Domain Walls and Massive Gauged Supergravity Potentials

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ABSTRACT

We point out that massive gauged supergravity potentials, for example those arising due to the massive breathing mode of sphere reductions in M-theory or string theory, allow for supersymmetric (static) domain wall solutions which are a hybrid of a Randall-Sundrum domain wall on one side, and a dilatonic domain wall with a run-away dilaton on the other side. On the anti-de Sitter (AdS) side, these walls have a repulsive gravity with an asymptotic region corresponding to the Cauchy horizon, while on the other side the runaway dilaton approaches the weak coupling regime and a non-singular attractive gravity, with the asymptotic region corresponding to the boundary of space-time. We contrast these results with the situation for gauged supergravity potentials for massless scalar modes, whose supersymmetric AdS extrema are generically maxima, and there the asymptotic regime transverse to the wall corresponds to the boundary of the AdS space-time. We also comment on the possibility that the massive breathing mode may, in the case of fundamental domain-wall sources, stabilize such walls via a Goldberger-Wise mechanism.

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The scalar potentials of gauged supergravity theories provide a natural testing-ground for studying domain-wall configurations within the framework of a basic theory. In general, such scalar potentials have isolated supersymmetric extrema with a negative cosmological constant. Within the AdS/CFT correspondence, these supersymmetric (BPS) domain walls play a role in elucidating the renormalization group flows and bound-state spectra of strongly coupled gauge theories (see, for example, [1]-[14] and references therein). A typical feature of gauged supergravity potentials is such that the supersymmetric extrema are maxima of the potential. The domain walls are therefore typically those with negative tension, and the metric transverse to the wall asymptotically ($z \to \infty$) approaches the boundary of the AdS space-time [11]. Another feature of these solutions is that the region near the wall ($z \to 0$) is in general singular; both the scalar field and the curvature generically exhibit singular behavior and thus the continuation across the wall region on the side $z < 0$ involves (within the effective theory) a continuation across a singular domain-wall regime (c.f. [11, 15]).

On the other hand, in recent months there has been a resurgence in the study of domain walls in asymptotically AdS space-times in $D = 5$ gravity theories. For special examples of such static domain walls, the gravity effects transverse to the wall are suppressed, which has interesting implications for the phenomenology of the world on the brane. (See, for example, [16]-[21] and references therein.) Non-static walls in $D = 5$ were also recently considered. (See [22]-[30] and references therein.)

A particular focus is on infinitely thin, static, $Z_2$-symmetric domain-wall solutions, constructed [16, 17] in a pure AdS gravity theory (the Randall-Sundrum scenario). Generalizations that incorporate the effects of additional compactified dimensions were given in [18, 20, 21]. These solutions have a repulsive gravity [34], for which the asymptotic regions ($z \to \pm \infty$) corresponding Cauchy horizons [35, 36]. They satisfy [16, 17] a specific relation between the domain-wall tension $\sigma$ and the cosmological constant $\Lambda$ of the AdS vacuum; this latter condition was subsequently shown [11] to be a consequence of supersymmetry. (These results are again completely parallel [37] with supersymmetric domain walls of N=1 supergravity theories in $D = 4$.) These types of wall are of Type II in the classification scheme of refs. [34, 11].

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1 It turns out [30] that the local and global space-time structure of vacuum domain walls ($(D - 2)$-brane configurations) in $D$ dimensions is universal, and thus the previous studies of domain walls in $D = 4$ (see, [31, 32] and references therein) are completely parallel to the domain-wall solutions in any other dimension $D$.

2 Another proposal for the origin of the five-dimensional domain wall was made in [33], which is dilatonic and can be viewed as M5-branes wrapped around the two-cycles of a Calabi-Yau manifold.
The main motivation of this paper is to provide a framework within gauged supergravity theories that has a chance of implementing the Randall-Sundrum scenario(s). As mentioned above, gauged supergravity theories tend to have potentials for the massless scalar modes that have isolated supersymmetric maxima and not minima. Thus the supersymmetric domain walls have negative tension (whose magnitude is the same as the tension of Type II walls). They have attractive gravity transverse to the wall, with the asymptotic regions \((z \to \pm \infty)\) corresponding to AdS space-time boundaries [11, 15]. These types of walls are referred to as Type IV walls [11] and are complementary to Type II walls.

In order to obtain Type II domain-wall solutions of the Randall-Sundrum scenario, the gauged supergravity potential would have to have two isolated supersymmetric minima. Since the potentials for the massless scalar fields in a gauged supergravity do not seem to have this feature, we turn in our analysis to include other scalar fields that do not lie in the massless supermultiplet.

We shall focus on the special classes of gauged supergravities that arise from sphere reductions of M-theory or string theory, with particular emphasis on the \(D = 5\) case. For examples in the Kaluza-Klein reduction of Type IIB string theory on a five-sphere \((S^5)\), there will be an infinite tower of massive supermultiplets in addition to the massless multiplet, and so one could consider the potentials for one or more of the massive scalar fields. In general, one cannot focus attention on a single such field in isolation, on account of its couplings to other fields. However, in certain special cases a consistent truncation to a single massive scalar can be performed. One such example is the “breathing mode” that parameterises the overall volume of the compactifying \(S^5\). (Unlike the breathing mode in a toroidal reduction, which is massless, the breathing mode in a spherical reduction is a member of a massive supermultiplet.)

The scalar potentials for the breathing-mode scalars in various Kaluza-Klein spherical reductions were studied in [38]. Although the breathing mode is a member of a massive multiplet, the truncation is nonetheless consistent since it is a singlet under the isometry group of the internal sphere. (It would not in general be consistent to turn on a finite subset of other fields as well.)

The resulting \(D\)-dimensional Lagrangians all turn out to have the following form:

\[
\mathcal{L}_D = e R - \frac{1}{2} e (\partial \phi)^2 - e V ,
\]

where the potential is given by [38]

\[
V = \frac{1}{2} g^2 \left( \frac{1}{a_1^2} e^{a_1 \phi} - \frac{1}{a_1 a_2} e^{a_2 \phi} \right) .
\]
The positive constants $a_1$ and $a_2$ are given by

$$a_1^2 = \frac{4}{N} + \frac{2(D - 1)}{D - 2}, \quad a_1 a_2 = \frac{2(D - 1)}{D - 2},$$  \hspace{1cm} (3)$$

where $N$ is a certain positive integer. For $D = 4, 7$ and $5$, this integer takes the value $N = 1$. These cases correspond to the $S^7$ and $S^4$ reductions of $D = 11$ supergravity, and the $S^5$ reduction of type IIB supergravity respectively. For $D = 3$ the integer $N$ can be equal to $1, 2$ or $3$, corresponding to the $S^1$ reduction of the Freedman-Schwarz model,\(^3\) the $S^3$ reduction of $D = 6$ simple (chiral) supergravity, and the $S^2$ reduction of $D = 5$ simple supergravity respectively. The explicit dilaton coupling constants $a_1$ and $a_2$ for the above cases are given in Table 1.

<table>
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<tr>
<th>$D$</th>
<th>$N$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$\frac{a_1}{a_2(D-1)}$</th>
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<tr>
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<td>3</td>
<td>$\frac{4}{\sqrt{3}}$</td>
<td>$\sqrt{3}$</td>
<td>$\frac{2}{3}$</td>
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</table>

Table 1: The values of the parameters $N$, $a_1$ and $a_2$ in diverse dimensions $D$ that enter the scalar potential (2).

Since $a_1 > a_2 > 0$, the potential has a minimum at $\phi = 0$, with

$$V_{\text{min}} = -\frac{g^2(D - 1)}{N(D - 1)a_1^2}.$$  \hspace{1cm} (4)$$

See Figure 1 for the shape of the potential. The potential can be expressed in terms of a “superpotential” $W_+$ or $W_-$ as follows:

$$V = \left(\frac{\partial W_\pm}{\partial \phi}\right)^2 - \frac{D - 1}{2(D - 2)} W_\pm^2,$$  \hspace{1cm} (5)$$

where

$$W_\pm = \sqrt{\frac{N}{2}} g \left( \frac{1}{a_1} e^{a_1 \phi/2} \pm \frac{1}{a_2} e^{a_2 \phi/2} \right).$$  \hspace{1cm} (6)$$

Let us now consider the following ansatz for a domain-wall metric:

$$ds^2 = e^{2A} dx^\mu dx^\nu \eta_{\mu\nu} + dz^2.$$  \hspace{1cm} (7)$$

The reduction in this case is of a generalised Scherk-Schwarz type, where the axion is allowed a linear dependence on the reduction coordinate.
The equations of motion are

\[
\begin{align*}
\phi'' + (D - 1) A' \phi' &= \frac{\partial V}{\partial \phi} \\
A'' + (D - 1)(A')^2 &= (D - 1) A'' + (D - 1) (A')^2 + \frac{1}{2} (\phi')^2 = -\frac{V}{D - 2}.
\end{align*}
\] (8)

These admit a first integral, given by

\[
\phi' = \sqrt{2} \frac{\partial W}{\partial \phi}, \quad A' = -\frac{1}{\sqrt{2(D - 2)}} W_\pm.
\] (9)

Here, we shall consider the choice \(W_-\) for the superpotential, since it has a supersymmetric minimum, i.e. \(\partial_\phi W_- = 0\), at \(\phi = 0\). From (9), we shall therefore have

\[
\phi' = \sqrt{\frac{N}{4}} g (e^{a_1 \phi/2} - e^{a_2 \phi/2}),
\]

\[
A' = = -\frac{g \sqrt{N}}{2(D - 2)} \left[ \frac{1}{a_1} e^{a_1 \phi/2} - \frac{1}{a_2} e^{a_2 \phi/2} \right].
\] (10)

Solving for \(\phi\) and \(A\), we find that \(A\) can be expressed as a function of \(\phi\), namely

\[
e^{(D-1)A} = c \frac{\partial W}{\partial \phi} e^{-\frac{1}{2}(a_1+a_2)\phi},
\] (11)

where \(c\) is an integration constant. For \(D > 3\), the solution for \(\phi\) is given by

\[
z - z_0 = \frac{4}{a_2 g \sqrt{N}} e^{-\frac{1}{2}a_2 \phi} 2F_1 \left[ \frac{a_2}{a_2 - a_1}, 1, 1 + \frac{a_2}{a_2 - a_1}; e^{\frac{1}{2}a_2 \phi} \right].
\] (12)

(For our specific examples mentioned above, we shall have \(N = 1\) and \(D = 4, 5\) or 7.) For \(D = 3\), we find that \(\phi\) is given by

\[
N = 1: \quad z - z_0 = \frac{\sqrt{8}}{g} \left( e^{-\frac{1}{\sqrt{2}} \phi} + \log(e^{-\frac{1}{\sqrt{2}} \phi} - 1) \right),
\]
\[ N = 2 : \quad z - z_0 = \frac{\sqrt{3}}{g} \left( e^{-\sqrt{3} \phi} + 2e^{-\frac{1}{\sqrt{3}} \phi} + 2 \log(e^{-\frac{1}{\sqrt{3}} \phi} - 1) \right), \quad (13) \]

\[ N = 3 : \quad z - z_0 = \frac{2}{g} \left( e^{-\frac{1}{\sqrt{3}} \phi} + \frac{2}{3} e^{-\frac{2}{\sqrt{3}} \phi} + 2e^{-\frac{1}{2 \sqrt{3}} \phi} + 2 \log(e^{-\frac{1}{2 \sqrt{3}} \phi} - 1) \right), \quad (13) \]

(The analogous solutions constructed using \( W_+ \) rather than \( W_- \) can also be easily obtained, but they seem not to be directly relevant for our present purposes.) These supersymmetric domain walls in a different coordinate system were given in [38], where their higher dimensional origins as M-branes and D3-branes were discussed. Note that the hypergeometric function \( _2F_1[a, 1, 1 + a, x] \) appearing in (12) is the Lerch transcendent \( a \Phi(x, 1, a) \). In fact, the solutions for \( D > 3 \) and for \( D = 3 \) can all be given by a single formula using the Lerch transcendent, namely

\[ z - z_0 = -\frac{a_1 \sqrt{N}}{g} e^{-\frac{1}{2} a_2 \phi} \Phi\left(e^{\frac{1}{2} (a_1 - a_2)} \phi, 1, \frac{a_2}{a_2 - a_1}\right). \quad (14) \]

The \( W_- \) solutions above all have two different branches. In one branch, \( \phi \) runs from 0 to \(+\infty\), with \( z \) running from \( z = -\infty \) to \( z = 0 \), where we have chosen the integration constant \( z_0 \) to be

\[ D > 3 : \quad z_0 = \frac{\pi a_1 \sqrt{N}}{g} \left( -i + \cot\left(\frac{\pi a_1}{a_1 - a_2}\right)\right), \]
\[ D = 3 : \quad z_0 = \frac{\pi a_1 \sqrt{N}}{g}. \quad (15) \]

(The imaginary part cancels the imaginary additive constant in the expressions (12) and (13).) When \( \phi \) is large, the solution takes the form

\[ e^{-\frac{1}{2} a_1 \phi} \sim -\frac{1}{4} a_1 \sqrt{N} g z, \]
\[ e^{(D-1) A} \sim c \sqrt{\frac{N}{8}} g e^{-\frac{1}{2} a_2 \phi} \sim c \sqrt{\frac{N}{8}} g \left( -\frac{1}{4} a_1 \sqrt{N} g z\right)^{\frac{a_2}{a_1}}. \quad (16) \]

In this branch, when the coordinate \( z \) reaches its limit at \( z = 0 \), the factor \( e^{2A} \) in the metric therefore goes to zero, and there is a power-law naked curvature singularity. (Note that in this regime the solution extends into large positive values of the potential (2) with a large cost to the energy density of the wall, and it thus terminates at a finite value of the transverse coordinate.)

As \( z \) approaches \(-\infty\), the functions \( \phi \) and \( A \) become

\[ \phi \sim e^{-\frac{a}{a_1 \sqrt{N}}} z, \quad A \sim \frac{g}{a_1 \sqrt{N} (D - 1)} z. \quad (17) \]

The metric asymptotically approaches the AdS space-time, described in horospherical coordinates with \( z \rightarrow -\infty \) corresponding to the Cauchy horizon [35, 36]. Note that on that
side of the wall the gravity is repulsive and provides “one half” of the Randall-Sundrum wall.

In the second branch, $\phi$ runs from 0 to $-\infty$, while $z$ runs from $z = -\infty$ to $z = +\infty$. The behaviour of the solution near $z = -\infty$ is the same as in the branch discussed previously, with the metric approaching asymptotically AdS. As $z$ approaches $+\infty$, the solution becomes

$$e^{-\frac{1}{2}a_2 \phi} \sim \frac{1}{4} a_2 \sqrt{N} g z,$$

$$e^{(D-1)A} \sim -c \sqrt{\frac{N}{8}} g e^{-\frac{1}{2}a_1 \phi} \sim -c \sqrt{\frac{N}{8}} g \left( \frac{1}{4} a_2 \sqrt{N} g z \right)^{\frac{a_1}{2}}. \quad (18)$$

(The constant $c$ is negative in this case.) This side describes one-side of a supersymmetric dilatonic domain wall [39]. Interestingly, it has no curvature singularity; as $z$ tends to $+\infty$ the curvature falls off as $1/z^2$, while the diverging dilaton $\phi \rightarrow -\infty$ approaches the weak coupling limit. Gravity on this side is attractive and for the null geodesics the affine parameter $\tau$ is infinite. Namely, $\tau \sim \int^{+\infty} e^{-A} dz \sim z^{1-\frac{a_1}{2}a_2(D-1)}|^{+\infty}$. Since for all the cases under consideration the ratio $\frac{a_1}{2}a_2(D-1) \leq 1$ (see Table 1), $\tau$ is indeed infinite and $z \rightarrow +\infty$ corresponds to the boundary of the space-time. ($D = 3$, $N = 1$ case is borderline with the affine parameter diverging logarithmically.)

For the purpose of constructing a domain-wall universe, it is the second of the two branches that is relevant. Thus this solution is a hybrid of the Type II vacuum domain wall and the dilatonic wall. The thickness of the wall is of the order of $1/g$. It is a non-singular solution, with repulsive gravity on the AdS-side ($z < 0$) and attractive gravity one on the dilatonic-side. While $z = -\infty$ corresponds to the AdS Cauchy horizons, $z \rightarrow +\infty$ is the time-like boundary of the space-time. The solutions for both the metric coefficient $e^{2A}$ and the breathing-mode scalar $\phi$, as functions of $z$, are sketched in Figure 2 below.

Thus within a pure field-theoretic framework, i.e. employing only the breathing-mode scalar field to construct the domain wall solution, we have been only partially successful; the massive gauged supergravity potential gave us one “supersymmetric AdS minimum” and another “run-away vacuum”, thus yielding a hybrid domain wall solution, and not the pure Type II vacuum domain wall that we were really after. Somewhat disappointing is the fact that on the dilatonic domain wall side gravity is attractive, and thus these domain walls cannot provide a phenomenologically viable scenario with a large transverse direction $z = \{-\infty, +\infty\}$; only the domain $z < 0$ can be taken large.

We may also explore another possibility, by adding a singular domain-wall source to this potential. The breathing-mode potential then provides a framework for implementing the
Figure 2: The functions $e^{2A}$ (upper line) and $\phi$ (lower line) for $D = 5$ as functions of the transverse coordinate $z$. Their forms for other dimensions are similar.

Goldberger-Wise scenario [40]. In this case, in the second branch the diverging behaviour of the dilaton is cancelled by a delta-function source for the domain wall at some finite value of $z$, say $z = z_*$. (Note that the source tension has to precisely balance that of the scalar contribution at the wall [41].) Then, the solution for $z > z_*$ can be replaced by a reflection of the solution for $z < z_*$, so that

$$-|z - z_*| + z_* = \frac{4}{a_2 g \sqrt{N}} e^{-\frac{1}{2} a_2 \phi} \, _2F_1\left[1, 1 + \frac{a_2}{a_2 - a_1}; e^{\frac{1}{2}(a_1 - a_2)\phi}\right].$$

(19)

The metric function $A$ is again given by substituting $\phi$ into (11). (The reason why a solution can be constructed in this way is because the original equations of motion (8) are invariant under $z \rightarrow -z$, and $z \rightarrow z + \text{constant}$.) Since $A$ is continuous at $z = z_*$, but its first derivative is not, it follows that there will be a delta-function curvature singularity there. This can be balanced by a domain-wall source term, in precisely the same way that one can balance the delta-function singularity on an electric string or $p$-brane soliton with an appropriate source term. The functions $e^{2A}$ and $\phi$ as a function of $z$ for this solution are plotted in Figure 3 below.

To summarise, in this paper we set out to explore the possibility of finding a supersymmetric AdS domain-wall solution, relevant for $D = 5$ for the Randall-Sundrum scenario, within massive gauged supergravity theories. By employing the potential for the massive breathing-mode scalar of the compactifying sphere in M-theory or string theory in diverse dimensions, we arrived at static (supersymmetric) domain walls which are of a hybrid type. On one side they correspond to the Randall-Sundrum wall with repulsive gravity, and on
Figure 3: The metric functions $e^{2A}$ (upper) and $\phi$ (lower) as a function of the transverse direction $z$ with the domain wall source added by hand. The solution provides a realization of the Goldberger-Wise [40] mechanism, where the massive breathing mode provides a potential that stabilizes the domain wall source and relaxes asymptotically to the AdS minimum of the potential.

the other side they are supersymmetric dilatonic walls [39].

Although these supergravity solutions *per se* do not possess all of the features needed for a Randall-Sundrum scheme, one can obtain a more satisfactory result by including also a fundamental (singular) domain-wall source. The massive scalar mode acts as a modulus stabilizing the domain wall (as in the Goldberger-Wise scenario), and it provides a repulsive (AdS) gravity transverse to the wall, as required for implementing the Randall-Sundrum scenario.

References


