/3 \) by reproducing the saturation density of normal
nuclear matter ($\rho_0 = 0.1533 \, fm^{-3}$), the volume energy coefficient for symmetric nuclear matter ($-16.1 \, MeV$), asymmetry energy coefficient ($34 \, MeV$), the surface energy coefficient of symmetric nuclear matter ($18.01 \, MeV$) and the energy dependence of the real part of the nucleon-nucleus optical potential. With the above choice of $n$, the incompressibility of normal nuclear matter turns out to be $260 \, MeV$. Also, the effective mass ratio ($m_N^*/m_N$) comes out to be $0.61$ at normal nuclear matter density in our calculation. The values of parameters for nucleon-nucleon interaction are presented in Table I.

Informations about nucleon-hyperon interactions are confined to hypernuclei data [21]. There is a large body of data on binding energies of $\Lambda$-hypernuclei. Analyses of those experimental data on hypernuclei indicate that the potential felt by a $\Lambda$ in normal nuclear matter is $\simeq -30 \, MeV$. With our two-body baryon-baryon interaction (equation (1)), we determine the strength of nucleon-hyperon and hyperon-hyperon interaction from equation (2) keeping two range parameters ($a$ and $b$) and the density dependence of the interaction same as that of the nucleon-nucleon interaction. Parameters of nucleon-$\Lambda$ interaction are shown in Table II.

Experimental data of $\Sigma$-hypernuclei are scarce and ambiguous because of the strong $\Sigma N \rightarrow \Lambda N$ decay. It is also assumed that the $\Sigma$ well depth [10] in normal nuclear matter is equal to that of a $\Lambda$ particle. Therefore, the strength of $\Sigma N$ interaction is the same as that of $\Lambda N$ interaction in our calculation and this is shown in Table II.

In emulsion experiments with $K^-$ beams, there are a few events attributed to the formation of $\Xi$-hypernuclei. These data can be explained in terms of a potential well of $\simeq -25 \, MeV$ for $\Xi$ particle in symmetric nuclear matter [22]. We obtain the strength parameter ($C_{BB}$) of $\Xi N$ interaction by fitting the single particle potential to the above mentioned value and present in Table II.

There are a few events of $\Lambda \Lambda$ hypernuclei. Analyses of those events indicate a rather strong hyperon-hyperon interaction. Schaffner et al. [10] constructed single particle potentials on the basis of one boson exchange calculations of Nijmegen group and the well depth of a hyperon in hyperon matter is estimated to be $\simeq -40 \, MeV$ and this is universal for all
hyperon-hyperon interactions. The parameters of hyperon-hyperon interaction are given in Table II.

In all cases, we notice that interactions involving hyperons, are weaker compared to nucleon-nucleon interaction.

The composition of a neutron star is constrained by charge neutrality and baryon number conservation. Also, constituents of matter are in beta-equilibrium. Baryon chemical potentials are related to neutron and lepton chemical potentials through the general relation given by

$$\mu_i = b_i \mu_n - q_i \mu_l,$$

where $b_i$ and $q_i$ are the baryon number and charge of $i$-th baryon species, respectively and $'l'$ stands for electrons and muons. Solving the above mentioned constraints, at a given density, self-consistently, we obtain effective masses, Fermi momenta or chemical potentials which determine the gross properties of neutron stars.

Particle abundances of nucleons-only matter relevant to a neutron star are shown in figure 1. Here, we notice that the proton (electron) fraction initially increases with density and decreases at higher densities. Such a behaviour of the proton fraction with density in nonrelativistic models was noted earlier by various authors [1–3]. They attributed it to the density dependence of nuclear symmetry energy. We will discuss about this later.

In figure 2, particle fractions in hyperon matter relevant to a neutron star are plotted with baryon density. The threshold condition for the appearance of hyperons depends not only on their masses but also on their charges and interaction strengths. The threshold condition is given by

$$\mu_n - q_B \mu_e \geq m_B^* + V_{B}^0 + V^2,$$

where $\mu_n$ and $\mu_e$ are neutron and electron chemical potentials respectively, $q_B$ is the charge of baryon $B$. The quantities on the right hand side of equation (15) are given by equations (4) - (7). When the left hand side equals to or exceeds the right hand side of equation (15),
baryon species B will be populated. Here, we notice that hyperons first appear at 1.5 times normal matter density. Also, it is noted that the electron fraction decreases monotonically because negatively charged hyperons make the neutron star almost charge neutral. On the other hand, the proton fraction is enhanced in hyperon matter compared to the situation in nuclear matter (see figures 1 and 2). Moreover, the proton fraction does not show any declining tendency at high density as it has been observed in nucleons-only system. We plot absolute proton density with baryon density in figure 3. The dashed line (curve $a$) denotes neutron-proton system whereas the solid line (curve $b$) represents hyperon system. We find that the proton density always increases with baryon density in hyperon environment. It may be attributed to the behaviour of nuclear symmetry energy with density in hyperon matter.

Violation of causality at high density is a problem in nonrelativistic models [7,16]. The speed of sound ($v^2 = \frac{dP}{d\epsilon}$) in nucleons-only matter becomes superluminal i.e. greater than the velocity of light at high density. With the appearance of additional degrees of freedom in the form of hyperons, Fermi momenta of neutrons and protons are reduced at high density in comparison to the situation with nucleons-only matter. As a consequence, the equation of state now respects causality at densities which might occur at the centers of neutron stars.

Nuclear symmetry energy is plotted with baryon density in figure 4. The dashed line (curve $a$) represents the calculation for nuclear matter whereas the solid line (curve $b$) implies that of hyperon matter. We find that the nuclear symmetry energy in nuclear matter increases initially with density and decreases later at high density. It was pointed out by many authors that the fall of the symmetry energy was due to the greater short-range repulsion in isospin singlet nucleon pairs than that of isospin triplet pairs at high density [1–3]. In our calculation, there are two strength parameters in the SBM nucleon-nucleon effective interaction i.e. $C_{nn}$ ($C_{pp}$) which represents isospin triplet state ($T=1$) and $C_{np}$ ($C_{pn}$) implying isospin singlet ($T=0$) state. It is evident from Table I that the strength parameter ($C_{np}$) in isospin singlet state is stronger than that of the triplet state ($C_{nn}$). It is the interaction term ($V_s$) in the symmetry energy (see equation (13)) that regulates the
behaviour of nuclear symmetry energy. At lower densities, the interaction term in $E_{\text{sym}}$ is positive because the repulsive first term in $V_s$ is larger than the attractive second term. Therefore, the symmetry energy is increasing at lower densities. On the other hand, the interaction term, $V_s$, becomes negative around $\sim 4\rho_0$ because the second term, in equation (13), which is attractive in nature, wins over the first term. Thus at high density, pure neutron matter is energetically favourable and protons disappear from the system.

We observe that the symmetry energy in nuclear matter (curve a in figure 4) starts falling around density $\sim 4\rho_0$. On the other hand, the appearance of hyperons is a possibility at about 2-3 times normal nuclear matter density. Therefore, it may not be justified to consider a system consisting only of nucleons at high density. Here, we discuss the density dependence of nuclear symmetry energy including hyperons in our nonrelativistic calculation. Nuclear symmetry energy in presence of hyperons is calculated using equations (11), (12) and (13). In hyperon matter, neutrons and protons couple to a hyperon with the same coupling strength. Therefore, those terms originating from nucleon-hyperon interaction cancel out in the calculation of nuclear symmetry energy in hyperon matter. The solid line (curve b) in figure 4 represents our calculation of the symmetry energy in hyperon matter. It increases with density. This may be attributed to the behaviour of the interaction term($V_s$) in $E_{\text{sym}}$ (see equation (13)) in a hyperon environment. Hyperons are produced at the cost of the energy of nucleons. Therefore, Fermi momenta of neutrons and protons are reduced with the appearance of hyperons compared to the case of nuclear matter. As a result, the repulsive first term of $V_s$ (equation (13)) dominates over the attractive second term leading to a rising nuclear symmetry energy for all densities. This behaviour of the symmetry energy in hyperon matter is also reflected in the proton fraction (curve b in figure 3). The proton fraction in neutron stars is crucial in the determining the direct URCA process which leads to the cooling of neutron stars [23,24]. The direct URCA process happens if the proton fraction exceeds the threshold value i.e. 11 percent. This happens in our calculation including hyperons around $\sim 3.0\rho_0$.

We have compared our nonrelativistic calculations with those of relativistic mean field
models. Nuclear symmetry energy calculated in RMF models increases monotonically with density [3]. In RMF models, the interaction part of the symmetry energy is related to the mean $\rho$ meson field which always increases with density. It is to be noted that the symmetry energy calculated in RMF models rises faster compared to nonrelativistic calculations [3]. This may be attributed to the fact that the different repulsive strengths in isospin singlet and isospin triplet nucleon pairs are not taken into account by RMF models.

In conclusion, we have studied nuclear symmetry energy in nuclear and hyperon matter relevant to neutron stars in the nonrelativistic Thomas-Fermi approximation using a momentum and density dependent finite range Seyler-Blanchard baryon-baryon effective interaction. In nuclear matter, the symmetry energy (proton fraction) initially increases and later it falls with density. With the appearance of hyperons, nuclear symmetry energy increases with density in hyperon matter. The proton fraction follows the same trend as that of the symmetry energy. The increasing symmetry energy or proton fraction might have important bearings on the mass-radius relationship and cooling properties of neutron stars. We will report on these aspects in a future publication.
REFERENCES


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TABLES

**TABLE I.** The different parameters of the effective nucleon-nucleon interaction as given in equation (1). The parameters - $a$ and $b$ define the ranges of the interaction in coordinate and momentum space, respectively; density dependence of the interaction is given by $d$ and $n$. The strengths of $nn(pp)$ and $np(pn)$ interactions are denoted by $C_{nn(pp)}$ and $C_{np(pn)}$, respectively.

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<th>$n$</th>
<th>$a$ ($fm$)</th>
<th>$b$ (MeV)</th>
<th>$d$ ($fm^{1/2}$)</th>
<th>$C_{nn(pp)}$ (MeV)</th>
<th>$C_{np(pn)}$ (MeV)</th>
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<td>0.827</td>
<td>254.2</td>
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**TABLE II.** The different parameters of the effective nucleon-hyperon and hyperon-hyperon interactions as given in equation (1). The parameters - $a$ and $b$ define the ranges of the interactions in coordinate and momentum space, respectively; density dependence of the interactions is given by $d$ and $n$. The strengths of $N\Lambda$, $N\Sigma$, $N\Xi$ and hyperon – hyperon interactions are denoted by $C_{N\Lambda}$, $C_{N\Sigma}$, $C_{N\Xi}$ and $C_{hh}$, respectively.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$a$ ($fm$)</th>
<th>$b$ (MeV)</th>
<th>$d$ ($fm^{1/2}$)</th>
<th>$C_{N\Lambda}$ (MeV)</th>
<th>$C_{N\Sigma}$ (MeV)</th>
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**Figure Captions**

FIG. 1. Particle abundances of nucleons-only matter as a function of normalised baryon density.

FIG. 2. Particle abundances of hyperon matter as a function of normalised baryon density.

FIG. 3. Proton density as a function of normalised baryon density in nucleons-only and hyperon matter.

FIG. 4. Nuclear symmetry energy as a function of normalised baryon density in nucleons-only and hyperon matter.