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A CONTRIBUTION TO SIXTH-ORDER ELECTRON
AND MUON ANOMALIES - III

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ABSTRACT

The sixth-order contribution to electron and muon anomalous magnetic moments, coming from second-order vacuum polarization insertion into the fourth-order ladder diagram, is analytically evaluated.

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Continuing previous work ^{1),2)}, we have analytically evaluated the contribution to the electron and muon magnetic moment anomalies a_e, a_μ coming from the graphs of Figs. 1 and 2. [The graphs are labelled according to the numeration of Ref. 3).]

The results are

$$a_e^{(9)} = \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{133}{216} - \frac{353}{54} \zeta(2) + 6 \zeta(2) \log 2 + \frac{403}{144} \zeta(3) + \left(\frac{119}{36} - 2 \zeta(2)\right) \log \frac{1}{m_e} \right] \quad (1)$$

$$a_e^{(10)} = \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{73}{216} - \frac{25}{6} \zeta(2) + \frac{28}{3} \zeta(2) \log 2 - \frac{31}{9} \zeta(3) - \left(\frac{119}{36} - 2 \zeta(2)\right) \log \frac{1}{m_e} \right] \quad (2)$$

$$a_{\mu}^{(9)} = \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{1}{6} \log^2 \frac{m_{\mu}}{m_e} + \left(-\frac{1}{8} + \frac{2}{9} \zeta(2)\right) \log \frac{m_{\mu}}{m_e} - \frac{1}{36} - \frac{17}{27} \zeta(2) - \left(\frac{25}{36} - \frac{1}{3} \log \frac{m_{\mu}}{m_e}\right) \log \frac{\lambda}{m_{\mu}} \right] \quad (3)$$

$$a_{\mu}^{(10)} = \left(\frac{\alpha}{\pi}\right)^3 \left[-\frac{1}{6} \log^2 \frac{m_{\mu}}{m_e} + \left(\frac{31}{72} + \frac{2}{9} \zeta(2)\right) \log \frac{m_{\mu}}{m_e} - \frac{95}{216} - \frac{2}{3} \zeta(2) - \frac{1}{9} \zeta(3) + \left(\frac{25}{36} - \frac{1}{3} \log \frac{m_{\mu}}{m_e}\right) \log \frac{\lambda}{m_{\mu}} \right] \quad (4)$$

so that

$$a_e^{(9)} + a_e^{(10)} = \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{103}{108} - \frac{289}{27} \zeta(2) + \frac{46}{3} \zeta(2) \log 2 - \frac{31}{48} \zeta(3) \right] = \left(\frac{\alpha}{\pi}\right)^3 [0.05327] \quad (5)$$

$$a_{\mu}^{(9)} + a_{\mu}^{(10)} = \left(\frac{\alpha}{\pi}\right)^3 \left[\left(\frac{11}{36} + \frac{4}{9} \zeta(2)\right) \log \frac{m_{\mu}}{m_e} - \frac{101}{216} - \frac{35}{27} \zeta(2) - \frac{1}{9} \zeta(3) \right] = \left(\frac{\alpha}{\pi}\right)^3 \left[\left(\frac{11}{36} + \frac{4}{9} \zeta(2)\right) \log \frac{m_{\mu}}{m_e} - 2.73348 \right] = \left(\frac{\alpha}{\pi}\right)^3 [2.79346] \quad (6)$$

(In a_{μ} terms vanishing with m_e/m_{μ} are always neglected.) Previous numerical results for the constants in square brackets are, for Eq. (5) :

$$0.0532 \quad (4) \quad \text{from Ref. 4)}$$

$$0.0522(10) \quad \text{from Ref. 5)}$$

and, for Eq. (6)

$$2.800 \quad (5) \quad \text{from Ref. 4)}$$

$$2.88 \quad (6) \quad \text{from Ref. 6)}$$

The agreement is good.

The method followed in deriving Eqs. (1)-(4), although still dispersive, is different from that used in Refs. 1) and 2). As in Ref. 1), given the Feynman amplitude of the vertex graphs in momentum representation for on-mass-shell electrons with momenta $(p+\Delta/2)$, $(p-\Delta/2)$, and an off-mass-shell photon

of momentum Δ , we project out the Pauli magnetic form factor $F_2(-\Delta^2)$ by means of standard algebra. The expression so obtained is an integral over the loop momenta q_i , whose integrand depends, besides the q_i 's, on p and Δ :

$$F_2(-\Delta^2) = \int \prod_{i=1}^n \left\{ \frac{i d^4 q_i}{(2\pi)^4} \right\} f_2(q_i; p, \Delta) \Big|_{(p+\frac{1}{2}\Delta)^2 = (p-\frac{1}{2}\Delta)^2 = -m^2}$$

where m is the external lepton mass.

The method we follow now to evaluate the anomaly $a \equiv F_2(0)$, consists in taking the static limit $\Delta \rightarrow 0$ at this level, before attempting any loop integration :

$$a = F_2(0) = \int \prod_{i=1}^n \left\{ \frac{i d^4 q_i}{(2\pi)^4} \right\} f_2(q_i; p, 0) \Big|_{p^2 = -m^2}.$$

The anomaly can then be considered as the value at $-p^2 = m^2$ of the function $G(-p^2)$

$$G(-p^2) \equiv \int \prod_{i=1}^n \left\{ \frac{i d^4 q_i}{(2\pi)^4} \right\} f_2(q_i; p, 0),$$

and it is evaluated by means of a dispersion relation in the variable p^2 .

The treatment of the vacuum polarization insertion, the analytic integrations and the working out of the algebra by means of the Schoonschip program are the same as in Refs. 1) and 2).

The method outlined above seems rather powerful, and its application to the evaluation of the remaining sixth order graph contributions coming from second order vacuum polarization insertions into fourth order graphs is in progress. It is likely to work for other classes of graphs too. Further details of it, as well as its application to an elementary evaluation of the lowest order electron anomaly, the Schwinger term $\alpha/2\pi$, can be found in Ref. 7).

For completeness, we give here the sum of the above results with those of Refs. 1) and 2), [again the various contributions are numbered as in Ref. 3)]:

$$\begin{aligned}
 & a_e^{(9)} + a_e^{(10)} + a_e^{(15)} + a_e^{(16)} + a_e^{(17)} + a_e^{(18)} = \\
 & = \left(\frac{\alpha}{\pi}\right)^3 \left[-\frac{287}{108} - \frac{25}{3} \zeta(2) + 12 \zeta(2) \log 2 + \frac{17}{8} \zeta(3) + \left(\frac{119}{18} - 4 \zeta(2)\right) \log \frac{\lambda}{m_e} \right] = \\
 & = \left(\frac{\alpha}{\pi}\right)^3 \left[-0.12864 + \left(\frac{119}{18} - 4 \zeta(2)\right) \log \frac{\lambda}{m_e} \right] \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 & a_\mu^{(9)} + a_\mu^{(10)} + a_\mu^{(15)} + a_\mu^{(16)} + a_\mu^{(17)} + a_\mu^{(18)} = \\
 & = \left(\frac{\alpha}{\pi}\right)^3 \left[-\frac{1}{3} \log^2 \frac{m_\mu}{m_e} + \frac{53}{36} \log \frac{m_\mu}{m_e} - \frac{56}{27} - \frac{7}{6} \zeta(2) + \left(-\frac{25}{18} + \frac{2}{3} \log \frac{m_\mu}{m_e}\right) \log \frac{\lambda}{m_\mu} \right] = \\
 & = \left(\frac{\alpha}{\pi}\right)^3 \left[-5.61948 + \left(-\frac{25}{18} + \frac{2}{3} \log \frac{m_\mu}{m_e}\right) \log \frac{\lambda}{m_\mu} \right] \quad (8)
 \end{aligned}$$

After the completion of this work, the authors became aware of a paper by K.A. Milton, W. Tsai and L.L. De Raad ⁸⁾, who use an improved Schwinger mass operator approach to evaluate the sum of the contributions to the electron anomaly coming from the graphs 9, 10, 11, 12, 15 and 16. Their results coincide with Eq. (7) of the present paper.

R E F E R E N C E S

- 1) D. Billi, M. Caffo and E. Remiddi, Lettere al Nuovo Cimento 4, 657 (1972).
- 2) R. Barbieri, M. Caffo and E. Remiddi, Lettere al Nuovo Cimento 5, 769 (1972).
- 3) B.E. Lautrup, A. Peterman and E. de Rafael, Phys.Reports 3, 4 (1972).
- 4) S.J. Brodsky and T. Kinoshita, Phys.Rev. D3, 356 (1971).
- 5) J. Calmet and M. Perrottet, Phys.Rev. D3, 3101 (1971).
- 6) B.E. Lautrup, Phys.Letters 32B, 627 (1970).
- 7) E. Remiddi, "A new dispersive technique for the $(g-2)$ ", presented at the third Colloquium on Advanced Computing Methods in Theoretical Physics, Marseille, 25-29 June, 1973.
- 8) K.A. Milton, W. Tsai and L.L. De Raad, preprint UCLA 73/TEP/98, November, 1973.