Sensitivity plots for WIMP modulation searches


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Prospects of WIMP searches using the annual modulation signature are discussed on statistical grounds, introducing sensitivity plots for the WIMP-nucleon scalar cross section.

1. INTRODUCTION

The annual modulation effect[1] provides a distinctive signature for the identification of a Dark Matter signal in the direct searches of WIMPS through their elastic scattering off the nuclei of a detector. Due to this effect the relative velocity between the detector and the WIMP Maxwellian distribution (assumed at rest in the Galactic rest frame) is given by:

\[ v_{\text{earth}} = v_{\text{sun}} + v_{\text{orb}} \sin \delta \cos[\omega(t - t_0)] \] (1)

where \( v_{\text{sun}} \) is the Sun’s velocity in the galactic rest frame, \( v_{\text{orb}} \approx 30 \text{ km sec}^{-1} \), \( \sin \delta \approx 0.51 \) (\( \delta \) is the angle between the Ecliptic and the Galactic plane), \( \omega = 2\pi/T \), \( T=1 \text{ year} \) and \( t_0 \approx 2^{nd} \text{ june} \).

2. EXTRACTING THE MODULATION SIGNAL

Given a set of experimental count rates \( N_{ik} \) representing the number of events collected in the i-th day and k-th energy bin, the mean value of \( N_{ik} \) (expressed in number of counts per unit of detector mass, time and interval of recoil energy) is:

\[ <N_{ik} > \equiv \mu_{ik} = \] (2)

\[ = [b_k + S_{0,k} + S_{m,k} \cos(\omega(t - t_0))] \cdot W_{ik} \] (3)

where the \( b_k \) represent the average background while \( S_{0,k} \) and \( S_{m,k} \) are the constant and the modulated amplitude of the WIMP signal respectively. The various parameters of the WIMP model are contained in \( S_{0,k} \) and \( S_{m,k} \). In particular they depend on the WIMP-nucleus elastic cross sections \( \sigma \) and the WIMP mass \( m_W \). The \( W_{ik} = M \Delta T_i \Delta E_k \) are the corresponding exposures, where \( M \) is the mass of the detector, \( \Delta E_k \) is the amplitude of the k-th energy–bin, while \( \Delta T_i \) represents the i-th time bin (in the following we will assume all \( \Delta T_i = 1 \text{ day} \)). For simplicity \( t_0 \) will be omitted in the following equations.

The general procedure to compare theory with experiment is by making use of the maximum-likelihood method. The combined-probability function of all the collected \( N_{ik} \), assuming that they have a poissonian distribution with mean values \( \mu_{ik} \), is given by:

\[ L = \prod_{ik} e^{-\mu_{ik}} \frac{\mu_{ik}^{N_{ik}}}{N_{ik}!} \] (4)

The most probable values of \( m_W \) and \( \sigma \) maximize \( L \) or, equivalently, minimize the function:

\[ y(m_W, \sigma) \equiv -2 \log L - \text{const} \] (5)

\[ = 2 \mu - 2 \sum_{ik} N_{ik} \log [b_k + S_{0,k} + S_{m,k} \cos(\omega t_i)] \]

where \( \mu \equiv \sum_{ik} \mu_{ik} \) and all the parts not depending on \( m_W \) and \( \sigma \) may be absorbed in the constant because are irrelevant for the minimization.

3. STATISTICAL SIGNIFICANCE OF THE SIGNAL

Once a minimum of the likelihood function has been found, a positive result excludes the absence of modulation at some confidence level probabil-
ity. This can be checked by evaluating the quantity \( \delta^2 = y(\sigma = 0) - y(m_W, \sigma)_{\text{min}} \) to test the goodness of the null hypothesis. In order to study the distribution of \( \delta^2 \) we make use of the asymptotic behaviour:

\[
\delta^2 \simeq \chi^2(\sigma = 0) - \chi^2_{\text{min}} \quad (6)
\]

\[
\chi^2(\sigma, m_W) \equiv \sum_k \frac{(S_{m,k}(m_W, \sigma) - X_k)^2}{\text{Var}(X_k)}
\]

\[
X_k \equiv \frac{\sum_i N_{ik} \cos \omega t_i - N_k \beta_k}{W_k (\alpha_k - \beta_k^2)} \quad (7)
\]

where \( \beta_k \equiv \sum_i W_{ik} \cos \omega t_i \), \( \alpha_k \equiv \sum_i W_{ik} \cos^2 \omega t_i \), and \( N_k \equiv \sum_i N_{ik} \). In the case of absence of a modulation effect numerical simulations show that the quantity \( \delta^2 \) belongs asymptotically to a \( \chi^2 \) distribution with two degrees of freedom. We explain this by the fact that once the cross section \( \sigma \) is set to zero the likelihood function \( L \) no longer depends on \( m_W \) (all the \( S_0 \) and \( S_\alpha \) functions vanish) and this is equivalent to fixing both the parameters of the fit at the same time. In the case of presence of a modulation, \( \delta^2 \) has the asymptotic distribution of a non central \( \chi^2 \) with one degree of freedom and with a mean value given by

\[
<\delta^2> = \frac{1}{2} \sum_k \frac{S_{m,k}(\sigma, m_W)^2 \Delta E_k}{b_k + S_{0,k}} MT\alpha + 2 \quad (8)
\]

where the same days of data taking have been assumed for all the energy bins, and the approximations \( \sum_i N_{ik} \cos^2 \omega t_i \simeq< N_{ik} > \sum \cos^2 \omega t_i \), \( < N_{ik} > \simeq W_k (b_k + S_0) \) have been made. In Eq.(8) we have also defined the factor of merit \( \alpha \equiv \frac{1}{2} \sum_i \cos^2 \omega t_i \) (\( \alpha =1 \) in case of a full period of data taking) and the terms depending on the \( \beta_k \) have been neglected.

Since the degree of overlapping between the distributions of \( \delta^2 \) in the two cases of absence and presence of modulation depends on \( <\delta^2> \), equation (8) allows to estimate the needed exposure \( MT\alpha \) in order to observe a modulation effect with a given probability: for instance, \( <\delta^2> = 14.9 \) (5.6) corresponds to a 90% (50%) probability to see an effect at least at the 95% (90%) C.L. Once a required \( <\delta^2> \) is chosen, a sensitivity plot may be obtained by showing the curves of constant \( MT\alpha \) in the plane \( m_W - \sigma \).

4. SENSITIVITY PLOTS AND QUANTITATIVE DISCUSSION

In Figures 1–2 we discuss the example of a Germanium detector with background \( b = 0.01 \) cpd/kg/keV (assumed constant with energy) and energy thresholds \( E_{\text{th}} = 2 \) keV, values not unrealistic, taken into account the recent performances of some Ge detectors. Parameter values used in the plots are the local halo mass density \( \rho = 0.3 \) GeV/cm\(^3\), \( v_{\text{loc}} = 220 \) km sec\(^{-1}\) (\( v_{\text{loc}} \) is the measured rotational velocity of the Local System at the Earth’s position), the WIMP r.m.s. velocity \( v_{\text{rms}} = \frac{3}{4} v_{\text{loc}} \) and \( v_{\text{ran}} \simeq (v_{\text{loc}} + 12) \) km sec\(^{-1}\).

In Figure 1 the sensitivity plots for \( <\delta^2> = 5.6 \) is shown in the plane \( m_W - \sigma \), where \( \sigma \) is the WIMP cross section \( \sigma \) rescaled to the nucleon by adopting a scalar–type interaction. The different curves correspond to values of \( MT\alpha \) from 10 kg year to 100 kg year in steps of 10 (from top to bottom). The closed contour and the cross indicate respectively the 2\( \sigma \) C.L. region singled out by the DAMA modulation search experiment and the minimum of the likelihood function found by the same authors[3]. Note that an exposure of 10 kg/year of a Ge detector of the above–quoted performances would explore almost totally the DAMA region.

In Figure 2 we show, as a function of \( m_W \), the minimal exposures required for the same germa-
Table 1
Summary of minimal exposures, all in kg · year. Values off (in) parenthesis refer to \(v_{\text{loc}} = 220(170) \text{ km sec}^{-1}\). 
\(E_{\text{th}}\) indicates the energy thresholds expressed in keV, \(b\) the background (assumed not dependent on energy) in cpd/kg/keV. Exposures are estimated for the WIMP mass range \(10^{10} m_W \lesssim 1000\) unless specified otherwise.

<table>
<thead>
<tr>
<th>Material</th>
<th>Exploration of not excluded regions</th>
<th>DAMA region (\delta^2 = 5.6)</th>
<th>DAMA region (\delta^2 = 15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge, (E_{\text{th}} = 2), (b=0.1)</td>
<td>80(50)</td>
<td>50(25)</td>
<td>175(90)</td>
</tr>
<tr>
<td>Ge, (E_{\text{th}} = 12), (b=0.01)</td>
<td>25(19)*</td>
<td>50(25)</td>
<td>190(95)</td>
</tr>
<tr>
<td>TeO(<em>2), (E</em>{\text{th}} = 5), (b=0.01)</td>
<td>40(25)</td>
<td>40(20)</td>
<td>150(80)</td>
</tr>
<tr>
<td>NaI, (E_{\text{th}} = 2), (b=0.1)</td>
<td>50†</td>
<td>180(100)</td>
<td>660(355)</td>
</tr>
</tbody>
</table>

* \(45 \text{ GeV} \lesssim m_W \lesssim 110 \text{ GeV}; \) † \(m_W < 70(125) \text{ GeV}^2\).

Figure 2. Minimal exposures \(M_{\alpha}\) for the \(< \delta^2 > = 5.6\) calculated for a germanium detector with threshold energy \(E_{\text{th}} = 2 \text{ keV}\).

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