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THE SLOPE OF THE PROTON-PROTON SCATTERING AMPLITUDE AT $t = 0$

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A B S T R A C T

A dispersion relation for the slope of a crossing symmetric amplitude is derived and applied in pp scattering at high energies and at $t=0$. The crossing symmetric amplitude is extracted from the pp amplitude with reasonable assumptions on the crossing antisymmetric part. It is found that the slope of the imaginary part is almost equal to the slope of the modulus. The derivative of the real part is much smaller than that of the imaginary part and shows a zero at NAL energies.

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1. - INTRODUCTION

In most investigations on elastic pp scattering at high energies it is assumed that the pp amplitude is pure imaginary (spin effects are generally neglected). For the forward amplitude the quality of this assumption can be checked from data as well as from dispersion calculations ¹⁾⁻³⁾. But practically nothing is known about the behaviour of the real part at $t \neq 0$. Any knowledge about that would be very helpful for testing models or, e.g., for verifying the results of the impact picture ^{4),5)}.

The aim of this work is to extract a further piece of information on the real part namely its derivative at $t = 0$. To do this a dispersion method - originally developed for the crossing odd πN amplitude ⁶⁾ - will be used which allows one to determine the derivative of the amplitude from the derivative of its modulus and from the amplitude itself at this t value. At $t = 0$ both quantities are known. The dispersion method is briefly described in the next section.

A problem arises from the fact that one needs the crossing symmetric part of the pp amplitude. The extraction of this part is described in a third section. The results and conclusions are discussed in a final section.

2. - THE METHOD

It is a well-known experimental fact that in pp scattering spin effects are negligible at least for high energies and small momentum transfer. So, pp scattering is described by one amplitude like the spinless case. Instead of the spin-averaged amplitude in this investigation the invariant function ^{*)}

$$F = F_s + \frac{v}{m} F_v \quad (1)$$

^{*)} Kinematical notations s, t, u are the usual Mandelstam variables, m is the nucleon mass. The lab. energy and momentum of the incident nucleon are $\omega = (s-2m^2)/(2m)$ and $p = \sqrt{\omega^2 - m^2}$, resp. The crossing variable is defined by $v = (s-u)/(4m) = \omega + t/(4m)$.

will be used for that. F_s and F_v are the Fermi invariants in the notation of Sharp and Wagner ⁷⁾, referring to scalar and vector couplings, resp. At $t = 0$ F is identical to the spin averaged amplitude.

For other t values F approaches this amplitude for $s \rightarrow \infty$. The total and differential cross-sections expressed by F are :

$$\sigma_{tot} = \frac{2m}{\rho} \text{Im } F(t=0) \quad (2a)$$

$$\frac{d\sigma}{dt} = \frac{m^2}{4\pi\rho^2} |F|^2 \quad (2b)$$

As shown by Goldberger et al. as well as by Amati et al. ⁸⁾, F is free of kinematical singularities so that Mandelstam analyticity can be assumed for it. Furthermore, F has nice properties under s - u crossing, so that one can establish crossing symmetric (+) and antisymmetric (-) combinations of the pp and $p\bar{p}$ amplitude :

$$F^{(\pm)} = \frac{1}{2} (F_{p\bar{p}} \pm F_{pp}) \quad (3)$$

According to what was said above, one can write a once-subtracted fixed t dispersion relation for $F^{(+)}$

$$\begin{aligned} \text{Re } F^{(+)}(v, t) &= \text{Re } F^{(+)}(v_0, t) + f_{\pi} \frac{v^2 - v_0^2}{(v_{\pi}^2 - v^2)(v_{\pi}^2 - v_0^2)} \\ &+ \frac{v^2 - v_0^2}{\pi} \int_{v(2\mu)}^{v_h} \frac{dv'}{v'^2 - v^2} \frac{v'}{v'^2 - v_0^2} \text{Im } F_{p\bar{p}}^{(+)}(v', t) + \frac{2}{\pi} (v^2 - v_0^2) \int_{v_h}^{\infty} \frac{dv'}{v'^2 - v^2} \frac{v'}{v'^2 - v_0^2} \text{Im } F^{(+)}(v', t) \quad (4) \end{aligned}$$

where $v_{th} = m + t/(4m)$, $v_{\pi} = (2\mu^2 - 4m^2 + t)/(4m)$ and $v(2\mu) = (8\mu^2 - 4m^2 + t)/(4m)$. μ is the mass of the pion, v_0 the point of subtraction - somewhere between the cuts - and $\text{Re } F^{(+)}(v_0, t)$ is the corresponding subtraction function. f_{π} is the residue of the pion pole in the u channel.

Forming now the derivative of Eq. (4) with respect to t at $t = 0$ keeping s fixed and eliminating $(\partial/\partial t)\text{Re } F^{(+)}$ with the aid of the obvious relation

$$|F^{(+)}| \frac{\partial}{\partial t} |F^{(+)}| = \operatorname{Re} F^{(+)} \frac{\partial}{\partial t} \operatorname{Re} F^{(+)} + \operatorname{Im} F^{(+)} \frac{\partial}{\partial t} \operatorname{Im} F^{(+)} \quad (5)$$

one arrives at a singular integral equation for the derivative of the imaginary part which can be solved by standard methods ⁹⁾ [some details are to be found in Ref. 6]

$$\begin{aligned} \frac{\partial}{\partial t} \operatorname{Im} F^{(+)} = \operatorname{Im} F^{(+)} \left\{ \frac{\partial}{\partial t} \ln |F^{(+)}| - \frac{\operatorname{Re} F^{(+)}}{|F^{(+)}|^2} (I + \tilde{I}) \right\} \\ - \frac{2}{\pi} \sqrt{\nu^2 - m^2} (\nu^2 - \nu_0^2) e^{\Gamma(\nu)} \frac{\operatorname{Re} F^{(+)}}{|F^{(+)}|} \times \\ \times \int_m^\infty \frac{d\nu'}{\nu'^2 - \nu^2} \frac{\nu' |F^{(+)}(\nu')| e^{-\Gamma(\nu')}}{\sqrt{\nu'^2 - m^2} (\nu'^2 - \nu_0^2)} \left[\frac{\partial}{\partial t} \ln |F^{(+)}(\nu')| - \frac{\operatorname{Re} F^{(+)}(\nu')}{|F^{(+)}(\nu')|^2} (I + \tilde{I}) \right] \quad (6) \end{aligned}$$

with

$$\Gamma(\nu) = \frac{2}{\pi} (\nu^2 - \nu_0^2) \int_m^\infty \frac{d\nu'}{\nu'^2 - \nu^2} \frac{\nu'}{\nu'^2 - \nu_0^2} \delta^{(+)}(\nu')$$

the phase representation ¹⁰⁾ of the forward amplitude $F^{(+)} = |F^{(+)}| \exp(i\delta^{(+)})$. $\tilde{I}$ is the sum over the derivatives of the pion pole term, of the subtraction function and of the integral on the unphysical cut between $\nu(2\mu)$ and ν_{th} . I is an integral over $\operatorname{Im} F^{(+)}(t=0)$ which originates from the fact that s is kept fixed and not ν if forming the derivative :

$$I = \frac{\nu - \nu_0}{2\pi m} \int_m^\infty \frac{d\nu'}{(\nu' + \nu)^2} \frac{2\nu' + \nu + \nu_0}{(\nu' + \nu_0)^2} \operatorname{Im} F^{(+)}(\nu', t=0) \quad (7)$$

In principle one may use Eq. (6) for calculations of the derivative of $F^{(+)}$ and indeed in other processes like πN scattering this can be done. But in the pp case considered here, this is impossible because of the lack of information on $F^{(+)}$ at low energies. Therefore, one has to confine oneself to high energies, say, to energies above $\bar{\nu} = 3 \text{ GeV}$. Then, for $(\bar{\nu}/\nu)^2 \ll 1$ Eq. (6) reduces to ^{*)}

*) The asymptotic expressions for the derivatives can also be derived from the modulus representation of analytic functions, studied for example by Odorico ¹¹⁾. However, the complete expressions are distinguished by the treatment of the complex zeros of $F^{(+)}$.

$$\frac{1}{\text{Im} F^{(+)}} \frac{\partial}{\partial t} \text{Im} F^{(+)} = b^{(+)} - \frac{2}{\pi} v s^{(+)} \int_{\bar{v}}^{\infty} \frac{dv'}{v'^2 - v^2} b^{(+)}(v') - \frac{\text{Re} F^{(+)}}{|F^{(+)}|^2} (I + \tilde{I}) \quad (8a)$$

Combining Eqs. (5) and (6), one receives the corresponding relation for the derivative of the real part :

$$\frac{1}{\text{Im} F^{(+)}} \frac{\partial}{\partial t} \text{Re} F^{(+)} = s^{(+)} b^{(+)} + \frac{2}{\pi} v \int_{\bar{v}}^{\infty} \frac{dv'}{v'^2 - v^2} b^{(+)}(v') + \frac{\text{Im} F^{(+)}}{|F^{(+)}|^2} (I + \tilde{I}) \quad (8b)$$

$b^{(+)}$ and $s^{(+)}$ are defined by

$$b^{(+)} = \frac{\partial}{\partial t} \ln |F^{(+)}| \quad (9a)$$

and

$$s^{(+)} = \frac{\text{Re} F^{(+)}}{\text{Im} F^{(+)}} \quad (9b)$$

respectively.

In general, the contributions of I and $\tilde{I}$ are also negligible because they tend to a constant or even powers of $\ln v$ for $v \rightarrow \infty$, depending on the asymptotic behaviour of the total cross-section. Moreover, I is numerically small compared with the other terms in Eq. (8). Nevertheless, if $F^{(+)}$ is dominated by a fixed Pomeron pole (this is, for example, not yet excluded in πN scattering) I and $\tilde{I}$ cannot be neglected. In this case one has

$$\frac{1}{\text{Im} F^{(+)}} \frac{\partial}{\partial t} \text{Im} F^{(+)} = b^{(+)} = \text{const.}$$

and

$$\frac{\partial}{\partial t} \text{Re} F^{(+)} = \text{const.} + I$$

I behaves like $\ln v$ for $v \rightarrow \infty$. Therefore, $\partial / \partial t \operatorname{Re} F^{(+)}$ is dominated by it at large energies.

In pp scattering this case does not seem to occur and therefore, I and $\tilde{I}$ will be neglected in what follows, keeping in mind that this gives corrections of order $1/v$ (besides logarithmic factors) to the final results.

3. - PREPARATION OF THE CROSSING SYMMETRIC AMPLITUDE

The basic equation (8) applies only to the crossing symmetric part of the pp amplitude. How this can be extracted from the data will be now discussed.

i) - The forward amplitude

Up to 50 GeV there exist total cross-section data for pp as well as for $p\bar{p}$ scattering ¹²⁾. Consider the difference of both cross-sections. One finds that this is well fitted from 3 to 50 GeV/c by a power law, according to a ω exchange

$$\sigma^{(-)} = \frac{1}{2} (\sigma_{p\bar{p}} - \sigma_{pp}) = \beta(0) (p/p_1)^{\alpha-1} \quad (10)$$

with $\beta(0) = (29.6 \pm 0.5) \text{mb}$, $\alpha = 0.38 \pm 0.02$ and $p_1 = 1 \text{ GeV/c}$. Assuming the validity of this law up to infinity, the whole amplitude $F^{(-)}$ is known at $t = 0$:

$$F^{(-)} = \frac{p_1}{2m} \beta(0) (p/p_1)^{\alpha} (i + \tan \frac{\pi\alpha}{2}) \quad (11)$$

It is now clear how to calculate $F^{(+)}$ and so $\sigma^{(+)}$, $\sigma^{(-)}$ from the pp data alone.

ii) - The slope of $|F^{(+)}|$

To determine $b^{(+)}$ one needs information on $F^{(-)}$ not only at $t = 0$ but also at small $|t|$. So far as $p\bar{p}$ differential cross-section data are ¹³⁾ available, one can calculate the component of $F^{(-)}$ parallel with respect to $F^{(+)}$ from

$$\frac{\frac{d\sigma}{dt}(p\bar{p}) - \frac{d\sigma}{dt}(pp)}{\sqrt{8\left(\frac{d\sigma}{dt}(p\bar{p}) + \frac{d\sigma}{dt}(pp)\right)}} \quad (12)$$

This quantity was computed for several energies between 3 and 40 GeV/c and the logarithmic derivative $b^{(-)}$ of $F_{||}^{(-)}$ at $t = 0$ was calculated ^{*)}. No significant energy dependence was found for it. Therefore, it was averaged : $b^{(-)} = (7.5 \pm 1.0)(\text{GeV}/c)^{-2}$. Because $F^{(+)}$ is dominantly imaginary, $F_{||}^{(-)}$ and $b^{(-)}$ should be good approximations for $\text{Im } F^{(-)}$ and its slope. Furthermore, with respect to the rather large error of $b^{(-)}$, it will be assumed that the slopes of $|F^{(-)}|$ and $\text{Re } F^{(-)}$ are also equal to $b^{(-)}$. Now, using these results and assumptions, one gets from the derivative of $d\sigma/dt$ the following formula which allows one to calculate $b^{(+)}$ from b_{pp} :

$$b^{(+)} = \frac{1}{1-\gamma} \left(\frac{\sigma_{pp}}{\sigma^{(+)}} \right)^2 \frac{1+\gamma^2}{1+\gamma^{(+)2}} b_{pp} + \gamma \frac{1-\gamma_{K0}^2 \pi/4}{1-\gamma} \quad (13)$$

where

$$\gamma = \frac{\sigma^{(-)}}{\sigma^{(+)}} \frac{1}{\sqrt{1+\gamma^{(+)2}}}$$

σ_{pp} and b_{pp} defined in analogy to Eq. (9).

The results for $b^{(+)}$ are given in Fig. 1. For comparison the corresponding values of b_{pp} are also shown at lower energies. For $s > 90 \text{ GeV}^2$ b_{pp} and $b^{(+)}$ agrees within their errors. $b^{(+)}$ can be fitted with a linear ansatz in $\ln s$ for $s > 7 \text{ GeV}^2$

$$b^{(+)} = c_1 + c_2 \ln s \quad (14)$$

$$c_1 = (4.25 \pm 0.02) \text{ GeV}^{-2}; \quad c_2 = (0.27 \pm 0.01) \text{ GeV}^{-2}$$

It is a remarkable fact that $b^{(+)}$ does not show such a rapid change of behaviour below $s \simeq 30 \text{ GeV}^2$ as b_{pp} does.

^{*)} This was done by a linear fit for $|t| < 0.12 (\text{GeV}/c)^2$, the t interval in which most of the pp slopes ¹⁴⁾ were determined.

4. - RESULTS AND CONCLUSIONS

Using the fit (14) for $b^{(+)}$ from 3 GeV up to infinity, one receives immediately from Eq. (8) the wanted results $[(\bar{v}/v)^2 \ll 1]$

$$\begin{aligned} \frac{1}{\text{Im } F^{(+)}} \frac{\partial}{\partial t} \text{Im } F^{(+)} &= b^{(+)} - \frac{\pi}{2} S^{(+)} c_2 \\ \frac{1}{\text{Im } F^{(+)}} \frac{\partial}{\partial t} \text{Re } F^{(+)} &= S^{(+)} b^{(+)} + \frac{\pi}{2} c_2 \end{aligned} \quad (15)$$

One sees that practically the slope of the imaginary part is equal to $b^{(+)}$. The same happens for the pp case ^{*}) if one goes back, using the assumption on $F^{(-)}$. So, there is no improvement with regard to the saturation of the MacDowell-Martin ¹⁵⁾ bound : the slope of the imaginary part is of the order of 10% higher than the bound at ISR energies and even more below these.

The results for $(1/\text{Im } F^{(+)}) (\partial/\partial t) \text{Re } F^{(+)}$ are given in Fig. 2. A zero of this quantity is seen at about 150 GeV², a lower energy than the zero of the forward real part occurs. This indicates that the zero point of $\text{Re } F^{(+)}$ travels with t . The results for $(1/\text{Im } F_{pp}) (\partial/\partial t) \text{Re } F_{pp}$ are also shown in Fig. 2. The qualitative behaviour is the same as for the crossing symmetric case even if for the latter the energy dependence is not so pronounced.

To get an idea of the behaviour of the real part near the forward direction in Fig. 3, $S_{pp} + t(1/\text{Im } F_{pp}) (\partial/\partial t) \text{Re } F_{pp}$ is represented for several energies. $|\text{Re } F_{pp}|$ decreases if one goes from the forward direction to small negative values with the exception of the energies between the zeros of $(\partial/\partial t) \text{Re } F_{pp}$ and $\text{Re } F_{pp}$ where $|\text{Re } F_{pp}|$ increases (and must have a maximum nearby). This behaviour may be very important in Coulomb interference experiments where actually the derivative of the amplitude is needed ¹⁷⁾ but usually it is assumed that the derivatives of $\text{Re } F_{pp}$ and $\text{Im } F_{pp}$ are equal to $(\partial/\partial t)|F_{pp}|$. On the other hand, these measurements could in principle be used for the experimental determination of $(\partial/\partial t) \text{Re } F_{pp}$, taking for example $\text{Re } F_{pp}(t=0)$ from dispersion relation to improve the results.

^{*}) Obviously, the derivative of F_{pp}^- is also determined. Because of the scarcity of $p\bar{p}$ data we do not discuss this quantity.

The derivative of the real part is also very important for testing models. For example, the naïve geometrical Pomeron model of Kane ¹⁸⁾ predicts the real part. At high energies the qualitative behaviour of $(\partial/\partial t)\text{Re } F_{pp}$ agrees with the results presented here but quantitatively it is larger. Henyey et al. ⁵⁾ have used in their determination of $\text{Im } F_{pp}$ in the impact parameter representation the real part of Ref. 18). From the above it seems that this contribution is too low. It would be very interesting to see whether a large real part changes their results or not. For example, small effects like the energy dependence of the impact parameter amplitude ⁴⁾ may be sensitive to the contributions of the real part.

It is important to remark that the results for $(\partial/\partial t)F$ depend on the assumed behaviour of $F^{(-)}$ mainly at lower energies and more for pp than for the crossing symmetric part. However, at least, the qualitative behaviour of them should be correct. The results depend also on the assumption of the asymptotic behaviour ^{*}) of $b^{(+)}$ but do not depend on that of the forward amplitude. Therefore, no problems arise so far as one looks only at the measured region of σ_{tot} ($s < 3000 \text{ GeV}^2$). But if one studies the asymptotic behaviour of $(\partial/\partial t)F$ one has to be aware of the fact that if σ_{tot} stays to rise as in the ISR region like $\ln^2 s$ up to infinity the MacDowell-Martin bound demands at least a rise like $\ln^2 s$ for the slope. This super-shrinkage is not seen in the current data. Therefore, something must happen above the ISR region: either σ_{tot} is increasing flatter at most like $\ln s$ or the differential cross-section shrinks more strongly (or both σ_{tot} and the slope change their behaviour). Assuming $\sigma_{\text{tot}}, \sigma_{\text{el}} \sim (\ln s)$ ($0 < \beta \leq 2$) for $s \rightarrow \infty$ (it is well known ¹⁹⁾ that in this case $\rho = \pi / \ln s$), assuming furthermore that b_{pp} has the minimal energy dependence allowed by the MacDowell-Martin bound for $s \rightarrow \infty$, one derives from Eq. (8) the asymptotic behaviour:

^{*}) If one splits the integral in Eq. (8), say at $\tilde{v} = 1500 \text{ GeV}$ into two parts and expands the denominator of the high energy part for $v < \tilde{v}$:

$$\frac{2}{\pi} v \int_{\tilde{v}}^{\infty} \frac{dv'}{v'^2 - v^2} b^{(+)} = \frac{2}{\pi} v \int_{\tilde{v}}^{\tilde{v}} \frac{dv'}{v'^2 - v^2} b^{(+)} + \frac{2}{\pi} v \sum_{n=0}^{\infty} v^{2n} \int_{\tilde{v}}^{\infty} \frac{dv'}{v'^{2n+2}} b^{(+)}$$

one sees that for $(v/\tilde{v})^2 \ll 1$ and realistic behaviours of $b^{(+)}$ only the first momentum $M = \int_{\tilde{v}}^{\infty} (dv'/v') b^{(+)}$ has any importance. Therefore, if $b^{(+)}$ changes its behaviour above $\tilde{v}$ modifications of the derivatives arise only via M and are of the order $(v/\tilde{v})$ besides possible logarithmic factors.

$$\begin{aligned}\frac{1}{\text{Im } F^{(+)}} \frac{\partial}{\partial t} \text{Im } F^{(+)} &= c_2 (\ln s)^\beta \\ \frac{1}{\text{Im } F^{(+)}} \frac{\partial}{\partial t} \text{Re } F^{(+)} &= \pi c_2 (1+\beta/2) (\ln s)^{\beta-1}\end{aligned}\quad (16)$$

($b = c_2 (\ln s)^\beta$) which means that the derivative of the real part at $t = 0$ is positive at high energies as well as $\text{Re } F^{(+)}(t=0)$, a result already stated for a pure Pomeron contribution by Bronzan²⁰). For the case $\beta = 2$ the ratio of both derivatives is essentially the same as that predicted by Martin.

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FIGURE CAPTIONS

Figure 1 : The slope parameter $b^{(+)}$ as a function of s . Below 90 GeV^2 the corresponding values of $b_{pp}^{(+)}$ are also given. The data are taken from Ref. 13) and Ref. 14). The solid line represents the best fit with a linear dependence in $\ln s$ [compare with Eq. (14)]. There seems to be a discrepancy between the NAL and the Serpukhov data. Therefore, the fit does not take into account the Serpukhov data. However, both data sets are compatible if one considers also the systematic errors of $0.3 (\text{GeV})^{-2}$ for the Serpukhov data and of $0.2 (\text{GeV})^{-2}$ for the NAL data.

Figure 2 : The derivative of $\text{Re } F^{(+)}$ at $t = 0$ divided by $\text{Im } F^{(+)}(t=0)$. The $\mathcal{S}_{pp}$ data are taken from Ref. 16) and corrected for $\mathcal{S}^{(+)}$ as described in the text. The $\mathcal{G}_{pp}$ data¹²⁾ needed for the correction are interpolated in energy if necessary. Above 1000 GeV^2 $\mathcal{S}^{(+)}$ is taken from dispersion calculations²⁾. The corresponding values of $(1/\text{Im } F_{pp})(\partial/\partial t)\text{Re } F_{pp}$ are denoted by crosses.

Figure 3 The real part of pp amplitude at small t in a linear approximation for several values of s .

Fig. 1

Fig. 2

Fig. 3