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SIXTH ORDER ELECTRON AND MUON $(g-2)/2$
FROM SECOND ORDER VACUUM POLARIZATION INSERTION

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A B S T R A C T

We complete the analytic evaluation of sixth order contributions to electron and muon anomalous magnetic moments coming from second order vacuum polarization insertions into all the fourth order graphs.

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Continuing previous work ^{1),2),3)} we have completed the analytic evaluation of the electron and muon anomalous magnetic moments a_e, a_μ due to the insertion of second order vacuum polarization into fourth order graphs. The relevant graphs are shown in Fig. 1. They are numbered as in Ref. 4).

For a_e , both the external and the internal loop leptons in Fig. 1 are electrons. We find

$$\begin{aligned}
 a_e^{(7)} + a_e^{(8)} &= \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{2005}{324} + \frac{854}{81} \zeta(2) - \frac{154}{9} \zeta(2) \log 2 - \frac{197}{54} \zeta(3) + \frac{2}{15} \zeta^2(2) \right] \\
 &= \left(\frac{\alpha}{\pi}\right)^3 \left[-0.00312135 \right] , \\
 a_e^{(11)} + a_e^{(12)} &= \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{599}{162} - \frac{205}{81} \zeta(2) - \frac{16}{9} \zeta(2) \log 2 + \frac{97}{54} \zeta(3) + \frac{2}{15} \zeta^2(2) \right] \\
 &= \left(\frac{\alpha}{\pi}\right)^3 \left[0.0274617 \right] , \\
 a_e^{(13)} + a_e^{(14)} &= \left(\frac{\alpha}{\pi}\right)^3 \left[-\frac{2641}{648} + \frac{305}{81} \zeta(2) - \frac{10}{9} \zeta(2) \log 2 - \frac{251}{54} \zeta(3) + \frac{26}{15} \zeta^2(2) \right. \\
 &\quad \left. + \left(-\frac{119}{18} + 4 \zeta(2) \right) \log \frac{\lambda}{m_e} \right] = \left(\frac{\alpha}{\pi}\right)^3 \left[-0.0458692 - 0.0313748 \log \frac{\lambda}{m_e} \right]
 \end{aligned} \tag{1}$$

For a_μ , the external lepton is a muon, the lepton of the vacuum polarization loop is an electron. We find

$$\begin{aligned}
 a_\mu^{(7)} + a_\mu^{(8)} &= \left(\frac{\alpha}{\pi}\right)^3 \left[-\frac{19}{18} - \frac{317}{54} \zeta(2) + \frac{116}{9} \zeta(2) \log 2 - 5 \zeta(3) + 5 c_4 + \left(\frac{2}{9} + \frac{26}{9} \zeta(2) - \right. \right. \\
 &\quad \left. \left. - \frac{20}{3} \zeta(2) \log 2 + \frac{5}{3} \zeta(3) \right) \log \frac{m_\mu}{m_e} \right] = \left(\frac{\alpha}{\pi}\right)^3 \left[2.14230 - 0.623527 \log \frac{m_\mu}{m_e} \right] \\
 a_\mu^{(11)} + a_\mu^{(12)} &= \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{43}{12} - \frac{11}{27} \zeta(2) - \frac{10}{9} \zeta(2) \log 2 + \frac{7}{18} \zeta(3) - c_4 + \frac{1}{3} \log^2 \frac{m_\mu}{m_e} \right. \\
 &\quad \left. + \left(-\frac{29}{12} + \frac{2}{9} \zeta(2) + \frac{4}{3} \zeta(2) \log 2 - \frac{1}{3} \zeta(3) \right) \log \frac{m_\mu}{m_e} \right] \\
 &= \left(\frac{\alpha}{\pi}\right)^3 \left[1.280005 + \frac{1}{3} \log^2 \frac{m_\mu}{m_e} - 0.931570 \log \frac{m_\mu}{m_e} \right]
 \end{aligned}$$

$$\begin{aligned}
 a_{\mu}^{(13)} + a_{\mu}^{(14)} &= \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{937}{216} - \frac{4}{3} \zeta(2) - \frac{16}{9} \zeta(2) \log 2 + \frac{10}{9} \zeta(3) - c_4 + \left(\frac{25}{18} - \right. \right. \\
 &\quad \left. \left. - \frac{2}{3} \log \frac{m_{\mu}}{m_e} \right) \log \frac{\lambda}{m_{\mu}} + \left(-\frac{67}{36} + \frac{2}{9} \zeta(2) + \frac{4}{3} \zeta(2) \log 2 - \frac{1}{3} \zeta(3) \right) \log \frac{m_{\mu}}{m_e} \right]^{(2)} \\
 &= \left(\frac{\alpha}{\pi}\right)^3 \left[0.619579 + \left(\frac{25}{18} - \frac{2}{3} \log \frac{m_{\mu}}{m_e} \right) \log \frac{\lambda}{m_{\mu}} - 0.376014 \log \frac{m_{\mu}}{m_e} \right]
 \end{aligned}$$

In Eqs. (2) we have introduced

$$\begin{aligned}
 c_4 &\equiv \frac{11}{18} \zeta^2(2) - \frac{4}{9} \zeta(2) \log^2 2 - \frac{1}{27} \log^4 2 - \frac{8}{9} a_4 = 0.83376809 \\
 a_4 &\equiv \sum_{n=1}^{\infty} \frac{1}{2^n n^4} = 0.51747906, \quad \zeta(2) = \frac{\pi^2}{6}
 \end{aligned}$$

Summing up with the results already obtained ^{1),2),3)}, all the graphs of Fig.1 give

$$\begin{aligned}
 a_e &= \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{227}{72} + \frac{31}{9} \zeta(2) - 8 \zeta(2) \log 2 - \frac{35}{8} \zeta(3) + 2 \zeta^2(2) \right] \\
 &= \left(\frac{\alpha}{\pi}\right)^3 \left[-0.15017 \right]
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 a_{\mu} &= \left(\frac{\alpha}{\pi}\right)^3 \left[\frac{115}{24} - \frac{79}{9} \zeta(2) + 10 \zeta(2) \log 2 - \frac{7}{2} \zeta(3) + 3 c_4 \right. \\
 &\quad \left. + \left(-\frac{31}{12} + \frac{10}{3} \zeta(2) - 4 \zeta(2) \log 2 + \zeta(3) \right) \log \frac{m_{\mu}}{m_e} \right] \\
 &= \left(\frac{\alpha}{\pi}\right)^3 \left[0.0487202 - 0.458889 \log \frac{m_{\mu}}{m_e} \right] = \left(\frac{\alpha}{\pi}\right)^3 \left[-2.39789 \right]
 \end{aligned} \tag{4}$$

(in all the above results for a_{μ} , terms vanishing with m_e/m_{μ} are dropped in the right-hand side).

Quite in the tradition of similar Q.E.D. calculations, the sum looks simpler than separate contributions. Agreement with previous numerical calculations ⁴⁾ is fairly good ^{5),*)}.

The method we used to derive Eqs. (1),(2) is a suitable extension of that of Ref. 3). A more complete description of the calculation will be given elsewhere.

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*) As an additional check, we have made a collapse of the electron bubble in each diagram and we found the 4th order results ⁶⁾.

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Figure Caption

Second order vacuum polarization insertions into fourth order vertex graphs.

FIG.1