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ANALYSIS OF MASS SHIFT DUE TO WEAK INTERACTIONS

Fayyazuddin

CERN - Geneva

and

Riazuddin

Institute of Physics, Islamabad

A B S T R A C T

Within the framework of light-cone algebra it is shown that there is no logarithmic divergence of order g_W^2 in the mass shift for a hadron due to weak interactions for currents having V-A structure, provided the weak interactions are mediated by vector bosons. As previously shown for electromagnetic mass shift, the source of this divergence is the mass term in the quark propagator. The contribution of this term to light-cone algebra vanishes due to the $V+A$ character of weak currents and this is the reason for absence of divergence of order g_W^2 in the weak mass shift. This result has important implications for unified gauge models for weak interactions based on semi-simple Lie groups of type $SU_L(n) \otimes U(1)$ or $SU_L(n) \otimes SU_R(n) \otimes U(1)$. For these models $g_W^2 \approx \alpha$, and the logarithmic divergence in the mass shift of a hadron can only arise from the current associated with $U(1)$. For mass difference of order α between an isospin multiplet to be finite and computable, $U(1)$ must be associated with an isoscalar quantity.

We also derive our main result by taking the Bjorken, Johnson and Low limit for the appropriate amplitudes. The structure of quadratic and logarithmic divergences of order g_W^2/m_W^2 in the mass shift is also discussed. These divergences arise due to non-conservation of weak currents.

*) On leave of absence from the University of Islamabad, Pakistan.

1. INTRODUCTION

A possible source of divergence in the electromagnetic self energy of a hadron is the deep inelastic region for electron-hadron scattering (the scaling region). This region in configuration space corresponds to light cone. The light-cone algebra is relevant for discussing this region. In a recent publication¹⁾ we have shown that within the framework of light-cone algebra, the electromagnetic mass shift comes out to be logarithmically divergent and that this logarithmic divergence arises precisely due to the mass term in the quark propagator. This divergence, therefore, can be removed by mass renormalization. The coefficient of this divergence is a scalar density which transforms as a singlet and octet under SU(3). In particular, proton-neutron mass difference is logarithmically divergent transforming as S_3 , but this divergence can be removed by quark mass renormalization. This may be helpful in understanding $\Delta I = 1$ mass differences.

If the weak interactions are mediated by intermediate vector bosons, then the contribution of weak interactions to self mass is in general of order g_W^2 rather than of order $G_F m^2$, where m is the typical hadron mass and G_F is the Fermi constant. Since g_W^2 can be of order e^2 [this is particularly true for unified gauge theories of weak interactions²⁾], the mass shift of a hadron due to isospin-breaking by weak interactions is expected to be of the same order as that due to electromagnetism. In particular, it may be possible that logarithmic divergence in the mass difference between members of an isospin multiplet due to electromagnetism is cancelled by the contribution from weak interactions, so that mass difference is finite and computable. This is what happens in the so-called³⁾ "natural" zeroth-order symmetries.

The source of logarithmic divergence for the case of electromagnetism is the contribution of quark mass term in the quark propagator to light-cone algebra. This term cannot contribute to the light-cone algebra for the weak currents which have pure $V - A$ structure (that is for weak currents having the form $V - A$ or $V + A$). The purpose of this paper is to show that there is no logarithmic divergence of order g_W^2 provided the weak currents have $V - A$ structure. Thus the radiative corrections of order g_W^2 due to weak interactions do not arise for such currents. For such currents we get a quadratic divergence and a logarithmic divergence of

order $G_F m^2$. These divergences entirely arise due to non-conservation of weak currents, the quadratic divergence being due to the so-called σ -term commutator.

The absence of logarithmic divergence of order g_W^2 in the mass shift has an important bearing on unified gauge theories of weak interactions. Consider a semi-simple gauge group G of type $SU_L(n) \otimes U(1)$ or $SU_L(n) \otimes SU_R(n) \otimes U(1)$. Since the currents associated with the parts $SU_L(n)$ or $SU_L(n) \otimes SU_R(n)$ have $V - A$ structure they do not give rise to a divergence of order $g_W^2 = \alpha$ in the self mass Δm . Only the current associated with the part $U(1)$ can give logarithmic divergence of order α . If the group structure is such that the current associated with the part $U(1)$ is not an isoscalar as is the case in the Salam-Weinberg²⁾ model, where $U(1)$ is generated by $Q - T_{L3}$, then the mass difference between members of an isospin multiplet is logarithmically divergent to order α . But this divergence can be removed by renormalizing the quark mass. If the current associated with $U(1)$ is isoscalar as is the case with the gauge group $SU_L(2) \otimes SU_R(2) \otimes U(1)$ considered by Weinberg³⁾, then the mass difference to order α is finite and computable. Thus any "natural" zeroth-order gauge-symmetry model based on semi-simple chiral gauge group must necessarily have group structure such that the current associated with $U(1)$ is isoscalar.

The plan of this paper is as follows: In Section 2, we write down light-cone algebra abstracted from the free quark model. In Section 3 we briefly discuss the electromagnetic mass shift and show that our previous result regarding logarithmic divergence in the electromagnetic mass shift holds if we discard the assumption that the amplitude T_L (defined in the text) satisfies an unsubtracted dispersion relation. In Section 4, we show that there is no logarithmic divergence of order g_W^2 in the self mass due to weak interactions provided weak currents have $V - A$ structure. In Section 5 we discuss the implication of this result for unified gauge models of weak interactions. In Section 6, we derive the same result by using the Bjorken, Johnson and Low limit.

2. LIGHT-CONE ALGEBRA

In the free quark model, the electromagnetic and weak hadronic current is defined in the familiar way

$$J_{\mu}^{em} = i \bar{\Psi} \gamma_{\mu} Q \Psi \quad (2.1a)$$

$$J_{\mu}^W = i \bar{\Psi} \gamma_{\mu} (1 + \gamma_5) W \Psi \quad (2.1b)$$

where ψ is the column matrix for the quark fields, Q is the charge matrix and W is the matrix for weak currents which will be specified later.

In order to write down the light-cone algebra, let us define

$$[W, W^{\dagger}]_{\pm} = R^{\pm} \quad (2.2)$$

and the bilocal operators

$$\begin{aligned} J_{\sigma}^R(z; S_A; x, y) &= \frac{1}{2} \left[i \bar{\Psi}(x) \gamma_{\sigma} (1 + \gamma_5) R \Psi(y) \right. \\ &\quad \left. \pm x \leftrightarrow y \right] \quad (2.3) \\ &= V_{\sigma}^R(S_A; x, y) + \gamma_5 A_{\sigma}^R(S_A; x, y) \end{aligned}$$

where R is any of the matrices, $R^{\pm}$, Q^2 introduced above and $r = \pm 1$.

The light-cone algebra satisfied by the commutators of the currents defined in Eq. (2.1) can be easily worked out in the free quark model and is given below:

$$\begin{aligned} [J_{\mu}^W(z), \bar{J}_{\nu}^W(0)] &\sim \\ &2 \left\{ \delta_{\mu\nu} \gamma_5 \left[J_{\sigma}^{R^-}(z, S; z, 0) + J_{\sigma}^{R^+}(z, A; z, 0) \right] \right. \end{aligned}$$

$$\begin{aligned}
 & + \epsilon_{\mu\nu\rho\sigma} \left[J_{\sigma}^{R-}(z, A; z, 0) + J_{\sigma}^{R+}(z, S; z, 0) \right] \Big\} \\
 & \times \frac{\partial}{\partial z_{\rho}} [D(z) + d(z)]
 \end{aligned}
 \tag{2.4a}$$

$$\begin{aligned}
 [J_{\mu}^{em}(z), J_{\nu}^{em}(0)] & \underset{z^2 \approx 0}{\sim} 2 \left[\epsilon_{\mu\nu\rho\sigma} V_{\sigma}^{Q^2}(A; z, 0) \right. \\
 & \left. + \epsilon_{\mu\nu\rho\sigma} A_{\sigma}^{Q^2}(S; z, 0) \right] \\
 & \times \frac{\partial}{\partial z_{\rho}} [D(z) + d(z)] \\
 & - 2i \left[\delta_{\mu\nu} J^{Q^2}(S; z, 0) \right. \\
 & \left. + i T_{\mu\nu}^{Q^2}(A; z, 0) \right] D(z),
 \end{aligned}
 \tag{2.4b}$$

where

$$\begin{aligned}
 J^{Q^2}(S; z, 0) & = \frac{1}{2} \left[\bar{\Psi}(z) M Q^2 \Psi(0) \right. \\
 & \left. + \bar{\Psi}(0) M Q^2 \Psi(z) \right]
 \end{aligned}
 \tag{2.5a}$$

$$T_{\mu\nu}^{Q^2}(A; z, 0) = \frac{1}{2} \left[\bar{\Psi}(z) \sigma_{\mu\nu} M Q^2 \Psi(0) - (z \leftrightarrow 0) \right] \quad (2.5b)$$

and

$$\delta_{\mu\nu} \delta_{\rho\sigma} = \delta_{\mu\rho} \delta_{\nu\sigma} + \delta_{\nu\rho} \delta_{\mu\sigma} - \delta_{\mu\sigma} \delta_{\nu\rho}.$$

In deriving Eqs. (2.4), we have made use of the anticommutation relation satisfied by free quark fields:

$$[\Psi(x), \bar{\Psi}(y)]_+ = -i S(x-y) \quad (2.6a)$$

where

$$S(x-y) = (\gamma_\mu \partial_\mu - M) \Delta(x-y) \quad (2.6b)$$

and

$$\Delta(z) \xrightarrow{z^2 \approx 0} -\frac{1}{2} \epsilon(z_0) \delta(z^2) \equiv \mathcal{D}(z). \quad (2.6c)$$

The next term to the leading light-cone singularity in Eq. (2.6c) is given by

$$\mathcal{d}(z) = -\frac{\mu^2}{2\pi} \frac{1}{2} \epsilon(z_0) \theta(z^2), \quad (2.6d)$$

where μ is a constant with dimension of mass and may be identified with the mean quark mass. This term has been included in the light-cone algebra given in Eqs. (2.4). In writing the light-cone algebra, one usually makes the approximation $S(x-y) \approx \gamma_\mu \partial_\mu \Delta(x-y)$. This approximation, we have not made in writing Eqs. (2.4). In Eq. (2.4a), since the weak currents J_μ^W and $\bar{J}_\mu^W$ have V - A structure, the mass term in the quark propagator (2.6b) does not contribute to the right-hand side of Eq. (2.4a). This is an important observation which is crucial to our results. In Eq. (2.4b), the second term on the right-hand side is due to the mass term in the quark propagator. It is precisely this term which gives rise to logarithmic divergence in the electromagnetic mass shift to order α . Note that M is quark mass matrix which is diagonal and so is Q^2 .

In order to use the algebra, let us define the spin summed matrix elements of the bilocal operators between one hadron states to be

$$\begin{aligned} & \frac{1}{2} (2\pi)^3 \frac{p_0}{m} \langle p | J_\epsilon^R(1; S_A; z, 0) | p \rangle \\ &= \tilde{G}_R^{S,A}(p \cdot z) \frac{p_\epsilon}{m} + i \tilde{h}_R^{S,A}(p \cdot z) z_\epsilon + \dots \end{aligned} \quad (2.7a)$$

$$= \frac{1}{2} (2\pi)^3 \frac{p_0}{m} \langle p | V_\epsilon^R(S_A; z, 0) | p \rangle$$

$$\begin{aligned} & \frac{1}{2} (2\pi)^3 \frac{p_0}{m} \langle p | J^{Q^2}(S; z, 0) | p \rangle \\ &= \tilde{S}_{Q^2}(p \cdot z) + \dots, \end{aligned} \quad (2.7b)$$

where z is light like and where ... denotes terms which vanish at $z^2 = 0$. Note that the axial vector bilocal operator does not contribute in the spin summed matrix elements and so is the case for the tensor bilocal operator if time-reversal invariance is assumed.

Let us now define Fourier transforms (omitting superscript S, A for brevity)

$$G_R(\xi) = \frac{1}{2\pi} \int d(p \cdot z) e^{i\xi(p \cdot z)} \tilde{G}_R(p \cdot z) \quad (2.8a)$$

$$\tilde{G}_R(p \cdot z) = \int d\xi e^{-i\xi(p \cdot z)} G_R(\xi) \quad (2.8b)$$

and similarly for $h(\xi)$ and $\tilde{h}(\xi)$. The Fourier transform of $\tilde{S}(p \cdot z)$ is given by

$$S_{Q^2}(\xi) = \frac{1}{2\pi} \int d(p \cdot z) e^{i\xi(p \cdot z)} \tilde{S}_{Q^2}(p \cdot z). \quad (2.8c)$$

We note that

$$\begin{aligned} \int d^4 z \, e^{-i q \cdot z} \tilde{G}_R(p, z) \frac{\partial}{\partial z_S} D(z) \\ = 2\pi (q_V + \xi p)_S (\xi/q^2) G_R(\xi) \end{aligned} \quad (2.9a)$$

$$\begin{aligned} \int d^4 z \, e^{-i q \cdot z} \tilde{G}_R(p, z) \frac{\partial}{\partial z_S} d(z) \\ = 2\pi \mu^2 \left[(q_V + \xi p)_S (\xi/q^2)^2 G'_R(\xi) \right. \\ \left. + O(1/q^4) \right] \end{aligned} \quad (2.9b)$$

$$\begin{aligned} \int d^4 z \, e^{-i q \cdot z} \tilde{h}_R(p, z) z_S \frac{\partial}{\partial z_S} D(z) \\ = - \frac{\partial}{\partial q_V} \left[2\pi (q_V + \xi p)_S \frac{\xi}{q^2} h_R(\xi) \right] \end{aligned} \quad (2.9c)$$

$$\begin{aligned} \int d^4 z \, e^{-i q \cdot z} \tilde{S}_{Q^2}(p, z) D(z) \\ = -i 2\pi (\xi/q^2) S_{Q^2}(\xi). \end{aligned} \quad (2.9d)$$

Here $G'_R(\xi) = (d/d\xi)G_R(\xi)$. We note that the two types of the non-leading terms arise from the non-leading terms in the light-cone algebra of Eqs. (2.4) namely $(\partial/\partial z_\rho)d(z)$ and $MD(z)$. The third type of the non-leading term arises from the coefficient of z_ρ in the spin-summed matrix elements of the bilocal operator defined in Eq. (2.7a). It is clear from Eq. (2.4b) that the explicit current conservation is satisfied to the leading order in the light-cone singularity. Since we have included all possible non-leading terms, it is appropriate to impose the condition of manifest gauge invariance for electromagnetic current to order $1/q^2$.

3. ELECTROMAGNETIC MASS SHIFT¹⁾

Let us define the absorptive part of the electroproduction amplitude:

$$A_{\mu\nu} = \frac{1}{2} (2\pi)^3 \frac{p_0}{m} \int d^4 z e^{-iq \cdot z} \langle p | [J_\mu^{em}(z), J_\nu^{em}(0)] | p \rangle$$

(3.1a)

$$= \frac{2\pi}{m^2} W_2(q^2, \nu) P_\mu P_\nu - 2\pi W_L(q^2, \nu) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right)$$

where

$$P_\mu P_\nu = p_\mu p_\nu + (p_\mu q_\nu + p_\nu q_\mu) / 2\xi + \frac{q^2}{4\xi^2} \delta_{\mu\nu}$$

$$W_L(q^2, \nu) = \frac{\nu^2}{q^2} W'_2(q^2, \nu) - W_1(q^2, \nu)$$

(3.1b)

$$W_L(q^2, \nu) = W_2(q^2, \nu) + W_L(q^2, \nu)$$

$$\nu = - \frac{p \cdot q}{m}, \quad \xi = \frac{q^2}{2m\nu}.$$

The scaling region ($q^2 \rightarrow \infty$, ξ finite) corresponds to light cone in the configuration space. Thus using Eqs. (2.4b), (2.7), (2.8) and (2.9), $A_{\mu\nu}$ in the deep inelastic region is given by (for brevity we have omitted the subscript Q^2)

$$\begin{aligned}
 A_{\mu\nu} \sim 2\pi \left\{ 2\xi \left[\frac{2\xi}{m} \left(\frac{P_\mu P_\nu}{q^2} \right) G^A(\xi) \right. \right. \\
 - \frac{\xi}{m} \frac{p^2}{q^2} \delta_{\mu\nu} G^A(\xi) \Big] \\
 + 2\xi \mu^2 \left[\frac{2\xi^2}{mq^2} \left(\frac{P_\mu P_\nu}{q^2} \right) G^{\prime A}(\xi) \right] \\
 - 2\xi \left[- \frac{2\xi_{\mu\nu}}{q^2} R^A(\xi) \right. \\
 + \frac{8\xi^2}{q^2} \left(\frac{P_\mu P_\nu}{q^2} \right) R^A(\xi) \\
 - \frac{4\xi}{q^2} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) R^{\prime A}(\xi) \\
 + \frac{8\xi^3}{q^2} \left(\frac{P_\mu P_\nu}{q^2} \right) R^{\prime A}(\xi) \\
 + \frac{2\xi}{q^2} \delta_{\mu\nu} R^{\prime A}(\xi) \Big] \\
 \left. - 2 \frac{\xi}{q^2} \delta_{\mu\nu} S(\xi) + O(1/q^4) \right\}, \quad (3.2)
 \end{aligned}$$

where $R^{\prime A}(\xi) = d/d\xi R^A(\xi)$.

From Eq. (3.2b) it is clear that we have gauge invariance to $O(1)$, and if we insist that the contribution to $O(1/q^2)$ should also be gauge invariant, then the terms which behave like $1/q^2$ and are not individually gauge invariant should combine to give a gauge invariant contribution to order $1/q^2$. This means (with $p^2 = -m^2$) that we must have

$$m \xi G^A(\xi) + 2 R^A(\xi) - 2 \xi R'^A(\xi) = S(\xi). \quad (3.3)$$

From Eqs. (3.1) and (3.2), using Eq. (3.3), we have

$$\begin{aligned} \nu W_2(q^2, \nu) = & 2 \xi G^A(\xi) - \frac{8 \xi^2}{q^2} m \left(R^A(\xi) + \xi R'^A(\xi) \right) \\ & + m^2 \frac{2 \xi^2}{q^2} G'^A(\xi) + O(1/q^4) \end{aligned} \quad (3.4a)$$

$$W_\ell(q^2, \nu) = - \frac{8 \xi^2}{q^2} R'^A(\xi) + O(1/q^4). \quad (3.4b)$$

Now with the usual definitions

$$\nu W_2(q^2, \nu) = F_2(\xi) + \frac{m^2}{q^2} H_2(\xi) + O(1/q^4) \quad (3.5a)$$

$$\begin{aligned} W_\ell(q^2, \nu) = & \frac{1}{m} \left[F_L(\xi) + \frac{m^2}{q^2} H_\ell(\xi) \right] \\ & + O(1/q^4) \end{aligned} \quad (3.5b)$$

$$\begin{aligned} W_L(q^2, \nu) = & \frac{1}{m} \left[F_L(\xi) + \frac{m^2}{q^2} H_L(\xi) \right] \\ & + O(1/q^4), \end{aligned} \quad (3.5c)$$

we have from Eqs. (3.4)

$$F_2(\xi) = 2\xi G^A(\xi) \quad (3.6a)$$

$$F_L(\xi) = 0 \quad (3.6b)$$

$$H_2(\xi) = -\frac{8\xi^2}{m} \left(R^A(\xi) + \xi R^{A'}(\xi) \right) + \frac{\mu^2}{m^2} 2\xi^2 G^{A'}(\xi) \quad (3.6c)$$

$$H_\ell(\xi) = -\frac{8\xi^2}{m} R^{A'}(\xi). \quad (3.6d)$$

Again using Eqs. (3.1), (3.5) and (3.6), we get

$$\begin{aligned} H_L(\xi) &\equiv 2\xi F_2(\xi) + H_\ell(\xi) \\ &= 4\xi^2 \left(G^A(\xi) - \frac{2}{m} R^{A'}(\xi) \right). \end{aligned} \quad (3.6e)$$

In general we expect the leading Regge behaviour for $H_L(\xi)$ or $H_\ell(\xi)$ to be $\xi^{-\alpha}$ ($\alpha = 1$) as $\xi \rightarrow 0$. In a special case $H_L(\xi)$ or $H_\ell(\xi)$ may not behave as $\xi^{-\alpha}$. This is the situation when the leading Regge contribution for W_1 and W_2 cancels in the linear combination (3.1b). For this case, the amplitude T_L (whose absorptive part is W_L) satisfies unsubtracted dispersion relation. This case has already been considered by us¹⁾. Here we consider the case when $H_L(\xi)$ behaves as $\xi^{-\alpha}$ as $\xi \rightarrow 0$. In this case T_L does not satisfy the subtracted dispersion relation.

To deal with the situation, when leading Regge behaviour for $H_L(\xi)$ is $\xi^{-\alpha}$, we multiply Eq. (3.3) by $e^{-i\xi\lambda}$ and integrate from $-\infty$ to $+\infty$

$$\begin{aligned}
 & m \int_{-\infty}^{\infty} e^{-i\xi\lambda} \xi G^A(\xi) d\xi \\
 & + 2 \int_{-\infty}^{\infty} e^{-i\xi\lambda} [R^A(\xi) - \xi R^A{}'(\xi)] d\xi \quad (3.7) \\
 & = \tilde{S}(\lambda).
 \end{aligned}$$

Equation (3.7) must be interpreted as a distribution, since we expect $\xi h^{A'}(\xi)$ or $h^A(\xi)$ to behave as $\xi^{-\alpha-1}$ ($\alpha = 1$) as $\xi \rightarrow 0$. First we note that

$$R^A{}'(\xi) = \frac{i}{2\pi} \int_{-\infty}^{\infty} e^{i\xi\lambda} \lambda \tilde{R}^A(\lambda) d\lambda \quad (3.8a)$$

and

$$i\lambda \tilde{R}^A(\lambda) = \int_{-\infty}^{\infty} e^{-i\xi\lambda} R^A{}'(\xi) d\xi. \quad (3.8b)$$

Differentiating with respect to λ , we have

$$\begin{aligned}
 & i \int_{-\infty}^{\infty} e^{-i\xi\lambda} [R^A(\xi) + \xi R^A{}'(\xi)] d\xi \\
 & = -i\lambda \frac{\partial \tilde{R}^A(\lambda)}{\partial \lambda}. \quad (3.8c)
 \end{aligned}$$

Now using Eqs. (3.8c) and (3.6d), we have from Eq. (3.7)

$$\begin{aligned}
 & \frac{1}{2} m \int_{-\infty}^{\infty} e^{-i\xi\lambda} F_2(\xi) d\xi \\
 & + \frac{1}{2} m \int_{-\infty}^{\infty} e^{-i\xi\lambda} \frac{H_0(\xi)}{\xi} d\xi \quad (3.9) \\
 & = \tilde{S}(\lambda) + 2\lambda \frac{\partial \tilde{R}^A(\lambda)}{\partial \lambda}.
 \end{aligned}$$

Parametrizing the leading Regge behaviour for $H_\ell(\xi)$ by

$$H_\ell(\xi) = \tilde{H}_\ell(\xi) + \sum_{\alpha > 0} \beta_\ell(\alpha) \xi^{-\alpha}, \quad (3.10)$$

we obtain

$$\begin{aligned} & \frac{1}{2} \pi \int_{-\infty}^{\infty} e^{-i\xi\lambda} F_2(\xi) d\xi \\ & + \frac{1}{2} \pi \int_{-\infty}^{\infty} e^{-i\xi\lambda} \frac{\tilde{H}_\ell(\xi)}{\xi} d\xi \\ & + \frac{1}{2} \sum_{\alpha > 0} \beta_\ell(\alpha) \int_{-\infty}^{\infty} e^{-i\xi\lambda} |\xi|^{-\alpha-1} d\xi \\ & = \tilde{S}(\lambda) + 2\lambda \frac{\partial \tilde{R}^A(\lambda)}{\partial \lambda}. \end{aligned} \quad (3.11)$$

But

$$\int_{-\infty}^{\infty} e^{-i\xi\lambda} |\xi|^{-\alpha-1} d\xi = - \frac{\Gamma(\alpha+1)}{\sin \pi\alpha/2} |\lambda|^\alpha. \quad (3.12)$$

Thus from Eq. (3.11), we have for $\lambda = 0$

$$\begin{aligned} & \frac{1}{2} \pi \int_{-\infty}^{\infty} F_2(\xi) d\xi + \frac{1}{2} \pi \int_{-\infty}^{\infty} \frac{\tilde{H}_\ell(\xi)}{\xi} d\xi \\ & = \tilde{S}(0). \end{aligned} \quad (3.13a)$$

The sum rule (3.13a) can also be expressed as

$$\begin{aligned} & \frac{1}{2} \pi \int_{-\infty}^{\infty} \frac{\tilde{H}_L(\xi)}{\xi} d\xi - \frac{1}{2} \pi \int_{-\infty}^{\infty} F_2(\xi) d\xi \\ & = \tilde{S}(0) \end{aligned} \quad (3.13b)$$

on using Eq. (3.6e). We note that $F_2(\xi) = 0$ for $|\xi| > 1$ and using the crossing property, we have⁴⁾

$$m \int_0^\infty \frac{\tilde{H}_L(\xi)}{\xi} d\xi - m \int_0^1 F_2(\xi) d\xi = \tilde{S}(0). \quad (3.14)$$

The electromagnetic mass shift can be written⁵⁾

$$\Delta m = - \frac{d}{8\pi^2} \int_0^\infty \frac{dq^2}{q^2} \int_{-1}^{+1} dy (1-y^2)^{1/2} \times q^2 F_{\mu\mu}(q^2, cy), \quad (3.15)$$

where

$$F_{\mu\nu}(q^2, \nu) = i(2\pi)^3 \frac{p_0}{m} \int d^4z e^{-iq \cdot z} \times \langle p | T(J_\mu^{em}(z) J_\nu^{em}(0)) | p \rangle \quad (3.16)$$

and $y = \nu/(q^2)^{1/2}$.

As is well known $F_{\mu\nu}$ can be decomposed as

$$F_{\mu\nu}(q^2, \nu) = \frac{1}{m^2} T_2(q^2, \nu) \left[p_\mu p_\nu - \frac{p \cdot q}{q^2} (p_\mu q_\nu + p_\nu q_\mu) + \frac{(p \cdot q)^2}{q^4} q_\mu q_\nu \right] + T_1(q^2, \nu) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right). \quad (3.17)$$

In the above equations

$$Abst T_i(q^2, \nu) = 2\pi W_i(q^2, \nu), \quad i = 1, 2. \quad (3.18)$$

We shall now make the assumption that the amplitude T_2 satisfies unsubtracted dispersion relation in v and T_1 satisfies one subtracted dispersion relation. Under these assumptions, it is possible to express the divergent part of the electromagnetic mass shift as

$$(\Delta m)_{\text{divergent}} = - \frac{3d}{4\pi} m \left\{ \left(\int_{q_m^2}^{\infty} \frac{d^4 q^2}{q^2} \right) \times \left[\int_0^1 F_2(\xi) d\xi - \int_0^{\infty} \frac{\tilde{H}_L(\xi)}{\xi} d\xi \right] \right\}. \quad (3.19)$$

The derivation of the above result is similar to the derivation given in the Appendix for the mass shift due to weak interactions. Using the sum rule (3.14), we get our previous result¹⁾

$$(\Delta m)_{\text{divergent}} = \frac{3d}{4\pi} \left(\int_{q_m^2}^{\infty} \frac{d^4 q^2}{q^2} \right) \tilde{S}(0), \quad (3.20)$$

where

$$\tilde{S}(0) = \frac{1}{2} (2\pi)^3 \frac{P_0}{m} \langle p | \bar{\Psi} M Q^2 \Psi | p \rangle.$$

We note that the source of this divergence in Δm is the contribution from quark mass in the quark propagator to the light-cone algebra (2.4b).

4. MASS SHIFT DUE TO WEAK INTERACTIONS

Let us define the absorptive part of the neutrino-production amplitude:

$$\begin{aligned}
 A_{\mu\nu}^{\bar{\nu}} &= \frac{1}{2} (2\pi)^3 \frac{p_0}{m} \int d^4x \, e^{iq \cdot x} \\
 &\quad \times \langle p | [J_\mu^W(x), \bar{J}_\nu^W(0)] | p \rangle \\
 &= \frac{2\pi}{m^2} W_2^{\bar{\nu}}(q^2, \nu) P_\mu P_\nu \\
 &\quad - 2\pi W_\ell^{\bar{\nu}}(q^2, \nu) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \\
 &\quad + \frac{2\pi}{2m^2} W_3^{\bar{\nu}}(q^2, \nu) \epsilon_{\mu\nu\rho\sigma} p_\rho q_\sigma \\
 &\quad + \frac{2\pi}{m^2} W_4^{\bar{\nu}}(q^2, \nu) q_\mu q_\nu + \frac{2\pi}{m^2} W_5^{\bar{\nu}}(q^2, \nu) \\
 &\quad \times (p_\mu q_\nu + p_\nu q_\mu).
 \end{aligned} \tag{4.1}$$

We are assuming T-invariance so that W_6 does not appear. Using light-cone algebra [Eq. (2.4a)] and following the method of Section 3, we get $A_{\mu\nu}^{\bar{\nu}}$ in the scaling limit

$$\begin{aligned}
 A_{\mu\nu}^{\bar{\nu}} &= 2\pi \frac{\xi}{q^2} 2 \left\{ \frac{1}{m} \left[G_{R^-}^S(\xi) + G_{R^+}^A(\xi) \right] \right. \\
 &\quad \times \left[2\xi P_\mu P_\nu - \xi p^2 \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q_\mu q_\nu}{q^2} \right) \right] \\
 &\quad \left. + \frac{1}{m} \left[G_{R^-}^A(\xi) + G_{R^+}^S(\xi) \right] \left[\epsilon_{\mu\nu\rho\sigma} p_\rho q_\sigma \right] \right\} \tag{4.2}
 \end{aligned}$$

$$\begin{aligned}
 & - \left[h_{R-}^S(\xi) + h_{R+}^A(\xi) \right] \left[-2\delta_{\mu\nu} + 2\frac{q_\mu q_\nu}{q^2} \right. \\
 & \quad \left. - 2\frac{q_\mu q_\nu}{q^2} + \frac{8\xi^2}{q^2} P_\mu P_\nu \right] \\
 & - \left[h_{R-}^{S'}(\xi) + h_{R+}^{A'}(\xi) \right] 2\xi \left[\frac{q_\mu q_\nu}{q^2} - \delta_{\mu\nu} \right. \\
 & \quad \left. + \frac{q_\mu q_\nu}{q^2} + \frac{4\xi^2}{q^2} P_\mu P_\nu \right] \quad (4.2) \\
 & + \mu^2 \left[\frac{2\xi^2}{m q^2} P_\mu P_\nu \left(G_{R-}^{S'}(\xi) + G_{R+}^{A'}(\xi) \right) \right. \\
 & \quad \left. + \frac{\xi}{m q^2} \epsilon_{\mu\nu\sigma\tau} P_\sigma q_\tau \left(G_{R-}^{A'}(\xi) + G_{R+}^{S'}(\xi) \right) \right] \\
 & \quad \left. + O(1/q^4) \right\}.
 \end{aligned}$$

From Eq. (4.2), we get

$$F_2^{\bar{\nu}, \nu}(\xi) = 2\xi \left[\pm G_{R-}^S(\xi) + G_{R+}^A(\xi) \right] \quad (4.3a)$$

$$\begin{aligned}
 H_2^{\bar{\nu}, \nu}(\xi) = & -\frac{8\xi^2}{m} \left[\pm h_{R-}^S(\xi) + h_{R+}^A(\xi) \right. \\
 & \left. \pm \xi h_{R-}^{S'}(\xi) + \xi h_{R+}^{A'}(\xi) \right] \quad (4.3b)
 \end{aligned}$$

$$+ \frac{\mu^2}{m^2} 2\xi^2 \left[\pm G_{R-}^{S'}(\xi) + G_{R+}^{A'}(\xi) \right]$$

$$F_L^{\bar{\nu}, \nu}(\xi) = 0 \quad (4.3c)$$

$$H_0^{\bar{\nu}, \nu}(\xi) = -\xi F_2^{\bar{\nu}, \nu}(\xi) - \frac{4\xi}{2\pi} \left[\pm R_{R^-}^S(\xi) + R_{R^+}^A(\xi) \right. \\ \left. \pm \xi R_{R^-}^{S'}(\xi) + \xi R_{R^+}^{A'}(\xi) \right] \quad (4.3d)$$

$$H_L^{\bar{\nu}, \nu}(\xi) = 2\xi F_2^{\bar{\nu}, \nu}(\xi) + H_0^{\bar{\nu}, \nu}(\xi) \quad (4.3e)$$

$$F_3^{\bar{\nu}, \nu}(\xi) = -2 \left[\pm G_{R^-}^A(\xi) + G_{R^+}^S(\xi) \right] \quad (4.3f)$$

$$\nu W_3^{\bar{\nu}, \nu}(q^2, \nu) = F_3^{\bar{\nu}, \nu}(\xi) - 2\mu^2 \frac{\xi}{q^2} \left(\pm G_{R^-}^A(\xi) + G_{R^+}^{S'}(\xi) \right) \\ + O(1/q^4) \quad (4.3g)$$

$$\nu^2 W_4^{\bar{\nu}, \nu}(q^2, \nu) \equiv F_4^{\bar{\nu}, \nu}(\xi) = \frac{m}{4\xi} F_2^{\bar{\nu}, \nu}(\xi) \\ + \frac{1}{\xi} \left[\pm R_{R^-}^S(\xi) + R_{R^+}^A(\xi) \mp \xi R_{R^-}^{S'}(\xi) - \xi R_{R^+}^{A'}(\xi) \right] \quad (4.3h)$$

$$\nu^2 W_5^{\bar{\nu}, \nu}(q^2, \nu) \equiv F_5^{\bar{\nu}, \nu}(\xi) = 0 \quad (4.3i)$$

Define $W_i^\pm = W_i^{\bar{\nu}} \pm W_i^{\nu}$. We get from Eqs. (4.3d), (4.3e), (4.3h) and (4.3i)

$$H_0^+(\xi) = -\xi F_2^+(\xi) - \frac{8\xi}{2\pi} \left[R_{R^+}^A(\xi) + \xi R_{R^+}^{A'}(\xi) \right] \quad (4.4a)$$

$$H_L^+(\xi) = 2\xi F_2^+(\xi) + H_0^+(\xi) \quad (4.4b)$$

$$F_4^+(\xi) = \frac{m}{4\xi} F_2^+(\xi) + \frac{2}{\xi} \left[R_{R^+}^A(\xi) - \xi R_{R^+}^{A'}(\xi) \right] \quad (4.4c)$$

$$F_5^+(\xi) = 0. \quad (4.4d)$$

It is easy to see that [integrating by parts and noting that at threshold $h_{R^+}^A(\xi = 1) = 0$]

$$\int_0^1 \xi R_{R^+}^{A'}(\xi) d\xi = - \int_0^1 R_{R^+}^A(\xi) d\xi. \quad (4.5)$$

Hence from Eq. (4.4a), we get the sum rule

$$\int_0^1 \left[\frac{H_L^+(\xi)}{\xi} + F_2^+(\xi) \right] d\xi = 0 \quad (4.6a)$$

and on using Eq. (4.4b), we get

$$\int_0^1 \left[\frac{H_L^+(\xi)}{\xi} - F_2^+(\xi) \right] d\xi = 0 \quad (4.6b)$$

This sum rule holds if the leading Regge behaviour for W_1 and W_2 cancels in the linear combination W_L . If this is not so, then we follow the method of Section 3 and using equation similar to Eq. (3.8c), we get from Eqs. (4.4a) and (4.4b), the sum rule

$$m \int_0^\infty \frac{\tilde{H}_L^+(\xi)}{\xi} d\xi - m \int_0^1 F_2^+(\xi) d\xi = 0 \quad (4.7)$$

The sum rules (4.6b) and (4.7) correspond, respectively, to the sum rule (18) of Ref. 1 and (3.14) for the electromagnetic case discussed in Section 3. Note that in the sum rules (4.6b) or (4.7), the right-hand side is zero, which is due to the fact that if weak hadronic currents have pure V - A structure, the mass term in the quark propagator does not contribute to the light-cone commutator of such currents.

The mass shift due to exchange of vector boson W coupled to weak current J_μ^W is given by

$$\begin{aligned} (\Delta m)^W &= i \frac{g_W^2}{2} \frac{1}{2(2\pi)^4} \int d^4 q \frac{\delta_{\lambda\mu} + \frac{q_\lambda q_\mu}{m_W^2}}{q^2 + m_W^2} \\ &\quad \times F_{\lambda\mu}^+(q^2, \nu) \\ &= - \frac{g_W^2}{4(2\pi)^3} \int \frac{d\bar{\nu}^2}{q^2 + m_W^2} \left[\delta_{\lambda\mu} + \frac{q_\lambda q_\mu}{m_W^2} \right] \\ &\quad \times \int_{-1}^{+1} dy (1-y^2)^{1/2} q^2 F_{\lambda\mu}^+(q^2, i y), \end{aligned} \quad (4.8)$$

where

$$F_{\lambda\mu}^+ = i (2\pi)^3 \frac{p_0}{2} \int d^4z \, e^{-iq \cdot z} \left[\langle p | T (J_\lambda^W(z) \bar{J}_\mu^W(0)) | p \rangle + (J^W \leftrightarrow \bar{J}^W) \right]. \quad (4.9)$$

Now $F_{\lambda\mu}^+$ can be decomposed as

$$F_{\lambda\mu}^+ = T_1^+ \delta_{\lambda\mu} + T_2^+ \frac{p_\lambda p_\mu}{m^2} + T_3^+ \frac{1}{2m^2} \epsilon_{\lambda\mu\sigma\tau} p_\sigma q_\tau + T_4^+ \frac{q_\lambda q_\mu}{m^2} + T_5^+ \frac{q_\lambda p_\mu + q_\mu p_\lambda}{2m^2}. \quad (4.10a)$$

Let $\bar{W}_i^+$ denote the absorptive parts of T_i^+ :

$$\text{Abs } T_i^+ = 2\pi \bar{W}_i^+. \quad (4.10b)$$

$\bar{W}_i$ are related to previously introduced W_i in Eq. (4.1) as follows:

$$\bar{W}_i = W_i, \quad i = 1, 2, 3 \quad (4.11a)$$

$$\bar{W}_4 = -\frac{m^2}{q^2} W_1 + \frac{m^2 v^2}{q^4} W_2 + W_4 \quad (4.11b)$$

$$\bar{W}_5 = \frac{2m v}{q^2} W_2 + 2W_5 \quad (4.11c)$$

We write the propagator in the form

$$\frac{\delta_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}}{q^2 + m_W^2} = \frac{\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}}{q^2 + m_W^2} + \frac{q_\mu q_\nu}{q^2} \frac{1}{m_W^2}. \quad (4.12)$$

The first part on the right-hand side of Eq. (4.12) projects out the contribution of the conserved part of the currents. The second part gives the contribution due to non-conservation of the currents. Only the first part can give divergence of order g_W^2 in the self mass. We first consider the contribution to $(\Delta m)^W$ from the conserved piece of the weak currents.

We note that

$$\begin{aligned} \left(S_{\lambda\mu} - \frac{q_\lambda q_\mu}{q^2} \right) F_{\lambda\mu}^+ &= 3T_1^+ - \left(1 + \frac{v^2}{q^2}\right) T_2^+ \\ &\equiv f^+ . \end{aligned} \quad (4.13)$$

Hence from Eq. (4.8), using Eq. (4.13), the contribution of the conserved piece of weak currents to the mass shift is given by

$$\begin{aligned} (\Delta m)^{Wc} &= - \frac{g_W^2}{4(2\pi)^3} \int \frac{d^4 q^2}{q^2 + m_W^2} \\ &\quad \times \int_{-1}^{+1} dy (1-y^2)^{1/2} q^2 f^+(q^2, y) . \end{aligned} \quad (4.14)$$

We now make the usual assumptions that T_2^+ satisfies unsubtracted dispersion relation in v for fixed q^2 , and T_1^+ satisfies one subtracted dispersion relation.

$$T_2^+(q^2, v) = 2 \int_{-\infty}^{\infty} \frac{W_2^+(q^2, v')}{v' - v} dv' \quad (4.15a)$$

$$\begin{aligned} T_1^+(q^2, v) &= T_1^+(q^2, 0) \\ &\quad + 2v \int_{-\infty}^{\infty} \frac{W_1^+(q^2, v')}{v'(v' - v)} dv' . \end{aligned} \quad (4.15b)$$

Using crossing and the variable $\xi = q^2/2m\nu$, we have in the scaling limit

$$T_2^+(q^2, \xi) = - \frac{8m\xi^2}{q^2} \int_0^1 \frac{F_2^+(\xi')}{\xi'^2 - \xi^2} d\xi' + O(1/q^4) \quad (4.16a)$$

$$T_1^+(q^2, \xi) = T_1^+(q^2, \xi = \infty) - \frac{4}{m} \int_0^1 \xi' \frac{[F_1^+(\xi') + \frac{m^2}{q^2} H_1^+(\xi')]}{\xi'^2 - \xi^2} d\xi' + O(1/q^4). \quad (4.16b)$$

As is clear from Eqs. (4.16), the divergence in $(\Delta m)^{W_c}$ given in Eq. (4.14) arises due to scaling behaviour of the structure function W_2 and W_1 .

From Eq. (4.14) using Eqs. (4.13) and (4.16), we have

$$(\Delta m)_{\text{divergent}}^{W_c} = - \frac{3g_W^2}{4(2\pi)^2} m \left\{ \int \frac{d^4 q^2}{q^2 + m_W^2} \times \left[\frac{q^2}{4m} T_1^+(q^2, \infty) - \int_0^1 (F_2^+(\xi) - \xi F_L^+(\xi)) d\xi \right] \right\}. \quad (4.17)$$

To determine the subtraction term $T_1^+(q^2, \infty)$, we consider two cases:

- i) The amplitude T_L satisfies the unsubtracted dispersion relation. This is clearly a possibility when the leading Regge contribution for W_1 and $(v^2/q^2)W_2$ cancels in the $-W_1 + (v^2/q^2)W_2$. For this case it is easy to see that

$$T_1^+(q^2, \infty) \underset{q^2 \rightarrow \infty}{\sim} - \frac{4}{m} \int_0^1 \left[F_L^+(\xi) + \frac{m^2}{q^2} (H_L^+(\xi) - 2\xi F_2^+(\xi)) \right] \frac{d\xi}{\xi}. \quad (4.18)$$

Hence from Eq. (4.17), we have, using the Callan-Gross⁶⁾ relation $F_L^+(\xi) = 0$,

$$(\Delta m)_{\text{divergent}}^{W_c} = - \frac{3g_W^2}{4(2\pi)^2} m \left\{ \int_0^\infty \frac{dq^2}{q^2 + m_W^2} \right. \\ \left. \times \left[\int_0^1 (F_2^+(\xi) - \frac{H_L^+(\xi)}{\xi}) d\xi \right] \right\}. \quad (4.19)$$

Now using the sum rule (4.6b), we have

$$(\Delta m)_{\text{divergent}}^{W_c} = 0. \quad (4.20)$$

This is our main result. There is no divergence of order g_W^2 in the mass shift $(\Delta m)^W$ provided the weak currents have V - A structure.

We now consider the other case:

- ii) The amplitude T_L does not satisfy the unsubtracted dispersion relation. For this case we have shown in the Appendix

$$(\Delta m)_{\text{divergent}}^{W_c} = - \frac{3g_W^2}{4(2\pi)^2} m \left\{ \int_0^\infty \frac{dq^2}{q^2 + m_W^2} \right. \\ \times \left[\int_0^1 F_2^+(\xi) d\xi - \int_0^\infty \frac{d\xi}{\xi} \tilde{H}_L^+(\xi) \right. \\ \left. \left. - \frac{2}{m} \int_0^\infty \tilde{F}_5^+(\xi) d\xi \right] \right\}. \quad (4.21)$$

Now using the sum rule (4.7) and Eq. (4.3g), we again have

$(\Delta m)_{\text{divergent}}^{W_c} = 0$. Although we have derived our main result (4.20), using light-cone algebra abstracted from the free quark model, our result holds in the quark-gluon model provided that the gluon is introduced in the gauge-invariant way. This is because the right-hand side of Eq. (4.21) also vanishes in the quark gluon model as discussed in Ref. 4.

We conclude that there is no divergence of order g_W^2 in the mass shift $(\Delta m)^W$ from the conserved piece of weak V - A currents; the only divergence in $(\Delta m)^W$ arises from the non-conservation of weak currents. This divergence is of order g_W^2/m_W^2 and as shown in the Appendix is given by

$$\begin{aligned}
 (\Delta m)_{\text{divergent}}^W &= - \frac{g_W^2}{16 (2\pi)^2} \frac{1}{m_W^2} \tilde{\Sigma}^+ \int_0^\infty dq^2 \\
 &\quad - \frac{g_W^2}{(2\pi)^2} \frac{m^2}{m_W^2} \left(\int_0^\infty \frac{dq^2}{q^2} \right) \\
 &\quad \times \left[-3 \int_0^1 \xi^2 \left(\xi F_4^+(\xi) - F_5^+(\xi) \right) d\xi \right. \\
 &\quad \left. + \int_0^\infty \left(\xi \tilde{H}_4^+(\xi) - \tilde{H}_5^+(\xi) \right) d\xi \right], \quad (4.22)
 \end{aligned}$$

where $\tilde{\Sigma}^+$ is the so-called σ -term

$$\begin{aligned}
 i \tilde{\Sigma}^+ \delta^3(\underline{z}) &= (2\pi)^3 \frac{p_0}{m} \left\{ \langle p | \left[\partial_\mu J_\mu^W(\underline{z}, 0), \bar{J}_0^W(0) \right] | p \rangle \right. \\
 &\quad \left. + (J^W \leftrightarrow \bar{J}^W) \right\} \\
 &= (2\pi)^3 \frac{p_0}{m} i \langle p | \bar{\Psi} \gamma_2 [M, R^+]_+ \Psi | p \rangle \delta^3(\underline{z}). \quad (4.23)
 \end{aligned}$$

Here M is the quark mass matrix. The structure functions H_4 and H_5 arise because we have assumed that structure functions W_4 and W_5 scale as

$$\nu^2 W_4(q^2, \nu) \sim F_4(\xi) + \frac{m^2}{q^2} H_4(\xi) \quad (4.24a)$$

$$\nu^2 W_5(q^2, \nu) \sim F_5(\xi) + \frac{m^2}{q^2} H_5(\xi). \quad (4.24b)$$

5. GAUGE THEORIES OF WEAK AND ELECTROMAGNETIC INTERACTIONS

We consider the unified gauge theories⁷⁾ based on the semi-simple group G of the form $SU_L(n) \otimes U(1)$ or $SU_L(n) \otimes SU_R(n) \otimes U(1)$. We shall assume that the fundamental strongly-interacting particles on which gauge symmetry acts are quark fields $\psi(x)$. The interaction Lagrangian with gauge bosons can be written

$$\mathcal{L}_\psi^W = \sum_a J_{\mu a} W_{\mu a}, \quad (5.1)$$

where $J_{\mu a}$ has the form

$$J_{\mu a} = i g \bar{\psi} \gamma_\mu (1 \pm \gamma_5) W_a \psi \quad (5.2a)$$

$$\text{or} \quad J_{\mu a} = i g \bar{\psi} \gamma_\mu W_a \psi. \quad (5.2b)$$

W_a are matrices representing the Lie algebra of group G . Note that $J_{\mu a}$ are in general not physical currents coupled to spin 1 bosons of definite mass. The physical bosons are some linear combinations of them.

The mass shift of order g^2 ($\approx \alpha$) arises for the conserved piece of these currents and is given by

$$\begin{aligned} (\Delta m)^{W_c} = & - \frac{1}{4(2\pi)^3} \int d^4 q^2 \int_{-1}^{+1} dy (1-y^2)^{1/2} q^2 \\ & \times \left[\delta_{\mu\lambda} - \frac{q_\lambda q_\mu}{q^2} \right] F_{\lambda\alpha, \mu\beta}(q^2, iy) \\ & \times (q^2 + m_W^2)^{-1}_{\alpha\beta} \end{aligned} \quad (5.3a)$$

where

$$\begin{aligned} F_{\lambda\alpha, \mu\beta} = & i(2\pi)^3 \frac{p_0}{m} \int d^4 z e^{-iq \cdot z} \\ & \times [\langle p | T(J_{\lambda\alpha}(z) J_{\mu\beta}(0)) | p \rangle] \end{aligned} \quad (5.3b)$$

We are interested in the divergent part of $(\Delta m)^W$. As far as the divergent part is concerned we can replace

$$(q^2 + m_W^2)^{-1}_{ab} = \delta_{ab} \frac{1}{q^2} \quad (5.4)$$

Then it is clear from the analysis of Sections 3 and 4 that the source of divergence in Eq. (5.3a) is a contribution of quark mass term to the light-cone commutator

$$[J_{\lambda a}(z), J_{\mu a}(0)]$$

For those currents which have V - A structure, this contribution is zero and hence such currents do not give a divergent contribution to $(\Delta m)^W$.

All the currents associated with group G are of this form except the current associated with the part U(1). This current can give a logarithmically-divergent contribution of order α to $(\Delta m)^W$. If this current is isoscalar, then the mass difference between members of an isospin multiplet to order α is finite and commutable. If this current is not an isoscalar, then the mass difference to order α is logarithmically divergent, but this divergence can be removed by quark mass renormalization. Thus the necessary condition for the so-called gauge models having "natural" zeroth order symmetry³⁾ is that they must have group structure such that the current associated with U(1) is isoscalar.

As an illustration of these remarks, we consider the following two models

- i) Weinberg Model³⁾: In this model the gauge group is $SU_L(2) \otimes SU_R(2) \otimes U(1)$. The interaction Lagrangian is given by⁸⁾

$$\begin{aligned} \mathcal{L}_\psi^W = & (J_{L\mu}^W \bar{W}_{L\mu} + h.c.) + (J_{R\mu}^W \bar{W}_{R\mu} + h.c.) \\ & + J_{L\mu}^{W^0} W_{L\mu}^0 + J_{R\mu}^{W^0} W_{R\mu}^0 + J_\mu^B B_\mu, \end{aligned} \quad (5.5)$$

where

$$J_{L,R\mu}^W = \frac{g_{L,R}}{2\sqrt{2}} i \bar{\Psi} \gamma_{\mu} (1 \pm \gamma_5) W \Psi \quad (5.6a)$$

$$J_{L,R\mu}^{W^0} = \frac{1}{4} g_{L,R} i \bar{\Psi} \gamma_{\mu} (1 \pm \gamma_5) W^0 \Psi \quad (5.6b)$$

$$J_{\mu}^B = \frac{1}{2} g' (2\alpha - 1) i \bar{\Psi} \gamma_{\mu} \Psi \quad (5.6c)$$

The currents given in Eqs. (5.6) are not the physical currents associated with bosons of definite masses. For example, the electromagnetic current in this model is given by

$$\frac{1}{e} J_{\mu}^{em} = \frac{1}{4} \left(\frac{1}{g_L} J_{L\mu}^{W^0} + \frac{1}{g_R} J_{R\mu}^{W^0} \right) + \frac{1}{g'} \frac{2\alpha - 1}{2} J_{\mu}^B \quad (5.7)$$

It is clear from the structure of the currents given in Eqs. (5.6), that in this model the mass difference to order α between members of an isospin multiplet is finite.

- ii) Salam-Weinberg model²⁾: In this model, the gauge group is $SU_L(2) \otimes U(1)$ generated by T_L and $Q - T_{L3}$, respectively. In this model, we do not expect mass difference to order α to be finite. The interaction Lagrangian is given by⁹⁾

$$\mathcal{L}_{\Psi}^W = J_{\mu}^W \bar{W}_{\mu} + \bar{J}_{\mu}^W W_{\mu} + J_{\mu}^{W^0} W^0 + J_{\mu}^B B_{\mu}, \quad (5.8)$$

where

$$J_{\mu}^W = \frac{g}{2\sqrt{2}} i \bar{\Psi} \gamma_{\mu} (1 + \gamma_5) W \Psi \quad (5.9a)$$

$$\bar{J}_{\mu}^W = \frac{g}{2\sqrt{2}} i \bar{\Psi} \gamma_{\mu} (1 + \gamma_5) W^{\dagger} \Psi \quad (5.9b)$$

$$J_{\mu}^{W^0} = \frac{g}{4} i \bar{\Psi} \gamma_{\mu} (1 + \gamma_5) W^0 \Psi \quad (5.9c)$$

$$J_\mu^B = \frac{g'}{4} i \bar{\Psi} \gamma_\mu (1 + \gamma_5) W^0 \Psi - g' i \bar{\Psi} \gamma_\mu Q \Psi. \quad (5.9d)$$

In both models (i) and (ii), ψ is the column matrix

$$\begin{bmatrix} \psi_4 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix}$$

W is the 4×4 matrix

$$\begin{bmatrix} 0 & U \\ 0 & 0 \end{bmatrix}$$

with

$$U = \begin{bmatrix} -\sin \theta_c & \cos \theta_c \\ \cos \theta_c & \sin \theta_c \end{bmatrix} \text{ and } W^0 = [W, W^\dagger].$$

Q is the charge matrix

$$\begin{bmatrix} a & & & \\ & a & & \\ & & a^{-1} & \\ & & & a^{-1} \end{bmatrix}.$$

In this model, currents given in Eqs. (5.9a, b) are the charged weak currents. The other two physical currents are given by

$$\begin{aligned} e J_\mu^{em} &= (g' J_\mu^{W^0} - g J_\mu^B) \frac{1}{(g^2 + g'^2)^{1/2}} \\ &= \frac{gg'}{(g'^2 + g^2)^{1/2}} i \bar{\Psi} \gamma_\mu Q \Psi \end{aligned} \quad (5.10a)$$

$$\frac{(g'^2 + g^2)^{1/2}}{2} J_\mu^Z = (g J_\mu^{W^0} + g' J_\mu^B) \frac{1}{(g^2 + g'^2)^{1/2}}$$

$$= \frac{(g^2 + g'^2)^{1/2}}{2} \left[\frac{1}{2} \bar{\Psi} \gamma_\mu (1 + \gamma_5) W^0 - 2 \sin^2 \theta_W J_\mu^{em} \right], \quad (5.10b)$$

where

$$\sin^2 \theta_W = \frac{g'^2}{g^2 + g'^2} \quad (5.11a)$$

$$e = g g' / (g^2 + g'^2)^{1/2} \quad (5.11b)$$

$$g'^2 = e^2 / \cos^2 \theta_W \quad (5.11c)$$

It is clear from Eqs. (5.9), that a logarithmic divergence of order α in Δm can only arise from the current J_μ^B given in Eq. (5.9c). To calculate $(\Delta m)_{\text{divergent}}$, we first write down the contribution of mass term in the quark propagator to the light-cone commutator for the currents of the form

$$J_\mu^W = i \bar{\Psi} \gamma_\mu (1 + \gamma_5) W \Psi \quad (5.12a)$$

$$\bar{J}_\mu^W = i \bar{\Psi} \gamma_\mu (1 + \gamma_5) W^\dagger \Psi \quad (5.12b)$$

This contribution is given by

$$\begin{aligned} & \left[J_\mu^W(x; z), \bar{J}_\nu^W(x'; 0) \right]_M \\ & \underset{z^2 \approx 0}{\sim} i \left\{ \left[\delta_{\mu\nu} J^{R+}(S; z, 0) + i T_{\mu\nu}^{R+}(A; z, 0) \right. \right. \\ & \quad \left. \left. + \delta_{\mu\nu} J^{R-}(A; z, 0) + i T_{\mu\nu}^{R-}(S; z, 0) \right] (1 - 2\hat{z}) \right. \\ & \quad \left. - \left[\delta_{\mu\nu} \tilde{J}^{R+}(A; z, 0) + i \tilde{T}_{\mu\nu}^{R+}(S; z, 0) \right. \right. \\ & \quad \left. \left. + \delta_{\mu\nu} \tilde{J}^{R-}(S; z, 0) + i \tilde{T}_{\mu\nu}^{R-}(A; z, 0) \right] (2 - \hat{z}) \right\} D(z), \end{aligned} \quad (5.13)$$

where $J^{R^\pm}$ and $T_{\mu\nu}^{R^\pm}$ have been defined in Eqs. (2.5) with Q^2 replaced by $R^\pm$. $\tilde{J}$ and $\tilde{T}_{\mu\nu}$ can be obtained from J and $T_{\mu\nu}$ by replacing M by $\gamma_5 M$. For the current J_μ^B , we get (writing only the terms which contribute to spin summed matrix elements of the commutator)

$$\begin{aligned} & [J_\mu^B(z), J_\nu^B(0)]_M \\ & \underset{z^2 \rightarrow 0}{\sim} -i g'^2 \left[-\frac{1}{2} \delta_{\mu\nu} J^X(s; z, 0) \right. \\ & \quad \left. + 2 J^{Q^2}(s; z, 0) \right] D(z), \end{aligned} \quad (5.14)$$

where we have used the relations

$$[W^0, Q]_+ = X, \quad [W^0, Q]_- = 0, \quad (5.15)$$

with

$$X = 2 Q^2 + 2 a (1-a) I. \quad (5.16)$$

Then from the analysis of Section 4, it is clear that the sum rule corresponding to (4.6d) for J_μ^B is given by

$$\begin{aligned} & m \int_0^1 \left[\frac{H_L^B(\xi)}{\xi} - F_2^B(\xi) \right] d\xi \\ & = \frac{1}{2} \left[\tilde{S}_{Q^2}(0) - a(1-a) \tilde{S}_I(0) \right]. \end{aligned} \quad (5.17)$$

Hence from Eq. (4.19), $(\Delta m)_{\text{divergent}}^{W_c}$ is given by

$$\begin{aligned} & (\Delta m)_{\text{divergent}}^{W_c} = \frac{3}{4\pi} \frac{d_c}{2 \cos^2 \theta_W} \\ & \times \left[\tilde{S}_{Q^2}(0) - a(1-a) \tilde{S}_I(0) \right]. \end{aligned} \quad (5.18)$$

But this divergence can be removed by quark mass renormalization. Finally, we note that for the physical neutral weak current J_μ^Z , the sum rule corresponding to Eq. (5.17) is given by

$$\begin{aligned} m \int_0^1 \left[\frac{H_L^Z(\xi)}{\xi} - F_2^Z(\xi) \right] d\xi \\ = -2 \sin^2 \theta_W (1 - 2 \sin^2 \theta_W) \tilde{S}_{Q^2}(0) \\ + 2a(a-1) \sin^2 \theta_W \tilde{S}_I(0) \end{aligned} \quad (5.19)$$

6. BJORKEN, JOHNSON AND LOW LIMIT

In this section we derive our main result by taking the Bjorken, Johnson and Low¹⁰⁾ (BJL) limit for the amplitude $F_{\lambda\mu}^+(q^2, \nu)$. In this limit

$$\begin{aligned} F_{\lambda\mu}^+ \underset{\nu_0 \rightarrow i\infty, \nu = 0}{\sim} - \frac{1}{q_0^2} (2\pi)^3 \frac{p_0}{m} \\ \times \int d^3z \left\{ \langle p | \left[[J_\lambda^W(z, 0), H], \bar{J}_\mu^W(0) \right] | p \rangle \right. \\ \left. + (J^W \leftrightarrow \bar{J}^W) \right\} \end{aligned} \quad (6.1)$$

In the free quark model

$$\mathcal{L} = -\bar{\Psi} \left(\gamma_\mu \frac{\partial}{\partial x_\mu} + M \right) \Psi \quad (6.2a)$$

$$\begin{aligned} H &= \bar{\Psi} (\gamma \cdot \underline{\partial} + M) \Psi \\ &= H_0 + H_M \end{aligned} \quad (6.2b)$$

Let us write

$$C_{\lambda\mu}^+ = \int d^3z \left\{ \left[\left[J_{\lambda}^W(z, 0), H_M \right], \bar{J}_{\mu}^W(0) \right] + (J^W \leftrightarrow \bar{J}^W) \right\} \quad (6.3)$$

$$O_{\lambda\mu}^+ = \int d^3z \left\{ \left[\left[J_{\lambda}^W(z, 0), H_0 \right], \bar{J}_{\mu}^W(0) \right] + (J^W \leftrightarrow \bar{J}^W) \right\}. \quad (6.4)$$

Now using Eqs. (6.2), we have [specifically for $J_{\mu}^W = i\bar{\psi}\gamma_{\mu}(1 + \gamma_5)\psi$]

$$C_{44}^+ = 2\bar{\psi} [M, R^+]_+ \psi + 2\bar{\psi} \gamma_5 [M, R^+]_- \psi \quad (6.5a)$$

$$C_{ij}^+ = -\delta_{ij} 2\bar{\psi} [M, R^+]_+ \psi - 2i\bar{\psi} \epsilon_{ij} \gamma_5 [M, R^+]_+ \psi \\ - 2i\bar{\psi} \epsilon_{ij} [M, R^+]_- \psi - \delta_{ij} 2\bar{\psi} \gamma_5 [M, R^+]_- \psi \quad (6.5b)$$

$$C_{4j}^+ = 2i\bar{\psi} \epsilon_{4j} \gamma_5 [M, R^+]_+ \psi \\ + 2i\bar{\psi} \epsilon_{4j} [M, R^+]_- \psi = -C_{j4}^+ \quad (6.5c)$$

$$O_{ij}^+ = -\bar{\psi} 2\Gamma_{ij}(1 + \gamma_5)R^+ \psi \quad (6.6a)$$

$$O_{44}^+ = 0, \quad O_{4j}^+ = -O_{j4}^+ = 0 \quad (6.6b)$$

where

$$\Gamma_{ij} = \left[\gamma_i \overleftrightarrow{\partial}_j + \gamma_j \overleftrightarrow{\partial}_i - 2 \delta_{ij} \gamma \cdot \overleftrightarrow{\partial} \right]. \quad (6.6c)$$

First we note that the Lorentz tensor structure of the spin summed matrix elements of $\bar{\psi} \gamma_{\mu} \overleftrightarrow{\partial}_{\nu} R^+ \psi$ is given by

$$(2\pi)^3 \frac{p_0}{m} \langle p | \bar{\psi} \gamma_{\mu} \overleftrightarrow{\partial}_{\nu} R^+ \psi | p \rangle = \quad (6.7a)$$

$$A \delta_{\mu\nu} + \frac{B}{m^2} p_{\mu} p_{\nu}.$$

From this, we have

$$(2\pi)^3 \frac{p_0}{m} \langle p | \bar{\psi} [M, R^+]_+ \psi | p \rangle \quad (6.7b)$$

$$\equiv \frac{1}{2} \tilde{\Sigma}^+ = 4A - B$$

and

$$(2\pi)^3 \frac{p_0}{m} \langle p | \bar{\psi} \gamma \cdot \overleftrightarrow{\partial} R^+ \psi | p \rangle$$

$$= 3A - B + \frac{p_0^2}{m^2} B \quad (6.7c)$$

$$= \frac{1}{4} \left[\frac{3}{2} \tilde{\Sigma}^+ - (1 + 4y^2) B \right].$$

Note that in this frame $p_0^2 = - \frac{m^2 v^2}{y^2} = -m^2 y^2.$

In the BJL limit, using Eqs. (6.1), (6.3), (6.4), (6.5), (6.6) and (6.7), we have

$$\begin{aligned}
 & \left(\delta_{\lambda\mu} + \frac{q_\lambda q_\mu}{m_W^2} \right) F_{\lambda\mu}^+ (q^2, y) \\
 & \sim \frac{1}{q^2} (2\pi)^3 \frac{p_0}{m} \langle B | \left[C_{\lambda\lambda}^+ + \frac{q^2}{m_W^2} C_{44}^+ \right. \\
 & \quad \left. + O_{\lambda\lambda}^+ + \frac{q^2}{m_W^2} O_{44}^+ \right] | B \rangle \\
 & = \left[-\frac{3}{q^2} + \frac{q^2 + m_W^2}{q^2 + m_W^2} \right] \tilde{\Sigma}^+ + \frac{1}{m_W^2} O(1/q^2) \\
 & \quad + \frac{1}{q^2} \left[3 \tilde{\Sigma}^+ - 2(1-4y^2) B \right] \\
 & \quad + O(1/q^4).
 \end{aligned} \tag{6.8}$$

Hence from Eq. (4.8), we have

$$\begin{aligned}
 (\Delta m)_{\text{divergent}}^W &= - \frac{g_W^2}{16(2\pi)^2} \frac{1}{m_W^2} \tilde{\Sigma}^+ \int d^4v \\
 &+ \frac{g_W^2}{m^2} \left[\text{A logarithmic divergent term} \right]
 \end{aligned} \tag{6.9}$$

This result is in agreement with our previous result as given in Eq. (4.22). Again, we see that a logarithmic divergent term of order g_W^2 is absent. Note that divergent terms in Eq. (6.9) arise entirely due to non-conservation of weak currents.

The logarithmic divergent term of order g_W^2/m_W^2 on the right-hand side of Eq. (6.9) can be explicitly calculated by noting that

$$\begin{aligned}
 q_\lambda q_\mu F_{\lambda\mu}^+ &= F^+ + (\sigma\text{-term}) \\
 &+ (\text{an equal time commutator linear in } q.)
 \end{aligned} \tag{6.10}$$

where

$$F^+ = (2\pi)^3 \frac{p_0}{m} i \int d^4 z \, e^{i q \cdot z} \times \left[\langle p | T \left(\partial_\lambda J_\lambda^W(z) \partial_\mu \bar{J}_\mu^W(0) \right) | p \rangle \right. \\ \left. + (J^W \leftrightarrow \bar{J}^W) \right]. \quad (6.11)$$

The contribution of the σ -term, we have already calculated in Eq. (6.9); the equal time commutator does not contribute to mass shift due to symmetric integration. Thus the logarithmic divergent contribution of order g_W^2/m_W^2 to mass shift can be calculated by taking the BJL limit of F^+ in Eq. (6.11).

Denoting this contribution by $(\Delta m)^{W}$, we have

$$(\Delta m)_{\text{divergent}}^W = - \frac{g_W^2}{4(2\pi)^3} \frac{1}{m_W^2} \int \frac{d^4 q^2}{q^2 + m_W^2} \\ \times \int_{-1}^{+1} dy (1-y^2)^{1/2} q^2 F^+(q^2, iy), \quad (6.12)$$

where

$$F^+(q^2, iy) \sim \frac{1}{q^2} \left\{ (2\pi)^3 \frac{p_0}{m} \right. \\ \times \int d^3 z \, \langle p | \left[\left[\partial_\lambda J_\lambda^W(z, 0), H \right], \partial_\mu \bar{J}_\mu^W(0) \right] | p \rangle \right. \\ \left. + (J^W \leftrightarrow \bar{J}^W) \right\}. \quad (6.13)$$

Now

$$\begin{aligned}
 & \int d^3z \left\{ \left[\left[\partial_\lambda J_\lambda^W(z, 0), H \right], \partial_\mu \bar{J}_\mu^W(0) \right] \right. \\
 & \quad \left. + (J^W \leftrightarrow \bar{J}^W) \right\} \\
 &= -\bar{\Psi} (\tilde{g} + \tilde{g}^\dagger + \tilde{f} + \tilde{f}^\dagger) \Psi \\
 & \quad + \bar{\Psi} \gamma_5 (g' - g'^\dagger - \tilde{f} + \tilde{f}^\dagger) \Psi \\
 & \quad + \bar{\Psi} \left[2 \underline{\gamma} \cdot \overset{\leftrightarrow}{\partial} (G + \tilde{G}) \right] \Psi \\
 & \quad + \bar{\Psi} \left[2 \underline{\gamma} \cdot \overset{\leftrightarrow}{\partial} \gamma_5 (G' + G'^\dagger) \right] \Psi,
 \end{aligned} \tag{6.14}$$

where

$$\tilde{g} = \left[\left[\tilde{w}, M \right]_+, \tilde{w}^\dagger \right]_+ \tag{6.15a}$$

$$\tilde{f} = - \left[\left[\tilde{w}', M \right]_-, \tilde{w}'^\dagger \right]_- \tag{6.15b}$$

$$\tilde{f} = \left[\left[\tilde{w}', M \right]_-, \tilde{w}^\dagger \right]_- \tag{6.15c}$$

$$g' = - \left[\left[\tilde{w}, M \right]_+, \tilde{w}'^\dagger \right]_+ \tag{6.15d}$$

$$G = \left[\tilde{w}, \tilde{w}^\dagger \right]_+ \tag{6.15e}$$

$$\tilde{G} = \left[\tilde{w}', \tilde{w}'^\dagger \right]_+ \tag{6.15f}$$

$$\tilde{w} = \left[w, M \right]_+ \tag{6.15g}$$

$$\tilde{w}' = \left[w, M \right]_- \tag{6.15h}$$

The Lorentz tensor structure of the spin summed matrix elements of $\bar{\psi} \gamma_{\mu} \overleftrightarrow{\partial}_{\nu} (G + \tilde{G}) \psi$ is given by

$$(2\pi)^3 \frac{p_0}{m} \langle p | \psi \gamma_{\mu} \overleftrightarrow{\partial}_{\nu} (G + \tilde{G}) \psi | p \rangle = h \delta_{\mu\nu} + \frac{h'}{m^2} p_{\mu} p_{\nu} \quad (6.16)$$

From this we have

$$(2\pi)^3 \frac{p_0}{m} \langle p | \bar{\psi} [M, (G + \tilde{G})]_+ \psi | p \rangle = 4h - h'. \quad (6.17a)$$

Hence we have

$$\begin{aligned} (2\pi)^3 \frac{p_0}{m} \langle p | 2 \bar{\psi} \gamma_{\mu} \overleftrightarrow{\partial}_{\nu} (G + \tilde{G}) \psi | p \rangle \\ = 2 \frac{1}{4} [3(4h - h') - h'(1 + 4y^2)] \end{aligned} \quad (6.17b)$$

From Eqs. (6.12), (6.13), (6.14) and (6.17b), we have

$$\begin{aligned} (\Delta m')_{\text{divergent}}^W &= - \frac{g_W^2}{16(2\pi)^2} \frac{1}{m_W^2} \left[\int \frac{d^4 q^2}{q^2 + m_W^2} \right] \\ &\times \left[(2\pi)^3 \frac{p_0}{m} \langle p | \bar{\psi} \left(\frac{3}{2} [M, (G + \tilde{G})]_+ \right. \right. \\ &\quad \left. \left. - (\tilde{g} + \tilde{g}^\dagger + f' + f'^\dagger) \right) \psi | p \rangle \right]. \end{aligned} \quad (6.18)$$

Finally from Eq. (6.9), we get

$$\begin{aligned} (\Delta m)_{\text{divergent}}^W &= - \frac{g_W^2}{16(2\pi)^2} \frac{1}{m_W^2} \\ &\times \left\{ \tilde{\Sigma}^+ \int d^4 q^2 + \left[\int \frac{d^4 q^2}{q^2 + m_W^2} \right] \right. \\ &\quad \left. \times \left[(2\pi)^3 \frac{p_0}{m} \langle p | \bar{\psi} \left(\frac{3}{2} [M, (G + \tilde{G})]_+ \right. \right. \right. \right. \\ &\quad \left. \left. \left. - (\tilde{g} + \tilde{g}^\dagger + f' + f'^\dagger) \right) \psi | p \rangle \right] \right\}. \end{aligned} \quad (6.19)$$

We conclude that divergence in the weak mass shift is of order g_W^2/m_W^2 for weak currents having V - A structure.

Now we briefly discuss the weak radiative corrections to any strong process $\alpha(p) \rightarrow \beta(p)$. First we note that for gauge theories which we have considered in Section 5, $R^+ = I$, where I is a unit matrix. As is clear from Eq. (6.8), the radiative correction to order $g_W^2 \sim \alpha$ is determined by

$$\begin{aligned} & \left(\delta_{\lambda\mu} - \frac{q_\lambda q_\mu}{q^2} \right) F_{\lambda\mu, \alpha \rightarrow \beta}^+ \\ & \sim \left\{ -\frac{6}{q^2} \left[(2\pi)^3 \frac{p_0}{m_\alpha} \langle \beta | 2 \bar{\Psi} M \Psi | \alpha \rangle \right] \right. \\ & \quad \left. + \frac{1}{q^2} \left[-(2\pi)^3 \frac{p_0}{m_\alpha} \langle \beta | \bar{\Psi} 2 \Gamma_{ii} (1 + \gamma_5) \Psi | \alpha \rangle \right] \right\} \end{aligned} \quad (6.20)$$

This shows that the weak correction of order α to any strong process is logarithmically divergent. Note that we have not taken the spin sum in Eq. (6.20). We now show that this divergence can be removed by wave-function renormalization. Let us now perform the transformation

$$\Psi \rightarrow Z_2^{1/2} \frac{1 + \gamma_5}{\sqrt{2}} \Psi. \quad (6.21)$$

Then

$$\begin{aligned} \mathcal{L} = & -\bar{\Psi} \gamma_\mu \frac{\partial}{\partial x_\mu} \Psi + (1 - Z_2) \\ & \times \bar{\Psi} \gamma_\mu \frac{\partial}{\partial x_\mu} \Psi - Z_2 \bar{\Psi} \gamma_\mu \gamma_5 \frac{\partial}{\partial x_\mu} \Psi, \end{aligned} \quad (6.22a)$$

$$\begin{aligned} H = & \bar{\Psi} \gamma_\mu \partial_\mu \Psi - (1 - Z_2) \\ & \times \bar{\Psi} \gamma_\mu \partial_\mu \Psi + Z_2 \bar{\Psi} \gamma_\mu \partial_\mu \gamma_5 \Psi. \end{aligned} \quad (6.22b)$$

Equation (6.20) now becomes

$$\left(\delta_{\lambda\mu} - \frac{q_\lambda q_\mu}{q^2} \right) F_{\lambda\mu, \alpha \rightarrow \beta}^+ \sim \frac{16}{q^2} \left[(2\pi)^3 \frac{p_0}{m_\alpha} \langle \beta | \bar{\Psi} \gamma_\mu (1 + \gamma_5) \Psi | \alpha \rangle \right] \quad (6.23)$$

Thus the weak radiative corrections of order $g_W^2 \approx \alpha$ to a strong process $\alpha \rightarrow \beta$ are in general logarithmically divergent. But this divergence can be cancelled by the counter terms available in Eq. (6.22). Note that in the single loop approximation which we are considering

$$(1 - Z_2)^{(2)} = -Z_2^{(2)}.$$

We conclude that any divergence of order α in the weak radiative corrections to any strong process can be removed by the renormalization procedure outlined above. Therefore, such divergences can have no effect on "natural" zeroth-order symmetry relations.

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APPENDIX

In this Appendix, we derive the formula (4.21). Let $T_{if}(q^2)$ denote the residue of possible fixed poles in amplitudes $T_i(q^2, \nu)$. We write

$$T_i(q^2, \nu) = \tilde{T}_i(q^2, \nu) + T_i^R(q^2, \nu) + T_{if}(q^2), \quad (A.1)$$

where T_i^R is the Regge contribution which determines the behaviour of amplitudes T_i as $\nu \rightarrow \infty$. Thus

$$\tilde{T}_i(q^2, \nu) \underset{\nu \rightarrow \infty}{\sim} 0. \quad (A.2)$$

and we can write

$$T_{if}(q^2) = \lim_{\nu \rightarrow \infty} [T_i(q^2, \nu) - T_i^R(q^2, \nu)]. \quad (A.3)$$

It is convenient to define two more amplitudes F_λ^+ and F^+ in addition to the amplitude $F_{\lambda\mu}^+$ defined in Section 4:

$$\begin{aligned} F_\lambda^+ &= (2\pi)^3 \frac{p_0}{m} i \int d^4 z \, e^{-i q \cdot z} \\ &\times [\langle p | T(\partial_\mu J_\mu^w(z) \bar{J}_\lambda^w(0)) | p \rangle \\ &\quad + (J^w \leftrightarrow \bar{J}^w)] \end{aligned} \quad (A.4)$$

$$= i [G_1^+ p_\lambda + G_2^+ q_\lambda]$$

$$\begin{aligned} F^+ &= (2\pi)^3 \frac{p_0}{m} i \int d^4 z \, e^{-i q \cdot z} \\ &\times [\langle p | T(\partial_\mu J_\mu^w(z) \partial_\lambda \bar{J}_\lambda^w(0)) | p \rangle \\ &\quad + (J^w \leftrightarrow \bar{J}^w)]. \end{aligned} \quad (A.5)$$

Let A_λ^+ and A^+ denote their absorptive part, respectively:

$$A_\lambda^+ = 2\pi i [g_1^+ p_\lambda + g_2^+ q_\lambda] \quad (\text{A.6})$$

$$A^+ = \frac{2\pi}{m^2} D^+. \quad (\text{A.7})$$

Then the conditions

$$\begin{aligned} i q_\mu A_{\mu\lambda}^+ &= A_\lambda^+ \\ -i q_\lambda A_\lambda^+ &= A^+ \end{aligned} \quad (\text{A.8})$$

give

$$\begin{aligned} g_1^+ &= \frac{q^2}{m^2} W_5^+ \\ g_2^+ + \frac{m\nu}{q^2} g_1^+ &= \frac{q^2}{m^2} W_4^+ \\ D^+ &= q^2 [q^2 W_4^+ - 2m\nu W_5^+]. \end{aligned} \quad (\text{A.9})$$

For amplitudes T_1^+ , T_4^+ , G_2^+ and F^+ , the asymptotic behaviour of their absorptive parts is of the form ν^α , $\alpha > 0$. These amplitudes, therefore, satisfy one subtracted dispersion relation in ν . Let us denote the subtraction constants in dispersion relations for these amplitudes by $T_{1,4}^+(q^2, 0)$, $G_2^+(q^2, 0)$ and $F^+(q^2, 0)$.

We note from Eq. (A.1)

$$T_{1,4}^+(q^2, 0) = \tilde{T}_{1,4}^+(q^2, 0) + T_{1,4f}^+(q^2) \quad (\text{A.10})$$

$$G_2^+(q^2, 0) = \tilde{G}_2^+(q^2, 0) + G_{2f}^+(q^2) \quad (\text{A.11})$$

$$F^+(q^2, 0) = \tilde{F}^+(q^2, 0) + F_f^+(q^2) \quad (\text{A.12})$$

But $\tilde{T}_{1,4}^+(q^2, 0)$, $\tilde{G}_2^+(q^2, 0)$ and $\tilde{F}^+(q^2, 0)$ are given by

$$\tilde{T}_1^+(q^2, 0) = 4 \int_0^\infty \frac{\tilde{W}_1^+(q^2, \nu)}{\nu} d\nu \quad (A.13)$$

$$\begin{aligned} \tilde{T}_4^+(q^2, 0) = & 4 \int_0^\infty \left[-\frac{m^2}{q^2} \tilde{W}_1^+(q^2, \nu) \right. \\ & \left. + \frac{m^2 \nu^2}{q^4} \tilde{W}_2^+(q^2, \nu) + \tilde{W}_4^+(q^2, \nu) \right] \frac{d\nu}{\nu} \end{aligned} \quad (A.14)$$

$$\begin{aligned} \tilde{G}_2^+(q^2, 0) = & 4 \int_0^\infty \left[\frac{q^2}{m^2} \tilde{W}_4^+(q^2, \nu) \right. \\ & \left. - \frac{\nu}{m} \tilde{W}_5^+(q^2, \nu) \right] \frac{d\nu}{\nu} \end{aligned} \quad (A.15)$$

$$\begin{aligned} \tilde{F}^+(q^2, 0) = & \frac{4q^2}{m^2} \int_0^\infty \left[q^2 \tilde{W}_4^+(q^2, \nu) \right. \\ & \left. - 2m\nu \tilde{W}_5^+(q^2, \nu) \right] \frac{d\nu}{\nu} \end{aligned} \quad (A.16)$$

Changing ν to the scaling variable ξ in Eqs. (A.13) to (A.16), we have from Eqs. (A.10) to (A.11) in the scaling limit

$$\begin{aligned} T_1^+(q^2, \infty) & \underset{q^2 \rightarrow \infty}{\sim} T_{1f}^+(q^2) \\ & + \frac{4}{m} \int_0^\infty \left[\tilde{F}_1^+(\xi) + \frac{m^2}{q^2} \tilde{H}_1^+(\xi) \right] \frac{d\xi}{\xi} \\ & + O(1/q^4) \end{aligned} \quad (A.17)$$

$$\begin{aligned} T_4^+(q^2, \infty) & \sim T_{4f}^+(q^2) + \frac{4m}{q^2} \int_0^\infty \left[\tilde{F}_L^+(\xi) \right. \\ & \left. + \frac{m^2}{q^2} \tilde{H}_L^+(\xi) + \frac{4m}{q^2} \xi^2 \tilde{F}_4^+(\xi) \right] \frac{d\xi}{\xi} + O(1/q^6) \end{aligned} \quad (A.18)$$

$$G_2^+(q^2, \infty) \sim G_{2f}^+(q^2) + \frac{8}{q^2} \int_0^\infty [2\xi \tilde{F}_4^+(\xi) - \tilde{F}_5^+(\xi)] d\xi + O(1/q^4) \quad (A.19)$$

$$F^+(q^2, \infty) \sim F_f^+(q^2) + 16 \int_0^\infty [\xi \tilde{F}_4^+(\xi) - \tilde{F}_5^+(\xi) + \frac{m^2}{q^2} (\xi \tilde{H}_4^+(\xi) - \tilde{H}_5^+(\xi))] d\xi + O(1/q^4). \quad (A.20)$$

We now assume that fixed poles have polynomial residues. If we assume that $T_1^+(q^2, \xi)$ scales in the same manner as its absorptive part, then it follows from Eqs. (4.16b) and (A.17), that $T_{1f}^+(q^2) = T_{1f}^+$. Moreover light-cone algebra also implies that $T_1^+(q^2, \infty) \sim O(1/q^2)$. Thus from Eq. (A.17), we have⁴⁾

$$\frac{4}{m} \int_0^\infty \frac{\tilde{F}_1^+(\xi)}{\xi} d\xi = -T_{1f}^+ \quad (A.21)$$

$$T_1^+(q^2, \infty) \sim \frac{4m}{q^2} \int_0^\infty \frac{\tilde{H}_1^+(\xi)}{\xi} d\xi + O(1/q^4). \quad (A.22)$$

Now we apply the similar consideration to $F^+(q^2, \xi)$ as we did to the amplitude T_1^+ . Then we have from Eq. (A.20)

$$16 \int_0^\infty [\xi \tilde{F}_4^+(\xi) - \tilde{F}_5^+(\xi)] d\xi = -F_f^+ \quad (A.23)$$

$$F^+(q^2, \infty) \sim \frac{16m^2}{q^2} \int_0^\infty [\xi \tilde{H}_4^+(\xi) - \tilde{H}_5^+(\xi)] d\xi \quad (A.24)$$

$$+ O(1/q^4).$$

We want to express $\tilde{H}_1^+(\xi)$ in terms of $\tilde{H}_L^+(\xi)$. For this purpose we use Ward identities. Assuming that there are no q-number Schwinger terms, we have

$$-\frac{\nu}{m} T_2^+ + \frac{q^2}{2m^2} T_5^+ = G_1^+ + \frac{4}{m} L \quad (A.25)$$

$$-\frac{\nu}{2m} T_5^+ + \frac{q^2}{m^2} T_4^+ + T_1^+ = G_2^+ \quad (A.26)$$

$$-m\nu G_1^+ + q^2 G_2^+ = F^+ + \tilde{\Sigma}^+, \quad (A.27)$$

where

$$2L \frac{p_\lambda}{m} \delta^3(\underline{z}) = (2\pi)^3 \frac{p_0}{m} \langle p | [J_0^W(\underline{z}, 0), \bar{J}_\lambda^V(0)] | p \rangle \quad (A.28)$$

and $\tilde{\Sigma}^+$ is defined in Eq. (4.23). From Eq. (A.27), for $\nu = 0$ ($\xi = \infty$), we have

$$q^2 G_2^+(q^2, \infty) = F^+(q^2, \infty) + \tilde{\Sigma}^+ \quad (A.29)$$

Then from Eqs. (A.19) and (A.24), it follows that

$$G_{2f}^+(q^2) = 0 \quad (A.30)$$

and we get the sum rule¹¹⁾

$$8 \int_0^\infty [2\xi \tilde{F}_4^+(\xi) - \tilde{F}_5^+(\xi)] d\xi = \tilde{\Sigma}^+, \quad (A.31)$$

Now from Eq. (A.26) for $v = 0$ ($\xi = \infty$), we have on using Eqs. (A.17), (A.21), (A.22), (A.18), (A.19) and (A.30)

$$\begin{aligned} \frac{4m}{q^2} \int_0^\infty \frac{\tilde{H}_1^+(\xi)}{\xi} d\xi &= - \frac{q^2}{m^2} T_{4f}^+(q^2) \\ &- \frac{4}{m} \int_0^\infty \left[\tilde{F}_L^+(\xi) + \frac{m^2}{q^2} \tilde{H}_2^+(\xi) \right. \\ &\quad \left. + \frac{4m}{q^2} \xi^2 \tilde{F}_4^+(\xi) \right] \frac{d\xi}{\xi} \quad (A.32) \\ &+ \frac{8}{q^2} \int_0^\infty \left[2\xi \tilde{F}_4^+(\xi) - \tilde{F}_5^+(\xi) \right] d\xi. \end{aligned}$$

Hence we have

$$T_{4f}^+(q^2) = 0 \quad (A.33)$$

and the sum rules

$$\int_0^\infty \frac{\tilde{F}_L^+(\xi)}{\xi} d\xi = 0 \quad (A.34)$$

$$\begin{aligned} m \int_0^\infty \left[\tilde{H}_1^+(\xi) + \tilde{H}_2^+(\xi) \right] \frac{d\xi}{\xi} \\ = -2 \int_0^\infty \tilde{F}_5^+(\xi) d\xi. \end{aligned} \quad (A.35)$$

Since

$$H_2(\xi) = \frac{1}{2\xi} H_2(\xi) - H_1(\xi), \quad (A.36)$$

we get the sum rule

$$m \int_0^\infty \frac{\tilde{H}_2^+(\xi)}{\xi^2} d\xi = -4 \int_0^\infty \tilde{F}_5^+(\xi) d\xi. \quad (A.37)$$

The sum rule (A.34) has been obtained by several authors⁴⁾. Sum rule (A.35) or (A.37) is a new sum rule.

From Eqs. (A.22) and (A.35), it follows that

$$\begin{aligned}
 T_1^+(q^2, \infty) &\sim -\frac{4m}{q^2} \int_0^\infty \frac{\tilde{H}_0^+(\xi)}{\xi} d\xi \\
 &\quad - \frac{8}{q^2} \int_0^\infty \tilde{F}_5^+(\xi) d\xi \\
 &= -\frac{4m}{q^2} \left[-\int_0^1 2 F_2^+(\xi) d\xi \right. \\
 &\quad \left. + \int_0^\infty \frac{\tilde{H}_L^+(\xi)}{\xi} d\xi + \frac{2}{m} \int_0^\infty \tilde{F}_5^+(\xi) d\xi \right] \\
 &\quad + O(1/q^4).
 \end{aligned} \tag{A.38}$$

Substituting (A.38) in Eq. (4.17), we get the formula (4.21).

Now we discuss the contribution to $(\Delta m)^W$ due to non-conservation of current. This contribution is due to $(q_\lambda q_\mu / q^2 m_W^2) F_{\lambda\mu}^+$. But

$$\begin{aligned}
 q_\lambda q_\mu F_{\lambda\mu}^+ &= F^+ + (\text{an equal time commutator linear in } q) \\
 &\quad - i(2\pi)^3 \frac{p_0}{m} \langle p | \sigma | p \rangle.
 \end{aligned} \tag{A.39}$$

The equal time commutator does not contribute to the mass shift due to symmetric integration. The σ -term is given by

$$\begin{aligned}
 (2\pi)^3 \frac{p_0}{m} \langle p | \sigma | p \rangle &= \int d^3z (2\pi)^3 \frac{p_0}{m} \\
 &\times \left\{ \langle p | [\partial_\mu J_\mu^W(z, 0), J_0^W(0)] | p \rangle \right. \\
 &\quad \left. + (J^W \leftrightarrow \bar{J}^W) \right\} = i \tilde{\Sigma}^+
 \end{aligned} \tag{A.40}$$

The contribution of the σ -term to $(\Delta m)^W$ can be calculated by using Eqs. (4.8) and (4.12) and is given by

$$(\Delta m)_\sigma^W = - \frac{g_W^2}{16(2\pi)^2} \frac{1}{m_W^2} \tilde{\Sigma}^+ \int^\infty d q^2. \quad (A.41)$$

Now $F^+(q, \nu)$ satisfies one subtracted dispersion relation in ν :

$$F^+(q^2, \nu) = F^+(q^2, 0) + \frac{4}{m^2} \nu^2 q^2 \times \int_{q^2/4m^2}^\infty \frac{[\nu^2 W_4^+(q^2, \nu') - 2m \nu' W_5^+(q^2, \nu')]}{\nu' (\nu'^2 - \nu^2)} d\nu'. \quad (A.42)$$

In the scaling limit $F^+(q^2, \nu)$ is given by

$$F^+(q^2, i y) \sim F^+(q^2, \infty) - 16 \times \int_0^1 \xi^2 \frac{[\xi F_4^+(\xi) - F_5^+(\xi)]}{\xi^2 + q^2/4m^2 y^2} d\xi + O(1/q^4). \quad (A.43)$$

Now from Eq. (4.8), using (4.12), (A.43) and (A.24), we have

$$(\Delta m')^W = - \frac{g_W^2}{(2\pi)^2} \frac{m^2}{m_W^2} \left(\int^\infty \frac{d q^2}{q^2} \right) \times \left[-3 \int_0^1 \xi^2 (\xi F_4^+(\xi) - F_5^+(\xi)) d\xi + \int_0^\infty (\xi \tilde{H}_4^+(\xi) - \tilde{H}_5^+(\xi)) d\xi \right]. \quad (A.44)$$

Combining Eqs. (A.41) and (A.44), we get the formula (4.22).

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