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SEMI-INCLUSIVE RAPIDITY CORRELATIONS IN
CLUSTER EMISSION MODELS

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ABSTRACT

From an approach to multiparticle production in which clusters of hadrons are emitted, the formalism is developed for an analysis of rapidity correlations in the central region at fixed multiplicity. Predictions are given for multiplicity, energy, and rapidity dependence of correlation functions.

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The idea that hadrons are produced in clusters provides a popular interpretation [1,2] of several features of multiparticle production data. In particular, the postulate of independent emission of isotropic clusters gives a facile imitation of data [3] on two-particle inclusive rapidity correlations in the central region. At present, it is not clear that clusters have intrinsic dynamical significance (e.g. generalised resonances) or whether they are primarily a phenomenological convenience. Taking them seriously as dynamical entities, we may enquire into their various properties, such as the isospin, charge, spin, mass, and multiplicity distributions within individual clusters. In this letter I discuss what can be learned from an experimental study of rapidity correlations in the central region at fixed final state hadron multiplicity (semi-inclusive correlations).

Based on the concept of independent emission of isotropic clusters, expressions are derived and presented here for two particle semi-inclusive rapidity correlations. A method of experimental analysis is suggested. The presentation is necessarily summary, but full details may be found elsewhere [4]. It is hoped that this approach will be useful for analysis of high-statistics second generation experiments now underway at NAL and the CERN ISR.

Semi-inclusive correlations provide several advantages and yield important information. First, contributions to the correlation function from competing reaction types are more readily separated, removing a source of ambiguity which plagues analyses of the fully inclusive correlations. Second, the postulate of isotropic cluster decay can be tested over a range of multiplicities. Third, possible dependence of the multiplicity distribution within clusters on the overall final state hadronic multiplicity can be investigated. A mean of $\langle K \rangle = 3$ to 4 total hadrons per cluster is determined from fully inclusive correlation functions. However, this figure results after an average over both the multiplicity distribution of hadrons within clusters and the multiplicity

distribution for production of clusters. Separation of the two effects is aided by fixing the final state hadronic multiplicity.

Despite oft-stated reservations, I present the discussion in terms of the rapidity correlation function, as usually defined, but for fixed final state multiplicity n .

$$C_n(y_1, y_2) = \frac{1}{\sigma_n} \frac{d^2\sigma_n}{dy_1 dy_2} - \left(\frac{1}{\sigma_n} \right)^2 \frac{d\sigma_n}{dy_1} \frac{d\sigma_n}{dy_2} . \quad (1)$$

Variables associated with transverse momenta p_T are suppressed. In principle, it would be valuable to study y correlations for fixed p_T . Thus, Eq. (1) can be understood as applicable at each set of fixed p_{Ti} , or as an expression for quantities integrated over the p_T 's. Multiplicity index n may denote total charged multiplicity, pion multiplicity, negatives ..., etc. Cross-section σ_n is the relevant partial cross-section for multiplicity n .

The single and two-particle inclusive y spectra satisfy

$$\frac{1}{\sigma_n} \int dy \frac{d\sigma_n}{dy} = n , \quad (2a)$$

$$\frac{1}{\sigma_n} \iint dy_1 dy_2 \frac{d^2\sigma_n}{dy_1 dy_2} = n(n-1) , \quad (2b)$$

and

$$\iint dy_1 dy_2 C_n(y_1, y_2) = -n \quad (2c)$$

It is agreed that at least two important reaction mechanisms contribute to hadron production. Denoting these by symbols D (diffractive) and ND (non-diffractive), we may decompose Eq. (1) as

$$C_n(y_1, y_2) = \alpha_n^{ND} C_n^{ND}(y_1, y_2) + \alpha_n^D C_n^D(y_1, y_2) + \alpha_n^D \alpha_n^{ND} C_n^{Cross}(y_1, y_2) . \quad (3)$$

Here $\alpha_n^{ND} = \sigma_n^{ND} / \sigma_n$; $\alpha_n^D = \sigma_n^D / \sigma_n$; $C_n^{ND}(y_1, y_2)$ and $C_n^D(y_1, y_2)$ are the rapidity correlation functions at fixed n for the purely non-diffractive and diffractive mechanisms, respectively.

$$C_n^{Cross}(y_1, y_2) = \left[\frac{1}{\sigma_n^{ND}} \frac{d\sigma_n^{ND}}{dy_1} - \frac{1}{\sigma_n^D} \frac{d\sigma_n^D}{dy_1} \right] \times \left[1 \leftrightarrow 2 \right] \quad (4)$$

An understanding of the correlation function C_n is reduced therefore to an understanding of each of the three terms on the right side of Eq.(3). Arguments are developed below to show that only the first term is important for $n \gtrsim \frac{1}{2} \langle n \rangle$.

The decomposition given in Eq. (3) is directly analogous to that for the fully inclusive case [1,2]. Even if both C_n^{ND} and C_n^D were of a purely short range character (vanish for $\Delta y = |y_1 - y_2| \gg \lambda$), the presence of C_n^{Cross} gives a long-range correlation component to C_n . It is argued below, however, that this effect may be neglected for $n \gtrsim \frac{1}{2} \langle n \rangle$.

A separation of σ_n and of $d\sigma_n/dy$ into "diffractive" and "non-diffractive" parts may be relatively clean at small n , but it is necessarily ambiguous at larger n where mechanisms overlap in phase space. Nevertheless, adopting reasonable operational procedures, experimenters at NAL estimate [5] that $\sigma_n^{(D)} / \sigma_n^{(ND)} < 20\%$ for $n \approx \frac{1}{2} \langle n \rangle$ and $< 10\%$ for $n \approx \langle n \rangle$. Similar figures may be obtained theoretically, for example, from a model [6] in which one assumes that the diffractive mechanism is represented by Pomeron exchange, with the Pomeron-proton subamplitude being non-diffractive in character. For small n , $d\sigma_n^D/dy$ surely peaks near $y = \pm \log \sqrt{s}$. However, as n increases, the chance increases that a diffractively produced system will populate the region near $y \approx 0$, perhaps even roughly uniformly [1]. This, taken together with the normalization condition (Eq. 2(a)), suggests that the difference given in square brackets in Eq. (4) vanishes in the central region as n increases. Combining these two arguments (σ_n and $d\sigma_n/dy$) and supported by model calculations [4],

I find that the net cross-term $\alpha_n^D \alpha_n^{ND} C_n^{\text{Cross}}$ in Eq. (3) may be ignored for $n \gtrsim \frac{1}{2} \langle n \rangle$.

Attention is now turned to the correlation functions C_n^{ND} and C_n^D in Eq. (3).

To obtain a parametrization of the form and magnitude of the non-diffractive C_n^{ND} , I adopt again the postulate of independent emission of isotropically decaying clusters [1]. (c.f. Fig. (1a). Essentially the same line of argument can be repeated here for fixed n as was used in the fully inclusive case [1,2]. The probability that N clusters are produced is $P_N = \sigma_N / \sigma$. The single-cluster rapidity density in an N cluster final state is

$$\rho_N^{(1)}(s, y_c) = \frac{1}{\sigma_N} \frac{d\sigma_N}{dy_c}.$$

In the independent cluster model, this takes the multiperipheral - like [7] form

$$\rho_N^{(1)} = N/Y \quad (5)$$

where $Y = \log s$. Similarly, the two-cluster density is

$$\rho_N^{(2)}(s, y_{c1}, y_{c2}) = \frac{1}{\sigma_N} \frac{d^2\sigma_N}{dy_{c1} dy_{c2}} \longrightarrow \frac{N(N-1)}{Y^2} \quad (6)$$

These equations express the fact that the cluster densities are plateaux extending over the full y space.

The cluster is assumed to decay isotropically, independent of the decay multiplicity. Single decay is expressed by the normalized function [1]

$$D^{(1)}(y, y_c) = \frac{1}{\delta\sqrt{2\pi}} \exp \left[-\frac{(y-y_c)^2}{2\delta^2} \right] \quad (7)$$

with dispersion $\delta \approx 0.9$. Two-hadron decay is postulated to follow

$$D^{(2)}(y_1, y_2, y_c) = D^{(1)}(y_1, y_c) D^{(1)}(y_2, y_c). \quad (8)$$

The probability that k hadrons of a specific type (charged, pions, anti-baryons, ...) decay from a given cluster is p_k whose k dependence is unknown and is to be determined, if possible, from data;

$\sum p_k = 1$, and $\sum k p_k = \langle k \rangle$. To simplify most of the equations in this letter, I take the extreme case: $p_k = 1$ if $k = \langle k \rangle$, and zero otherwise. The general case is treated fully elsewhere [4] [c.f. also Eq. (17)].

Combining statements above, we easily find the non-diffractive single hadron y distribution

$$\frac{1}{\sigma_n} \frac{d\sigma_n}{dy} = \int dy_c \rho_N^{(1)}(y_c) \langle K \rangle D^{(1)}(y_c, y) = \frac{\langle K \rangle N}{Y} = \frac{n}{Y}. \quad (9)$$

The correlation function in the non-diffractive component is

$$\begin{aligned} C_n^{ND}(y_1, y_2) &= \int dy_c \rho_N^{(1)}(y_c) \langle K(K-1) \rangle D^{(2)}(y_1, y_2; y_c) \\ &+ \int dy_{c1} \int dy_{c2} \rho_N^{(2)}(y_{c1}, y_{c2}) \langle K \rangle^2 D^{(1)}(y_1, y_{c1}) D^{(1)}(y_2, y_{c2}) \\ &- \left(\int dy_{c1} \rho_N^{(1)}(y_{c1}) \langle K \rangle D^{(1)}(y_1, y_{c1}) \right) \times \left(\int dy_{c2} \rho_N^{(1)}(y_{c2}) \langle K \rangle D^{(1)}(y_2, y_{c2}) \right) \\ &= \frac{N \langle K(K-1) \rangle}{2 \delta \sqrt{\pi} Y} \exp \left[- \frac{(y_1 - y_2)^2}{4 \delta^2} \right] - \frac{N \langle K \rangle^2}{Y^2}. \quad (10) \end{aligned}$$

It is useful to reexpress Eq. (10) in terms of observables as

$$\begin{aligned} C_n^{ND}(y_1, y_2) &= \frac{\langle K(K-1) \rangle}{\langle K \rangle^2} \frac{1}{2 \delta \sqrt{\pi}} \left(\frac{1}{\sigma_n} \frac{d\sigma_n}{dy} \right)_{y=0} \exp \left[- \frac{(y_1 - y_2)^2}{4 \delta^2} \right] \\ &- \frac{\langle K \rangle}{n} \left(\frac{1}{\sigma_n} \frac{d\sigma_n}{dy_1} \right) \left(\frac{1}{\sigma_n} \frac{d\sigma_n}{dy_2} \right). \quad (11) \end{aligned}$$

Although Eq. (11) was derived under the assumption that both y_i are in the central region, we may conjecture that it remains valid outside this region, provided that we employ a realistic (i.e. the observed) y dependence of $\sigma_n^{-1} d\sigma_n/dy_i$ in the second term. Further remarks about the n , y , and s dependences of Eq. (11) are made after the diffractive term C_n^D is disposed of.

To estimate the magnitude and form of $C_n^D(y_1, y_2)$ in the central region, I use the model (sketched in Fig. 1(b)) in which Pomeron exchange couples to a system of mass M , whose internal structure is given by the non-diffractive amplitude of Fig. 1(a). This model has been studied previously [1,6] to obtain inclusive spectra; I employ it here for semi-inclusive quantities. Expressions (5) and (6) now apply at fixed M , with $Y = \log M^2$. The cross-section for production of system M is $d\sigma/dM^2 = a/M^2$; $\sigma_D = 2aY$. The value of a (≈ 0.5) can be estimated from triple Regge fits to ISR data [1].

Expressions for $\sigma_n^{(D)}$, $d\sigma_n^{(D)}/dy$ and $d^2\sigma_n^{(D)}/dy_1 dy_2$, obtained after integration over M^2 , can all be expressed in terms of incomplete gamma functions [4]. Near $y_1 \approx y_2 \approx 0$, the correlation function takes on the approximate form

$$C_n^D(y_1, y_2) \approx \frac{\langle K(K-1) \rangle}{2\langle K \rangle \delta \sqrt{\pi}} \left(\frac{1}{\sigma_n^D} \frac{d\sigma_n^D}{dy} \right)_{y \approx 0} \exp \left[- \frac{(y_1 - y_2)^2}{4\delta^2} \right] - \frac{\langle K \rangle}{n} \left(\frac{1}{\sigma_n^D} \frac{d\sigma_n^D}{dy_1} \right) \left(\frac{1}{\sigma_n^D} \frac{d\sigma_n^D}{dy_2} \right). \quad (12)$$

This expression is quite similar in structure to the non-diffractive result, Eq. (11). Essential corrections [4] to Eq. (12) do occur for $y_i \rightarrow \pm \log \sqrt{s}$, but I confine my remarks here to the central region (e.g. $|y_i| < 1$). The important point to notice is that for small y ,

$C_n^D(y_1, y_2)$ is comparable in magnitude and form to $C_n^{ND}(y_1, y_2)$. For large n , however, as discussed above $\sigma_n^{ND} \gg \sigma_n^D$. Thus, for $n \gtrsim \frac{1}{2} \langle n \rangle$, I conclude that

$$C_n(y_1, y_2) \rightarrow C_n^{ND}(y_1, y_2) \quad (13).$$

This is the essential result of the analysis, and I now discuss the expected n , y , and s dependences of $C_n(y_1, y_2)$ for $n \gtrsim \frac{1}{2} \langle n \rangle$.

The structure of Eq. (11) is that of a positive term, which is invariant with respect to translations in rapidity (dependence only on $|y_2 - y_1|$), riding upon a negative "background". The positive (Gaussian) term expresses the correlation owing to the existence of clusters. It is suggestive therefore, that one attempt to fit data to an expression of the form given in Eq. (11), that is, to a sum of two terms:

$$C_n^{\text{Exp}}(y_1, y_2) = A \exp \left[- \frac{(y_1 - y_2)^2}{4\delta_{\text{exp}}^2} \right] - B \left(\frac{1}{\sigma_n} \frac{d\sigma_n}{dy_1} \right) \left(\frac{1}{\sigma_n} \frac{d\sigma_n}{dy_2} \right). \quad (14)$$

According to the model discussed here, one should find that

- (1) Correlation length $2\delta_{\text{exp}}$ is independent of n and s , with $\delta_{\text{exp}} \approx 0.9$.
- (2) The ratio $A / \left(\frac{1}{\sigma_n} \frac{d\sigma_n}{dy} \right)_{y=0}$ is independent of n and s , provided that the mean number of particles per cluster $\langle K \rangle$ is so independent, as would seem reasonable. Thus, this ratio gives a direct measure of $\langle K(K-1) \rangle / \langle K \rangle$ as a function of n .
- (3) Coefficient B is equal to $\langle K \rangle / n$, independent of s .

Because in the independent cluster model [1], $\left(\frac{1}{\sigma_n} \frac{d\sigma_n}{dy} \right) \approx \frac{n}{y}$,

the following additional remark may be added:

$$(4) \quad C_n(0, 0) \approx \frac{n}{\log s} \left[\frac{\langle K(K-1) \rangle}{2\delta\sqrt{\pi} \langle K \rangle} - \frac{\langle K \rangle}{\log s} \right] \quad (15)$$

Reexpressed in terms of the popular ratio $R_n(y_1, y_2) = C_n / \left(\frac{1}{2} \frac{d\sigma_n}{d\sigma_n} \frac{d\sigma_n}{dy_1} \frac{d\sigma_n}{dy_2} \right)$, statement (4) becomes *

$$R_n(0, 0) \approx \frac{\log s}{n} \left[\frac{\langle K(K-1) \rangle}{2\delta\sqrt{\pi} \langle K \rangle} - \frac{\langle K \rangle}{\log s} \right] \quad (16)$$

These n and s dependences of $R_n(0,0)$ are in respectable agreement with preliminary ISR results [8] (at which energies $R_n(0,0)$ is positive). Taking Eq. (16) (too) seriously, we note its prediction that $R_n(0,0) \approx 0$, for all n , when $\log s_{\text{crit}} = \frac{2\langle K \rangle^2 \delta\sqrt{\pi}}{\langle K(K-1) \rangle}$. Accepting the estimate [1] from analyses of fully inclusive charged-charged (cc) correlations that $3 \approx 2\delta\sqrt{\pi} \langle K \rangle / \langle K(K-1) \rangle$, I find $\log s_{\text{crit}} = 3\langle K \rangle$. Recent 205 GeV/c NAL data [9] show $R_n^{\text{CC}}(0,0)$ within two standard deviations of zero for all n . Although their statistical precision deserves to be improved, these data (at $\log s_{\text{crit}} \approx 6$) would then suggest $\langle K \rangle = 2$ charged particles per cluster, or $\langle K \rangle = 3$ total hadrons per cluster, somewhat lower than previous estimates [1,2]. High statistics studies of central region semi-inclusive correlations between $2\pi^-$ or $2\pi^+$ (particle type and momentum identification) are needed to further verify these estimates. The reduced value of $\langle K \rangle$, implying $\langle \pi^\pm \rangle \approx 1$, per cluster, suggests that the coefficient of the Gaussian term in Eq. (11) is rather small for $2\pi^-$ or $2\pi^+$ correlations.

In Fig. 2 a sketch is given of the expected shape of $C_n(y,0)/(\sigma_n^{-1} d\sigma_n/dy)_{y=0}$ for $\sqrt{s} = 62$ GeV and $n \approx \langle n \rangle$. Note that the positive part of this ratio is universal, independent of n and s , if $\langle K \rangle$ is so independent. These expectations are qualitatively consistent with data [8,9], but it would be instructive to verify in detail these predictions for n , s , and y dependences of $C_n(y_1, y_2)$ (statements 1-4, above).

* A similar result was derived by A. Morel, private communication.

The results given above pertain to $n \gtrsim \frac{1}{2} \langle n \rangle$, where estimates suggest that diffractive effects can be neglected. Thus, the semi-inclusive correlations for these values of n should allow direct insight into the "non-diffractive" mechanism. For $n < \frac{1}{2} \langle n \rangle$ complications arise [4], and statements are dependent on what one believes about the diffractive mechanism. However, one simple statement is that, just as for the fully-inclusive $C(y_1, y_2)$ the Gaussian part of $C_n(y_1, y_2)$ will sit above a long-range positive "background". Thus, fits to Eq. (14) for $n < \frac{1}{2} \langle n \rangle$ will give values of δ_{exp} which should decrease with n to the n independent values taken on for $n \gtrsim \langle n \rangle$.

As a technical remark, one may add that it is in principle not necessary to employ a 4π solid angle detector to carry out the analyses suggested here. It should suffice, for example, to determine multiplicities n in a counter set-up which covers the central region, say, to the extent of 2π . Finally, for a general distribution $p(k)$ of hadrons from cluster decay, the ratio $\langle K(K-1) \rangle / \langle K \rangle$ in Eq. (11) is replaced by

$$\frac{1}{n} \sum_N \langle k(k-1) \rangle_{N,n} \frac{N P_N q_N(n)}{N P_N q_N(n)} \quad (17)$$

where

$$q_N(n) = \sum_{k_i} \tilde{p}_i(k_i; N, n); \quad (18)$$

$$q_N(n) \langle k(k-1) \rangle_{N,n} = \sum_{k_i} k_i(k_i-1) \tilde{p}_i(k_i; N, n); \quad (19)$$

and

$$\tilde{p}_i(k_i; N, n) = \sum_{k_N} \dots \sum_{k_2} \sum_{k_1=1}^N \prod_{j=1}^i p_j(k_j). \quad (20)$$

The sums in Eq. (20) are made subject to the requirement that $\sum_{j=1}^N k_j$. No sum is made over k_i . These expressions can be evaluated numerically for various choices of $p(k)$ [4].

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FIGURE CAPTIONS

Fig.1 (a) Diagram illustrating an amplitude in which clusters of hadrons are produced via some multiperipheral-like exchange mechanism. The exchanges are denoted by dashed lines.

(b) Single diffractive excitation diagram, in which a single Pomeron link (wavy line) is present.

Fig.2 The solid line is a sketch of the shape expected for the correlation function $C_n(0,y)$ divided by the single particle density $(\frac{1}{\sigma_n} \frac{d\sigma_n}{dy})$ at $y = 0$. The energy was taken to be $\sqrt{s} = 62$, and $n = \langle n \rangle$. The dashed line shows the universal Gaussian part of the correlation function, owing to the existence of clusters. From this, the term $(\langle K \rangle / n) \sigma_n^{-1} d\sigma_n / dy$, calculated with the Monte Carlo method of ref. [1], is subtracted to obtain the solid curve.

$$C_u(y, 0) = \frac{b_p^u \left(\frac{\gamma_p}{b_p^u} - 1 \right)}{\gamma_p}$$