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THE SCHWINGER MECHANISM IN NON-ABELIAN GAUGE THEORIES

S. Sarkar
CERN - Geneva

A B S T R A C T

An alternative to the Higgs-Kibble mechanism for the formulation of spontaneously broken gauge theories has been recently proposed. It is now shown how the new procedures can be generalized to the case when non-Abelian gauge groups are present.

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Introduction:

Until very recently gauge theories using spontaneous symmetry breaking have been described by Lagrangians which contain scalar fields [1]. The symmetry group of the lagrangian is spontaneously broken by giving some of these scalar fields a non-zero vacuum expectation value. The classic examples of such lagrangians have been discussed by Higgs and Kibble. The scalar fields in the formal theory are very attractive, but we are forced however to consider other schemes for two reasons. Firstly, in the construction of specific models [2] it is necessary, often, to have many of these scalar fields which do not correspond to observed particles. Secondly, when scalar fields are present it is difficult to arrange to have asymptotic freedom [3] and, in particular, so far no theories which are both asymptotically free and which contain no massless vector mesons in perturbation theory have been found. It is then useful to look for schemes of spontaneous symmetry breaking which do not involve these scalar fields. Such a scheme was invented some time ago by Jona-Lasinio and Nambu [4]. Recently Jackiw and Johnson [5], and independently Cornwall and Norton [6], have extended these old ideas to give vector mesons mass without requiring the existence of canonical scalar fields in the lagrangian.

It is the purpose of this paper to show that the procedures followed by [5,6] hold when non-abelian gauge groups are involved. Apart from the possible applications to weak and strong interactions, the non-abelian case is of interest in its own right. As in the abelian case we make crucial use of the simple observation first made by Schwinger [7] that an apparently massless vector meson can acquire a mass if its vacuum polarisation tensor

2.

$$\Pi_{\mu\nu}(q) = -i (g_{\mu\nu} q^2 - q_\mu q_\nu) \Pi(q^2)$$

" is such that $\Pi(q^2)$ behaves like m^2/q^2 as q^2 tends to zero for some positive m .

2. The Non-Abelian Model :

The model we shall consider is the most simple generalisation of the abelian model discussed by Jackiw and Johnson.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \bar{\psi} \left[i \gamma^\mu \partial_\mu - g \gamma^\mu A_\mu \frac{(1 - i \gamma_5)}{2} \right] \psi \quad (1)$$

$$\begin{aligned} \text{where } F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c, \\ A_\mu &= \frac{1}{2} \lambda_a A_\mu^a \\ \text{and } [\frac{1}{2} \lambda^a, \frac{1}{2} \lambda^b] &= i f^{abc} \frac{1}{2} \lambda^c \end{aligned}$$

The vector meson and fermion fields are denoted by A_μ^a and ψ_i respectively where a and i are group indices. It may be necessary to add a further vector interaction to the Lagrangian. A somewhat crude calculation,^[5] to be discussed later, seems to indicate the necessity of such an interaction. It is probable that the second vector meson thus introduced plays a role similar to that of the phonon in the theory of superconductivity [6].

$\mathcal{L}$ is invariant under the following infinitesimal transformation:

$$\begin{aligned} \delta A_\mu^a(x) &= \partial_\mu \theta^a(x) + g f_{abc} \theta^b(x) A_\mu^c(x) \\ \delta \psi(x) &= -ig \theta^a(x) \frac{1}{2} \lambda^a \frac{1}{2} (1 - i \gamma_5) \psi(x) \end{aligned} \quad (2)$$

where $\{\theta^a(x)\}$ is the set of infinitesimal group parameters. Naturally we will deal with compact groups. The equations of motion give us

$$\begin{aligned} \partial^\nu J_\nu^a &= 0 \\ \text{where } J_\nu^a &= \bar{\psi} \gamma_\nu \frac{1}{2} (1 - i \gamma_5) \frac{1}{2} \lambda^a \psi \end{aligned} \quad (3)$$

Hence the current associated with (2) is conserved; unfortunately

4.

there are anomalies and so the currents are not conserved. The Lagrangian will have to be modified in order to remove the anomalies. This can be done by doubling the number of fermion fields appropriately. We shall, however, ignore this aspect of (1) from now on.

Our gauge will be given by

$$\partial^\mu A^\alpha_\mu(x) = b^\alpha, \quad \alpha = 1, \dots, N \quad (4)$$

where N is the order of the gauge group and b^α are x -independent. Via, by now, the standard techniques of Faddeev-Popov (as interpreted by 't Hooft) we have a theory with vertices

$$- \frac{1}{4} i g \gamma^\mu (1 - i \gamma_5) \lambda^a_{ji}$$

$$- i g^2 \left(f_{gab} f_{gcd} (\delta_{\alpha\gamma} \delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma}) + f_{gac} f_{gbd} (\delta_{\alpha\beta} \delta_{\gamma\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma}) + f_{gad} f_{gbc} (\delta_{\alpha\beta} \delta_{\gamma\delta} - \delta_{\alpha\gamma} \delta_{\beta\delta}) \right)$$

$$- i g f_{abc} (\delta_{\beta\gamma} (q-p)_\alpha + \delta_{\delta\alpha} (k-q)_\beta + \delta_{\alpha\beta} (p-k)_\gamma)$$

Finally we have the ghost vertex

$$-ig f^{abc} k_\mu \quad (8)$$

where $\text{-----}\rightarrow\text{-----}$ denotes the ghost particle. The renormalised theory is given essentially via the substitutions

$$\begin{aligned} A_\mu^a &= Z_3^{\frac{1}{2}} (A_\mu^a)_r \\ g &= g_r (Z_1/Z_3^{\frac{1}{2}}) \\ c &= Z_3^{\frac{1}{2}} c_r \\ \psi_a &= Z_2^{\frac{1}{2}} (\psi_a)_r \end{aligned} \quad (9)$$

where 'r' denotes renormalised. As in the abelian theory an important role is played by the Ward- Taylor -Slavnov identities [9]. We shall give the steps in the derivation of the following Ward type identity in an appendix :

$$-ig f_{b\beta c} \quad -ig f_{b\beta c} \quad -ig f_{b\beta c} \quad -ig f_{b\beta c}$$

$$-ig \quad -ig \quad -ig \quad -ig$$

$$\frac{(1+i\gamma_5)}{2} \frac{\lambda_{ei}^\beta}{2} \quad -ig \frac{(1-i\gamma_5)}{2} \frac{\lambda_{if}^\beta}{2}$$

6.

+ = 0

'.....' denotes diagrams which contain vector meson tadpoles as factors
e.g.

We have used the convention that represents a one-particle irreducible Green's function while will represent a connected Green's function. One particle irreducible Green's functions have amputated legs. If we have an amputated leg coming out of a vertex we will denote the amputated line by . The vertices occurring here, but not in the lagrangian are straightforward diagrammatic representations of some terms in the mathematical expression for the Ward identity given in the appendix. There are of course other identities between diagrams, but from the point of view of the Schwinger mechanism (10) is the most useful. It is convenient to multiply (10) by the inverse ghost propagator. Our spontaneous symmetry-breaking condition is

$$\begin{aligned}
 & -\frac{1}{4} g \quad \begin{array}{c} i \\ \text{---} \text{---} \text{---} \end{array} \text{---} \text{---} \text{---} \begin{array}{c} f \\ \text{---} \text{---} \text{---} \end{array} \quad \lambda_{ei}^{\delta} (1 + i \gamma_5) \\
 & + \frac{1}{4} g (1 - i \gamma_5) \quad \begin{array}{c} e \\ \text{---} \text{---} \text{---} \end{array} \text{---} \text{---} \text{---} \begin{array}{c} i \\ \text{---} \text{---} \text{---} \end{array} \quad \lambda_{if}^{\delta} \\
 & \neq 0
 \end{aligned}$$

It is easy to check that a symmetry conserving solution indeed demands that the quantity in (11) vanishes. We choose a renormalisation scheme in which tadpole terms vanish. Moreover, all diagrams in which there are dressed three or four point vertices involving ghost fields vanish in the limit q goes to zero owing to (8).

Condition (11) then demands that

has a pole as q vanishes. Since we have Goldstone excitations, we realise them as bound states of the fermion and anti-fermion channel. Explicitly we assume that

(13)

where P_α term denotes the contribution due to the α th bound state and the R term is the regular part of the amplitude. Hence

(14)

The right hand side of (14) is of the form

$$q^\mu I_{\alpha}^{\delta} (q^2) P_{\alpha}^{e e'} (p+q, p)$$

where we are using the summation convention. The pole contribution

to the above vertex is thus proportional to q^μ / q^2 , just as in the abelian case. We will now look at the vacuum polarisation tensor to see how the Schwinger mechanism works. The pole of the vacuum polarisation tensor is given by

$$\left(\begin{array}{c} \text{Diagram 1} + \text{Diagram 2} \\ + \text{Diagram 3} + \text{Diagram 4} \end{array} \right)^2 \quad (16)$$

where $(\quad)^2$ indicates that we have terms such as

and etc.

This gives

$$\Pi_{\alpha\beta}^{\mu\nu}(q^2) \sim -i q^\mu q^\nu / q^2 \eta_{\alpha'}^\alpha \eta_{\alpha'}^{\beta} \quad (17)$$

From (14) we see that η is non-zero if P is; so from the Schwinger mechanism we obtain a non-trivial mass matrix for the vector mesons.

Although we have introduced bound state poles we do not want the S-matrix elements to have these poles. We shall now demonstrate how these poles cancel with vector meson poles for on-shell amplitudes. With obvious notation we have

10.

$$(18)$$

The external fermions are of course on mass shell; so the $q^\mu q^\nu$ term of the vector meson propagator in (18) gives zero. This follows principally from the Ward identity (11). This is taken between on-shell spinors for the fermion leg. Use of the Dirac equation gives that the vertex is orthogonal to the vector meson momentum. Now

$$(19)$$

since the pole part of the vertex is proportional to q and so annihilates the vertex 'opposite' it. Similarly we have

$$(20)$$

Hence we can deduce that

$$(21)$$

So the pole part of the S matrix element is

$$\begin{aligned}
 & - \sum_{\alpha} \text{diagram 1} - \text{diagram 2} \\
 & + \text{diagram 3}
 \end{aligned} \tag{22}$$

Diagram 1: Two shaded circles labeled P_{α} connected by a thick horizontal line. Each circle has two external lines with arrows pointing away from it.

Diagram 2: Two shaded circles labeled P connected by a wavy horizontal line. Each circle has two external lines with arrows pointing away from it.

Diagram 3: Two shaded circles labeled "pole" connected by a wavy horizontal line. Each circle has two external lines with arrows pointing away from it.

Now

$$\begin{aligned}
 & \text{diagram 3} \\
 = & \text{diagram 4} \\
 + & \text{diagram 5} \\
 + & \dots
 \end{aligned}$$

Diagram 4: Diagram 3 with a loop of two arrows on the thick line between the P_{α} circles.

Diagram 5: Diagram 3 with a loop of two arrows on the wavy line between the "pole" circles.

$$= P_{\alpha} \frac{(-iq^{\mu})}{q^2} \eta_{\mu}^{\delta} \left(\frac{g_{\mu\nu}}{(q^2 - q^2 \Pi(q^2))_{\delta\kappa}} \right) - iq^{\nu} \eta_{\beta}^{\kappa} P_{\beta} \tag{23}$$

$$\text{But } (q^2 \Pi(q^2))_{\delta\kappa} \sim -i \eta_{\delta\alpha'} \eta_{\alpha'\kappa} \quad q^2 \rightarrow 0$$

and so we have

$$- \sum_{\alpha} \text{diagram 1} + \text{diagram 3} = 0 \tag{24}$$

This shows that the four point scattering amplitude is free from bound state poles; however it turns out that the arguments presented for this special case are powerful enough to demonstrate that the general scattering amplitude also is free from bound state poles. If the above argument is examined closely, it will emerge that provided

n particle state with all
particles on mass shell

is zero for all n then everything goes through exactly as before.

However this is just a statement of the gauge invariance of the theory.

Throughout we have been assuming the existence of bound state poles. It is possible, as in the abelian case, to consider Bethe-Salpeter equations for the three point function and deduce from them homogeneous integral equations for the P vertices. However, beyond this point the analysis is difficult to carry out in a gauge-invariant way; in fact if we take for the kernel of the integral equation the second order four fermion function with a single vector meson exchange we find a non-trivial solution for $P_\alpha(p,p)$ provided we introduce an extra vector coupling. In order not to spoil gauge invariance the additional vector is taken to be scalar under (2). Fortunately this does not introduce any extra anomalies. In this solution $P_\alpha(p,p)$ has a behavior $(p^2/m^2)^{-\epsilon}$ for large p^2 where m and ϵ are positive as in [5,6]. Now that the mechanism suggested in [5,6] has been generalised to non-abelian gauge theories the next important step is to develop non-perturbative calculational schemes. This is particularly important since field theories with scaling force us to have non-abelian gauge theories and very probably dynamical methods of spontaneous symmetry breaking too [3].

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Appendix:

We shall use the formalism given in [10]. In particular Γ will represent the generating functional for one-particle irreducible vertices and Q^α_μ , Ψ^i and $\bar{\Psi}^j$ will denote the classical fields corresponding to the vector bosons and fermions.

The generalised Ward identities in the theory can all be obtained by functionally differentiating the following equation [10]:

$$\begin{aligned}
 & - \partial^\mu Q^\alpha_\mu(x^a) - \int dx^\beta \int dx^b \frac{\delta \Gamma}{\delta Q^\nu_b(x^b)} \left[\delta_{\beta b} \frac{\partial}{\partial (x^b)^\nu} \delta(x^b - x^\beta) + g f_{b\beta c} \delta(x^b - x^\beta) \right] \{ Q^\nu_c(x^b) \\
 & + i \int dy \hat{G}_{c,\nu,x^b;d,\mu,y} \frac{\delta}{\delta Q^\mu_d(y)} + i \int dy \hat{G}_{c,\nu,x^b;\bar{d},y} \frac{\delta}{\delta \bar{\Psi}_d(y)} \\
 & + i \int dy \hat{G}_{c,\nu,x^b;d,y} \frac{\delta}{\delta \bar{\Psi}_d(y)} \}] G_{\beta,a}(x^\beta, x^a) \\
 & - \int dx^\beta \int dx^i \frac{\delta \Gamma}{\delta \Psi_i(x^i)} (ig) \frac{(1+i\gamma_5)}{2} \frac{\lambda^\beta_{ij}}{2} \delta(x^i - x^\beta) (\Psi_j(x^i) \\
 & + i \int dy \hat{G}_{x^i,j;\bar{y},d} \frac{\delta}{\delta \bar{\Psi}_d(y)} + i \int dy \hat{G}_{x^i,j;y,\bar{d}} \frac{\delta}{\delta \Psi_d(y)} \\
 & + i \int dy \hat{G}_{x^i,j;y,d,\mu} \frac{\delta}{\delta Q^\mu_d(y)}) G_{\beta,\alpha}(x^\beta, x^a) \\
 & - \int dx^\beta \int dx^i \delta(x^i - x^\beta) (\bar{\Psi}_j(x^i) + i \int dy \hat{G}_{x^i,j;y,d} \frac{\delta}{\delta \bar{\Psi}_d(y)} \\
 & + i \int dy \hat{G}_{x^i,j;y,\bar{d}} \frac{\delta}{\delta \Psi_d(y)} + i \int dy \hat{G}_{x^i,j;y,d,\mu} \frac{\delta}{\delta Q^\mu_d(y)}) \\
 & \times (-ig) \frac{1}{2} (1-i\gamma_5) \frac{1}{2} (\lambda^\beta)_{ij} \frac{\delta \Gamma}{\delta \bar{\Psi}_i(x^i)} G_{\beta,a}(x^\beta, x^a) \\
 & = 0
 \end{aligned}$$

In the above, $\hat{G}_{i,j}$ denotes the propagator when the relevant fields are constrained to the vacuum expectation values A_μ^a , Ψ^i and $\bar{\Psi}^j$. Also $G_{\beta,\alpha}$ denotes the Faddeev-Popov ghost propagator. Now, in order to obtain the Ward identity given in (10) we just consider

$$\left. \frac{\delta}{\delta \Psi^e(x^e)} \frac{\delta}{\delta \bar{\Psi}^f(x^f)} \left(\text{L.H.S. of the above equation} \right) \right|_{A=\Psi=\bar{\Psi}=0} = 0$$

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