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NON-SCALING BEHAVIOUR IN PARTON MODELS FOR LARGE
TRANSVERSE MOMENTA

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A B S T R A C T

The possibility of non-scaling contributions to large transverse momentum processes is discussed within the context of simple parton-parton scattering models. The models are extended slightly to include the "quasi-exclusive" production of π 's as a threshold effect in the function $G(y)$ which describes the evolution of partons into hadrons. The resulting non-scaling $P_{\perp}^{-8}$ contribution is particularly enhanced in large $P_{\perp}$ processes, compared to the usual $P_{\perp}^{-4}$ term, and the dominance of this term in the present data can be explained if the scale which now appears has a value $M_0 \geq 4$ GeV or if naïve Bloom-Gilman duality, as applied here, is broken.

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Recent data ¹⁾ on the process $p+p \rightarrow \pi+X$, where the π has large transverse momentum, are consistent with a behaviour for the invariant cross-section of the form

$$E_{\pi} \frac{d^3\sigma}{d^3P_{\pi}} \simeq P_{\pi}^{-N} \cdot f(P_{\pi}/\sqrt{s}, \theta^*) \quad (1)$$

where N is approximately 8 in the CCR data and closer to 10 for the Chicago-Princeton experiment. This result is in contradiction with the prediction of naïve, scale free, parton models. For example, simple parton-vector gluon models ²⁾⁻⁴⁾, although they predicted the general form of Eq. (1), would suggest the dimensional analysis result $N=4$. Such simple scaling behaviour is suggested by the process $e+p \rightarrow e+X$ which exhibits naïve scaling in the form

$$E_e \frac{d^3\sigma}{d^3P_e} \simeq \frac{\alpha^2}{s^2} \cdot f(M_x^2/s, t/s) \quad (2)$$

in the limit $s \rightarrow \infty$, t/s and M_x^2/s fixed. This behaviour is explained, in the parton picture, in terms of point-like hadron constituents, which absorb the exchanged photon without the appearance of a form factor, i.e., without introducing a scale, and which subsequently evolve into hadrons, again in a scale invariant fashion. Of course, any individual exclusive channel will have an inherent scale and hence should fall faster than $1/s^2$. However, the inclusive sum of such channels seems to be scale free.

The extension of these ideas to purely hadronic reactions via the assumption of some simple, scale free, parton-parton interaction leads to the prediction $N=4$. The fact that N is greater than 4 in the present data indicates the presence of a scale in the problem contrary to the naïve expectation. In the following note, we shall attempt to motivate the presence of such a non-scaling contribution while remaining within the context of the usual parton picture and without sacrificing the $N=4$ contribution as an asymptotic limit.

Recall that in simple parton-parton scattering models, there are three basic ingredients: 1) the scale invariant distribution of partons within the initial hadron ($F(x)$); 2) the scale invariant distribution of hadrons from the evolution of the scattered parton ($G(y)$); and 3) the scale invariant parton-parton cross-section ($d\sigma/d\hat{t}$). It has already been suggested ⁵⁾ that the latter should in fact not be scale invariant but instead should be calculated as if the underlying theory were ϕ^3 with a dimensional coupling

constant. We shall not pursue this idea here as it violates our constraint that P_L^{-4} behaviour still be present ⁶⁾. In the case of ingredient 1), the parton distribution, one can in fact anticipate the appearance of a scale, i.e., a non-scaling contribution, in the limit that $x \rightarrow 1$ and the kinematic variables are large but not yet infinite (i.e., in the real world). This is discussed for example in the paper by Bloom and Gilman ⁷⁾. In the limit $x \rightarrow 1$, one finds structure due to the individual contributions of specific exclusive channels which, as mentioned earlier, contain explicit scales. For example, from the elastic proton channel, there is a contribution to $F(x)$ of the approximate form [to first order in $(M_0/Q)^2$] $\delta(1-x)F_{E.M.}^2(Q^2)$, where $F_{E.M.}$ is the electromagnetic form factor of the proton and clearly introduces an explicit scale - the size of the proton. Recall also the interesting feature that, if we assume $F_{E.M.}(Q^2) \sim (M_0^2/Q^2)^2$ (plus higher order corrections in M_0^2/Q^2) for large Q^2 and we assume that the scale invariant part of $F(x)$ behaves as $(1-x)^3$, then there is consistency with $F(x)$ describing, on the average, even the non-scaling contributions near threshold ($x=1$). For example, an idealized version of the complete distribution might be (Q^2 large but finite)

$$F(x, Q^2) \simeq 4(1-x)^3 \Theta(1-x - \frac{M_0^2}{Q^2}) + (\frac{M_0^2}{Q^2})^4 \tilde{\delta}(1-x) \quad (3)$$

such that $\int dx F(x, Q^2) = 1$ independent of Q^2 for large Q^2 . In the second term, we have ignored terms from $F_{E.M.}^2$ which are higher order in (M_0^2/Q^2) . Note that only the zeroth moment of $F(x, Q^2)$ is Q^2 independent [to order $(M_0^2/Q^2)^4$]. Higher moments will contain non-scaling terms. Such details are not generally of interest for parton calculations but they are essential in the present work.

Explicit calculations in parton-parton scattering models ⁴⁾ indicate that the momentum fractions of the participant partons in the initial state are, on average, very near to the momentum fraction $(2P_\pi/\sqrt{s} = x_\pi)$ of the observed π (i.e., $\langle x_1 \rangle \sim \langle x_2 \rangle \sim x_{L\pi}$) which is far from 1 in present data. However, the same calculations indicate that the pion has only slightly less momentum than its parent parton ($P_\pi/P_{\text{parton}} = y \sim 1$). This is a straightforward result of the rapid cut-off in momentum of both the initial parton distributions and the basic scattering process $d\sigma/d\hat{t}$. Hence the threshold ($y \rightarrow 1$) behaviour of the function $G(y)$ is of central importance for the study of large p_L processes, at least as the experiments are presently performed. It is generally thought that the scale invariant threshold behaviour of the function $G(y)$ ($y \rightarrow 1$) should be similar to the behaviour of the

corresponding $F(x)$ ⁸⁾ [e.g., $F_\pi \sim (1-x)$ and $G^\pi(y) \sim (1-y)$]. We shall here go one step further and suggest that $G^\pi(y)$ also exhibits non-scaling contributions near $y=1$ just as we discussed above for $F(x)$. Although we may again expect that such effects are of small importance when studying low moments of the distribution, in the case of large $P_\perp$ processes we are explicitly looking at a quite high moment because of the rapid cut-off of the parton distribution and, as we shall see, such effects can be greatly enhanced.

By analogy with Eq. (3) above, we will study the consequences of using the following idealized form for $G(y)$

$$G(y, \hat{t}) \simeq 2 \cdot (1-y) \Theta(1-y - \frac{M_0^2}{\hat{t}}) + C \cdot (\frac{M_0^2}{\hat{t}})^2 \sqrt{1-y} \quad (4)$$

where $\hat{t}$ is the momentum transfer between the scattered partons [$\langle \hat{t} \rangle \sim 2P_\perp^2$] and M_0 sets the scale of the exclusive channels (π, π^* , etc.) which we have lumped together in the final term. This form is illustrated in Fig. 1. We have taken $F_{E.M.}^\pi(\hat{t})$ to behave as $(M_0^2/\hat{t})$ for $\hat{t}$ large. The parameter C , which describes the relative strength of the non-scaling to scaling contributions will be left free for now, but it would take the value $C=1$ if we assumed that a form of Bloom-Gilman duality held here also. One may ask whether Eq. (4) can be motivated by anything other than analogy with Eq. (3). There are, in fact, some limited data which are suggestive of Eq. (4). Recent data on the process $e+p \rightarrow e+\pi+X$ ⁹⁾ evidence clear resonance behaviour near the edge of phase space. The area under these bumps is consistent with $1/Q^4$ behaviour but the data are not particularly suggestive of Bloom-Gilman duality. The $1/\hat{t}^2$ behaviour is also consistent with the data on hadronic multiplicities in $e+p \rightarrow e+X$ ¹⁰⁾ which suggest a form $\langle n_{ch} \rangle \sim 1.33 (\ln M_X^2/Q^2 + \ln Q^2)$ where the second term is identified with the fragmentation of the photon. In order for this region (both the hole and parton fragmentation regions) to be unoccupied, as is appropriate to the non-scaling term in Eq. (4), we must pay the price $e^{-\langle n \rangle_V} \sim e^{-2 \ln Q^2} \sim 1/Q^4$ (take $\langle n_0 \rangle \sim \frac{1}{2} \langle n_{ch} \rangle$). This last identification is made more apparent if we note that the process corresponding to the second term in Eq. (4) is the "quasi-exclusive" production of a pion which we may write symbolically as (see Fig. 2)

$$p + p \rightarrow \pi \uparrow (\text{or } \pi^* \uparrow) + X \downarrow \quad (5)$$

where the arrows denote the effective direction of the large $P_\perp$ components.

The "quasi-exclusive" quality of the process gives a natural explanation of the presence of a scale and suggests that there should be a clear experimental signal if the present model is correct. Namely, the large $P_{\perp}$ pion which is used as a trigger in present large $P_{\perp}$ experiments should be appearing unaccompanied by other large $P_{\perp}$ hadrons ($P_{\perp} \gtrsim 0.5$ GeV/c) in the same hemisphere of phase space. Thus "same side" correlations should be very weak compared to "opposite side" correlations. This is consistent with present data and should be easily tested when data are available on the full momentum distribution of the accompanying hadrons. One can also test the picture by triggering on total energy deposition at large angle, instead of a single π ¹¹⁾. In this case the function $G(y)$ would not enter the problem directly and the scaling $(P_{\perp}^{-4})$ term (if present) would presumably dominate.

The fact that we identify the $P_{\perp}^{-8}$ behaviour with process (5) allows us to make connection with the "parton interchange" model of Brodsky, Blankenbecler and Gunion ¹²⁾ who in fact predicted that N would be equal to 8 prior to the appearance of the data. In their model, they are led naturally to calculate the process (5) and hence predict non-scaling behaviour. However, the procedure for calculating process (5) given in Ref. 12) differs in certain details from the present approach ¹³⁾. In Ref. 12) one of the incident protons is first decomposed into hadronic constituents including at least one pion. This pion then suffers an elastic, large angle scattering (via parton interchange) with a parton from the other proton and appears with large $P_{\perp}$. Thus to go from the present approach to that of Ref. 12) one of the initial parton distributions is replaced by a pion distribution and $d\sigma/d\hat{t}|_{\text{parton-parton}} \cdot G^{\pi}$ is replaced by $d\sigma/d\hat{t}|_{\pi\text{-parton}}$. To the extent that these functions differ, the models will give different results for the function $f(P_{\perp}/\sqrt{s}, \theta^*)$ in Eq. (1). The presence (or absence) of a scaling contribution is left as an open question in Ref. 12), although presumably such a term will arise from explicit summation of "quasi-exclusive" terms.

To proceed to calculate with Eq. (4), we need only substitute it into the parton-parton scattering model ²⁾⁻⁴⁾ formula (the complete form will include also a $\hat{u}$ channel term)

$$E \frac{d^3\sigma}{d^3P_{\perp}} \simeq \frac{4}{\pi X_{\perp}^2} \iint dx_1 dx_2 F(x_1) F(x_2) \frac{d\sigma}{d\hat{t}} G\left(\frac{x_1(1+\eta)}{2X_{\perp} \tan^2 \theta^*/2}, \hat{t}\right) \frac{\eta}{(1+\eta)^2} \quad (6)$$

$$+ (\hat{t} \rightarrow \hat{u})$$

where we have used the notation of Ref. 4); $X_{\perp} = 2P_{\perp}/\sqrt{s}$,

$$\hat{t} = - \frac{x_1^2 s \tan^2 \theta^*/2}{(1+\eta)}, \quad \eta = (x_1/x_2) \cdot \tan^2 \theta^*/2$$

and $F(x)$ is taken from ep data [see Ref. 4)]. We will use $d\sigma/d\hat{t}$ for a single gluon exchange but any scale free form will do as well. We express the result in terms of the ratio of the $N=4$ contribution to the $N=8$ contribution at $\theta^* = \pi/2$.

$$R_{\text{ATIO}}\left(\frac{\sigma_N=4}{\sigma_N=8}\right) = \frac{S^2}{CM_0^4} \cdot R(x_1) \quad (7)$$

The quantity $R(x_1)$ is plotted in Fig. 3¹⁴⁾. We see that we can understand the dominance of the non-scaling term at present energies ($s \leq 3000 \text{ GeV}^2$) if the quantity CM_0^4 satisfies $CM_0^4 \geq 2 \times 10^2$. For example, assume the data behave as P_1^{-8} at $x_1 = 0.2$, where $R = 2 \times 10^{-5}$, up to $S = 3000 \text{ GeV}^2$. This gives the above number for CM_0^4 . Hence if the present model is correct, either M_0 is fairly large, $M_0 \geq 4 \text{ GeV}$, with $C=1$ or $C \gg 1$ and Bloom-Gilman duality as interpreted here is not satisfied. In any case, it is difficult to imagine that we are far from the region where P_1^{-4} behaviour should dominate.

In Fig. 4 we indicate that the present naïve model for the non-scaling term (solid line) offers a reasonable description of the CCR data [the quantity $CM_0^4 \alpha_{\text{eff}}^2 \tilde{F}^2 G$, $\tilde{F} = \int dx F(x)$, has been given the value $\sim 0.3 \text{ GeV}^4$].

The quantity $R(x_1)$ is fairly insensitive to variations in the form of $F(x)$, varying by much less than an order of magnitude as $F(x)$ varies from $(1-x)^1$ to $(1-x)^5$. However, the cross-section itself is quite sensitive to the detailed forms of the various distributions. Any serious attempt to fit the data should include a more realistic form for $G(y)$, presumably to be found in high Q^2 lepton-hadron data, and should keep both scaling and non-scaling terms. The present work was intended only to illustrate the basic ideas. For fun, we have also indicated in Fig. 4 (dashed line) the amusing, but presumably content free, point that the naïve form $F(x) \sim 4(1-x)^3$ gives a better description of the data than a form more consistent with the observed e-p data which have a maximum around $x \approx 0.2$. This result might be interpreted either as a warning against being overly satisfied with naïve models or as an indication that the e-p data at small x have not yet reached their asymptotic (scaling) limit.

In conclusion we have seen that one can understand the "non-scaling" behaviour (P_1^{-8}) of the current large transverse momentum data within the context of the usual parton scattering model by including non-scaling contributions at threshold for the function $G(y)$. These contributions are not unexpected and their effects are particularly enhanced in this process. The actual numerical description of the data requires either a scale parameter of

magnitude $M_0 \geq 4$ GeV or a breakdown of naïve Bloom-Gilman duality for the function $G(y)$. In any case, the general features of the model which arise from the identification of the non-scaling behaviour with process (5) and the continued presence of a scaling $(P_{\perp}^{-4})$ term should be easily tested via good multiparticle correlation data for large $P_{\perp}$ events, single particle data at higher s and/or $x_{\perp}$, and data which result from experiments involving different triggering techniques. Finally, it is amusing to note that the ideas discussed above might also be directly applicable to lepton induced processes if these processes were studied in a non-standard fashion. For example, if one triggered on a large $P_{\perp}$ pion (in the ep centre-of-mass system) in the process $ep \rightarrow \pi + X$ instead of triggering on the scattered electron¹⁵⁾ (i.e., $ep \rightarrow e + \pi + X$), the analysis in the parton model would follow closely the discussion above with one parton replaced by the electron and one distribution $F(x)$ replaced by $\delta(1-x)$. Hence one would expect a sizeable non-scaling contribution from the "quasi-exclusive" production of the pion.

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NOTE ADDED IN PROOF

At the conclusion of the present work, the author received the preprint by M. Bander, R.M. Barnett and Dennis Silverman (UC Irvine), wherein a somewhat different formalism leads to essentially the same interpretation of the present data.

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- 11) This idea is discussed in detail by
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- 12) R. Blankenbecler, S.J. Brodsky and J.F. Gunion, Phys.Letters 42B, 961 (1972); Phys.Rev. D6, 2652 (1972).
- 13) The subtle question of the relationship between these two pictures will be treated in a separate paper.
- 14) The general shape in Fig. 3 is easily understood if we consider a scattered parton distribution of the form $z_1^{-n}(1-z_1)^m$. Convolution with Eq. (4) yields $R(x_1)$ of the form x_1^4 for $x_1 \ll 1$ and $x_1^2(1-x_1)^2$ for x_1 large.

- 15) Here care must be taken to avoid coherent contributions from the decay of large mass N^* 's which may be produced via the exchange of a low Q^2 photon (small angle electron scattering). The relative contribution of such effects may be larger here than in the purely hadronic process, although within the present framework they are assumed small in both processes. We thank Professor W.K.H. Panofsky for helpful discussions on this point.

FIGURE CAPTIONS

- Figure 1 : Idealized form of the function $G(y, Q^2)$.
- Figure 2 : Pictorial representation of the "quasi-exclusive" process $p+p \rightarrow \pi \uparrow + X \downarrow$.
- Figure 3 : The quantity $R(x_1)$ versus $x_1 = 2P_1/\sqrt{s}$, $\theta^* = \pi/2$.
- Figure 4 : Comparison of present model for non-scaling term with the CCR data, first paper in Ref. 1). The solid line is for $F(x) = 3.8(1-x)^3 + 14.9(1-x)^4 - 17.7(1-x)^5$ and the dashed line is for $F(x) = 4(1-x)^3$.

FIG. 1

FIG. 2

FIG.3

FIG.4