THE NUCLEON ELECTROMAGNETIC POLARIZABILITIES

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A B S T R A C T

Theoretical values for the electric and magnetic polarizabilities of the nucleon are derived and give $\alpha_p = 4 \times 10^{-4} \text{ fm}^3$ and $\beta_p = 10 \times 10^{-4} \text{ fm}^3$. We show that the intrinsic polarizabilities give major contributions to the susceptibility of nuclei and neutron stars. Better experimental determination of α and β is consequently badly needed.

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It is well known that the Compton scattering amplitude, up to second order in the energy of the photon, is determined by two structure constants, apart from mass, charge and magnetic moment. These constants are usually called the electric and magnetic polarizabilities and they can be considered as representing the scattering produced by time-varying induced dipoles driven by the electric and magnetic fields of the photon. The magnitude typical for α and β , electric and magnetic polarizabilities, respectively, is given by the exact forward dispersion relation ¹⁾

$$\alpha + \beta = \frac{1}{2\pi^2} \int_{\omega_0}^{\infty} \frac{d\omega}{\omega^2} \sigma_T(\omega) = (14.2 \pm .3) \times 10^{-4} \text{ fm}^3 \quad (1)$$

where $\sigma_T(\omega)$ is the total photo-absorption cross-section on protons, known up to sufficiently large energies.

The only experimental measurement of such fundamental parameters known to us was made a decade ago by Goldanskii et al. ²⁾, who investigated Compton scattering on protons in the backward hemisphere, at an average energy of 56 MeV. The cross-section, in the laboratory frame, is given by

$$\begin{aligned} \frac{d\sigma}{d\Omega}(\theta) = & \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{m^2} \left(\frac{\omega'}{\omega}\right)^2 \left\{ \frac{1 + \cos^2\theta}{2} \right. \\ & \left. + \left(\frac{\omega}{m}\right)^2 \left[\frac{(1 - \cos\theta)^2}{2} + 21.4 - 17.3 \cos\theta - 1.6 \cos^2\theta \right] \right. \\ & \left. - \left(\frac{4\pi}{e^2}\right) m \omega^2 \left[\frac{\alpha + \beta}{2} (1 + \cos\theta)^2 + \frac{\alpha - \beta}{2} (1 - \cos\theta)^2 \right] \right\}, \end{aligned} \quad (2)$$

where the numerical constants come from the anomalous magnetic moment and ω' is the final photon energy in the laboratory frame. We note that, in the energy range covered by the experiment of Ref. 2), the polarizability contribution (Rayleigh scattering) to the differential cross-section is expected to be, from Eqs. (1) and (2), of the order of 10% of the energy independent term (Thomson scattering). This figure is at the limit of the systematic uncertainties ($\sim 8\%$) quoted in Ref. 2). Their results suggest that the magnetic polarizability β of the proton is very small as compared with the electric one α ; they give

$$\alpha = (9 \pm 2) \times 10^{-4} \text{ fm}^3 ; \quad \beta = (2 \pm 2) \times 10^{-4} \text{ fm}^3. \quad (3)$$

These findings are extremely surprising. One knows from experiment that the $\Delta(1236)$ excitation of the nucleon is the dominant contribution ($\sigma \sim 500 \mu\text{b}$) to the sum rule (1) and this excitation is of magnetic nature. So one expects that the nucleon should be essentially a good paramagnetic object and then $\beta > \alpha$. Furthermore, in a naïve quark model the resonance corresponds to a spin reorientation of the quarks, i.e., to the effect an external magnetic field would have on the nucleon. However, no corresponding simple electric excitation exists, which again argues for an important value of β as compared to α .

In order to clarify to what extent the experimental conclusion (3) that $\alpha > \beta$ is compelling, we have reanalyzed the data of Ref. 2) on cross-sections at the angles 75° , 90° , 120° , 135° and 150° , including all sources of uncertainties. The values of α and β have been constrained to satisfy Eq. (1). The corresponding fit gives

$$\begin{aligned}\alpha &= (10 \pm 2) \pm 5 \times 10^{-4} \text{ fm}^3 \\ \beta &= (4 \mp 2) \mp 5 \times 10^{-4} \text{ fm}^3,\end{aligned}\tag{4}$$

where the first error (± 2) is statistical and the second one (± 5) is systematic, coming from the 8% uncertainty in the normalization of the cross-section. We observe that from Eq. (4) a value of β larger than α cannot be rejected, at the present level of precision. The question of $\alpha > \beta$ or $\alpha < \beta$ is still an open one.

We will now give theoretical predictions for the structure constants α and β , with the hope that present-day experimental techniques will allow a much more precise clarification of this fundamental question. It is known that in Compton scattering, continuum contributions are of second order in the frequency for the spin averaged amplitudes; they are of third order (or higher) in the energy for the remaining amplitudes. Therefore, up to second order, one only needs to discuss the spin averaged part of the scattering matrix. If one disregards the part of the tensor which does not contribute when transverse photons are contracted, we can expand the amplitude in the gauge invariant way suggested in Ref. 3). Then the tensor reads

$$\begin{aligned}T^{\mu\nu} &= [2K^2 g^{\mu\nu} - k^\mu k'^\nu] A_1(s, u) \\ &+ [2K^2 P^\mu P^\nu - (P \cdot K)(P^\mu k'^\nu + k^\mu P^\nu) + (P \cdot K)^2 g^{\mu\nu}] A_2(s, u)\end{aligned}\tag{5}$$

where the index μ (ν) is contracted to the final (initial) photon of momentum k' (k). The four-momentum $P = \frac{1}{2}(p+p')$ is the average of the initial and final nucleon momenta and the same holds for the averaged photon momentum $K = \frac{1}{2}(k+k')$. The amplitudes A_1 and A_2 are symmetric under crossing, $A_1(s,u) = A_1(u,s)$. In a dispersive approach, the continuum contributions to these amplitudes are related to the structure constants appearing in Eq. (2) through

$$\frac{1}{2m} [P^2 A_2]^C = \alpha + \beta \quad (6a)$$

$$\frac{1}{2m} [2A_1 + P^2 A_2]^C = \alpha - \beta + \frac{1}{2m^3} (2Z\kappa + \kappa^2) \left(\frac{e^2}{4\pi} \right), \quad (6b)$$

where the superscript C means the limit $s \rightarrow m^2$ of the dispersive integral, at fixed scattering angle, and Z and κ are the charge and anomalous magnetic moment in natural units. If the left-hand side of Eq. (6a) is calculated in the forward direction, the absorptive part of $P^2 A_2$ is just proportional to the total cross-section, since it is the physical amplitude in that case and one can use the optical theorem. Thus, one gets Eq. (1). A similar sum rule for $\alpha - \beta$ could be, in principle, written down starting from Eq. (6b), but in the forward case A_1 reggeized like s^j , with $j \leq 1$, and in general a subtraction is required. This would introduce a new unknown constant. We therefore investigate the left-hand side of Eq. (6b) at backward angles, where the amplitudes reggeize in a more convenient fashion: A_1 and $P^2 A_2 \sim s^{j-\frac{1}{2}}$, $j < \frac{1}{2}$. The combination $2A_1 + P^2 A_2$ is the physical spin-averaged amplitude at 180° . Its absorptive part is obtained from that of the forward physical amplitude by changing the sign of the non-parity flip multipoles (induced by $M1, E2, \dots$ radiation). In that way we arrive to the following sum rule

$$\begin{aligned} \alpha - \beta = & - \left(\frac{e^2}{4\pi} \right) \frac{1}{2m^3} \kappa (2Z + \kappa) \\ & + \frac{1}{2\pi^2} \int_{\omega_0}^{\infty} \frac{d\omega}{\omega^2} \left(1 + 2 \frac{\omega}{m} \right)^{\frac{1}{2}} \left[\sigma(\Delta P = \gamma ES) - \sigma(\Delta P = N_0) \right] \\ & + (\text{annihilation channel } \gamma\gamma \rightarrow N\bar{N} \text{ contribution}) \end{aligned} \quad (7)$$

The last term comes from physical states in $\gamma\gamma \rightarrow N\bar{N}$ and is due to the backward character of the dispersion relation. This is the price one has to pay in order to obtain good superconvergence properties. However, in many cases one can guess the order of magnitude of the annihilation channel, for example from the knowledge of radiative decays of the meson coupled to

the $N\bar{N}$ pair. For nucleon-antinucleon annihilation the $f_0(1260)$ is presumably the dominant contribution. However, in our sum rule, it is coupled to helicity flip vertices. There exists some phenomenological evidence that such couplings are small for the f_0 meson⁴⁾. Consequently, we shall neglect the annihilation channel in the following considerations. Equations (1) and (7) can be combined to give separate sum rules for α and β . It is clear that in the static approximation ($m \rightarrow \infty$) the integrand of depends exclusively on $\sigma(\Delta P = \text{NO})$, i.e., predominantly M1 excitations and, to a smaller extent, E2, M3, ... multipoles. This confirms the strong dominance of β by $\Delta(1236)$ excitation. In the same approximation α depends exclusively on $\sigma(\Delta P = \text{YES})$, i.e., predominantly E1, but also on M2, E3, ... multipoles. There is no fundamental problem in maintaining the complete expression without any approximations.

We have evaluated the integrands appearing in the corresponding sum rules using the most recent π photoproduction multipolar analysis of Moorhouse et al.⁵⁾, the $\pi\Delta$ photoproduction fit of Lücke⁶⁾ and an estimate of other photoproduction channels through inelastic resonance decay. These functions are plotted in Fig. 1 versus the laboratory energy, in such a way that their integrals I_α , I_β give directly the values of α and β through

$$\alpha = I_\alpha - \left(\frac{e^2}{4\pi}\right) \kappa (2Z + \kappa) / (4m^3)$$

$$\beta = I_\beta + \left(\frac{e^2}{4\pi}\right) \kappa (2Z + \kappa) / (4m^3) . \quad (8)$$

We obtain for the proton

$$\alpha_p = 4 \times 10^{-4} \text{ fm}^3 \quad ; \quad \beta_p = 10 \times 10^{-4} \text{ fm}^3 . \quad (9)$$

The neutron polarizabilities, although less well predicted, are expected to be rather similar to the proton ones. Within the accuracy of the numerical input, the electric polarizability for neutrons is 20% higher than α_p , while $\alpha_n + \beta_n \approx \alpha_p + \beta_p$. The convergence is much better for the magnetic sum rule. As we can see from Fig. 1, the integral I_β is completely dominated by the $\Delta(1236) \frac{3}{2}^+$ resonance. In fact, if we estimate that integral in the narrow width approximation, the result is

$$I_\beta \approx 2 \left(\frac{m}{m_\Delta}\right) \frac{\mu^{*2}}{m_\Delta - m} , \quad (10)$$

where μ^* is the magnetic dipole transition moment, equal to $1.20\mu_{\text{proton}}$, and one reproduces the value quoted above. The leading contribution to I_α is, on the contrary, due to non-resonant excitation. As the energies near threshold are enhanced in the sum rules, about 50% of the contribution to I_α comes from the photodissociation of the nucleon into a pion-nucleon pair in S wave orbital motion.

It is amusing to note that the large value of the magnetic polarizability β due to the importance of $\Delta(1236)$ excitation is quite the contrary to the situation in nuclear physics. In nuclei, polarizabilities are dominated by excitation of the E1 giant dipole resonance which nearly saturates the sum rules and leads to a large electric polarizability. Similarly, the electric polarizabilities of atoms are very large. The nucleon provides an exceptional case of large magnetic polarizability.

The above predictions for the nucleon polarizabilities have not been experimentally confirmed and are even weakly at variance with present experiments. What would be the theoretical consequences if a new generation of experiments confirm that $\alpha > \beta$? The most conservative attitude would be to question the validity of the backward sum rule (7), which possibly could have significant contributions from the annihilation channel.

It should be noted that, from our calculation, the values of α and β are very close for neutrons and protons. It may be asked what the experimental situation is on the neutron polarizabilities. Limits to the electric polarizability have been given as $-50 \times 10^{-4} \text{ fm}^3 < \alpha_n < 60 \times 10^{-4} \text{ fm}^3$ by Aleksandrov et al. ⁷⁾ in a beautiful experiment drifting slow neutrons in the Coulomb field of lead nuclei. This limit is still a magnitude above the expected value. The magnetic polarizability is, to our knowledge, not determined even as a remotely interesting limit. Apart from the experimental difficulty of using a neutron target, the unpolarized differential cross-section for Compton scattering on the neutron is not adequate to determine the polarizabilities to second order in the frequency. In fact, to that order the cross-section is completely given in terms of the anomalous magnetic moment, due to the lack of interference with the Thomson term, which is zero.

The interest in having good values for α and β is not only a question of elementary particle physics. We originally found that these quantities were poorly determined when asking the question how much of the nuclear polarizabilities is due to the constituent nucleons ⁸⁾. For example, the magnetic polarizability of a spin-saturated nucleus is approximately

$$\chi_N \approx N\beta_n + Z\beta_p - \frac{Z\langle r^2 \rangle}{6m} \left(\frac{e^2}{4\pi} \right). \quad (11)$$

Even with Goldanskii's low value for β_p the nucleon terms are as important as the last term (nuclear diamagnetism). As a matter of fact, in ${}^4\text{He}$ the magnetic polarizability of the constituents would completely dominate over the diamagnetic term ($-13.5 \times 10^{-4} \text{ fm}^3$) if we assume for β the theoretically expected value, Eq. (9), which yields $\chi_{{}^4\text{He}} = 40 \times 10^{-4} \text{ fm}^3$.

Finally, the magnetic susceptibility of neutron stars has been vigorously debated lately in connection with pulsars. At densities for which the neutrons still maintain their identity, their intrinsic polarizability contributes an additional amount

$$\chi_n = \beta_n S = \beta_n \frac{k_F^3}{3\pi^2} \quad (12)$$

to the susceptibility. This equals the Pauli paramagnetic susceptibility of a Fermi gas

$$\chi_F = \mu^2 \frac{m_n k_F}{\pi^2} \quad (13)$$

for $k_F \approx 2 \text{ fm}^{-1}$, using $\beta_n \approx 10^{-3} \text{ fm}^3$. Figure 2 shows the recent calculations of χ_F/χ by Pandharipande et al.⁹⁾ as compared with $\chi_F/(\chi + \chi_n)$. It is clear that in this particular case, the intrinsic neutron polarizability is the dominant contribution to the neutron star susceptibility and that it overwhelms the interaction contribution for the interesting density region. Therefore, the intrinsic neutron polarizability cannot be neglected in such discussions.

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FIGURE CAPTIONS

Figure 1   The input to the polarizability sum rules, Eqs. (8).

Figure 2   Effect of the intrinsic magnetic polarizability β_n on a recent calculation of the paramagnetic susceptibility of dense neutron matter, adapted from Ref. 9).

FIG.1

FIG. 2