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PROTON NUCLEUS SCATTERING AT MEDIUM ENERGIES

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CONTENTS

1. - INTRODUCTION	1
2. - DIRECTORY OF EXPERIMENTAL RESULTS	3
3. - THE EXPERIMENTAL PROBLEMS	4
3.1. - INTRODUCTION	4
3.2. - THE MAGNETIC SPECTROMETER	5
3.3. - KINEMATICS AND BEAM ENERGY DISPERSION	6
3.4. - ANGULAR RESOLUTION AND COUNTING RATE	6
3.5. - BLOCK DIAGRAM OF AN EXPERIMENT	7
3.6. - VARIOUS EXPERIMENTAL SET-UPS	7
4. - THEORETICAL MODELS	9
4.1. - GLAUBER THEORY	9
4.2. - OPTICAL POTENTIALS	15
5. - ELASTIC SCATTERING	17
6. - POLARISATION EXPERIMENTS AND THE EFFECTS OF SPIN DEPENDENCE	21
7. - THE EXCITATION OF DISCRETE NUCLEAR LEVELS	25
8. - SUM RULES AND CORRELATIONS	32
9. - CONCLUSIONS	34
TABLE 1	36
TABLE 2	37
TABLE 3	40
TABLE 4	41
REFERENCES	42
FIGURE CAPTIONS	47

1. - INTRODUCTION

It has long been argued (1) that probing the nucleus with high energy protons could be much more fruitful for nuclear structure research than analogous experiments at lower energies. One obvious reason is that a 500 MeV proton has a wave-length $\lambda = 0.2F$, which is much smaller than the internucleon spacing in the nucleus. The probe can therefore "see" the individual nucleons, which would not be the case at 20 MeV ($\lambda = 1F$). Of greater importance though, when a low energy proton is introduced into a nucleus there is little to distinguish it from one of the target nucleons. It participates in the many-body dynamics. On the other hand a high energy particle passing through a nucleus is on the average hardly deflected. Its momentum is much greater than the typical Fermi momenta in the nucleus, so that it will have left the nucleus before the nuclear rearrangement it has induced can take place. In the main the target nucleons are just static spectators, so that the scattering problem is in the first approximation a sequence of two-body ones.

The same type of argument can be given for electron scattering, but in that case the basic interaction with a single nucleon is well understood, and relatively weak so that Born approximation can be used as a first rough description of such data. Clearly any analysis of proton scattering results is much more model dependent since the basic force is strong and in many of its details still largely unknown ! What therefore is to be gained by using incident protons ? There are firstly a couple of practical points. Away from the ground state, nuclear energy levels tend to be clustered quite closely together, so that electron scattering with its attendant bremsstrahlung problems has some difficulty separating them in a quantitative manner. The relative weakness of the electromagnetic interaction is largely compensated by the high beam intensities available at electron machines. This is not true in the very interesting limit of high momentum transfer q , since even neglecting the form factors, the basic Rutherford cross-section falls off like q^{-4} . By comparison the nucleon-nucleon amplitude is very smooth. In many ways the information content is also different. The proton interacts with the target

neutrons and protons about equally, so that we determine matter densities rather than charge. This is just one reason why we might expect to excite different nuclear states in (p,p') reactions compared to (e,e') . The strength of the nucleon-nucleon interaction is such that the proton will interact with several nucleons on its traverse of the nucleus and it will be influenced by any correlations among the target nucleons. These cause virtual excitation of the nucleus (dispersion effects) which should be more important for protons also because of the short range of the nucleon-nucleon force. Correlations can also play a significant rôle in inelastic (p,p') cross-sections summed over all energy loss, where they influence the interference resulting from the scattering from different nucleons. Such sum rules are difficult to construct for electrons due to bremsstrahlung complications.

There have in the past been many high energy proton scattering experiments but with poor energy resolution on the final nuclear state. The modern era dates from the work of Palevsky and friends at the Cosmotron (2), who showed that it was possible to resolve distinct final nuclear levels excited by a 1 GeV proton, with counting rates sufficient to explore several orders of magnitude in variation of the differential cross-sections. This work has proved the model for other laboratories, and a substantive list of such experiments is to be found in the directory of the next section. In the subsequent section is a critical survey of the different major spectrometers from which it is possible to deduce the quality of the data we can expect over the next few years. It must be stressed that the subject is changing quite rapidly on the experimental side. The results so far available from the new Saclay spectrometer (3) confirm the broad outlines of the Cosmotron work, but differ in details which are of great theoretical interest. The Los Alamos meson facility should provide even more precise information. Advances on the theoretical side are comparatively slow, and almost all the present day interpretations are in terms of either Glauber theory (4) or the optical potential proposed by Kerman, McManus and Thaler (KMT) (5). We shall concentrate on the former

approach, referring the reader to the accompanying article by Silbar and Sternheim (6) on pion-nucleus scattering for a derivation of the optical model. All the theoretical calculations must be classed as preliminary since, unlike the pion case, the input nucleon amplitudes are very uncertain. This must hopefully change over the next few years, but at the time of writing we know very little about the spin dependence of these amplitudes.

2. - DIRECTORY OF EXPERIMENTAL RESULTS ¹

In table 1 can be found a list of proton and neutron total cross-section measurements (7-20) of various nuclei in the energy range 500 MeV to 2 GeV. Since the extrapolation in momentum transfer is smaller, the total can normally be measured more accurately than the reaction cross-section. However, for the heavier nuclei, the Coulomb distortion problem is severe in the former case when the incident particle is charged.

It is clear from looking at the list of differential cross-section and polarisation experiments (2,3,21-38) in table 2 that these fall into three basic groups. For light nuclei, elastic scattering and the corresponding polarisation has been studied in recoil and coincidence experiments. Then, up to 1965 there were many experiments with an energy resolution greater or equal to 20 MeV, where the purely elastic cross-section could not be identified. Measurements were confined to quite small angles where the elastic is a large fraction of the total cross-section. Lastly, after 1967 there have been experiments with energy resolution ranging from 3 MeV down to 100 keV, which permits an investigation of elastic and excitation cross-sections out to quite large angles. This should soon be extended to include precise polarisation data.

¹ Review of bibliography attempted up to December 1973.

Since the thorough review by Seagrave (39) in 1969, there have been several measurements of elastic proton-deuteron scattering (40-42). Furthermore the proton polarisation has been determined at two energies (24,43), and the asymmetry in the scattering from a polarised deuteron target at 600 MeV (44). Deuteron alignment effects have been found (45) by a measurement of deuteron double scattering in hydrogen at 3.6 GeV/c (equivalent to 1 GeV proton scattering).

3. - THE EXPERIMENTAL PROBLEMS

3.1. - Introduction

As can be seen from the experimental directory, we have information from total and reaction cross-sections, the angular distribution of differential cross-sections, and from polarisation experiments. Detailed descriptions (13,46) of total cross-section experiments show that the basic techniques have not changed over the past decade and we shall not review them. Up to now, all polarisation experiments performed in our energy range have involved double scattering (25,31,34), which is limiting in both quality and quantity. However, in 1974 the Saturne synchrotron in Saclay should accelerate polarised deuterons, and some time afterwards protons. The Los Alamos meson physics factory is also planning to receive a polarised proton source. Using such beams, most of the polarisation experiments will be very similar to the differential cross-section ones discussed in detail below.

In a single arm scattering experiment, the number of beam protons scattered through a given angle, but leaving the recoil nucleus in a variety of excited states, is measured. Much nuclear structure information depends upon the clean identification of this final nuclear state, and this can be done purely by an energy measurement of the outgoing proton providing that the absolute energy resolution ΔE is smaller than the separation between neighbouring energy levels. This is well known from low energy experiments, and crudely it ranges from some MeV in light or closed shell nuclei, down to some keV in heavy or

deformed nuclei. Thus for a wide range of two-body reactions of interest, ΔE must be of the order of a few tens of keV. This is very difficult if the energy E is of the order of 500 MeV, since it implies a relative energy resolution of better than 10^{-4} . Another important reason for improving the energy resolution is that the smaller ΔE is, the easier it is to detect and measure a narrow energy level situated on a background continuum of either experimental or physical origin. The major rôle of good energy resolution being shown, let us now see how we can attain it. We shall not discuss experiments where it is the recoil nucleus which is detected, since this method is restricted to certain light nuclei which are not stopped in the target (22).

3.2. - The Magnetic Spectrometer

In our energy range, the only way to measure with precision the energy of the scattered proton is to analyse its momentum P with a magnetic spectrometer. Calculations, showing the relationship between trajectories and momenta of particles in a magnetic field, may be found elsewhere (47-49) and we just outline the features necessary to yield a $\Delta E/E \sim 10^{-4}$. A magnet is characterised by its focusing and dispersive properties. Suppose that particles with momentum P elastically scatter from a target of width T at an angle $\theta \pm d\theta$, where $2d\theta$ is the angular aperture of the magnet (Figure 1). They focus at some point F with an image width $I = M_s T$, where M_s is the spectrometer magnification. Now if the particles leave the nucleus in an excited state, they will have momentum $P - dP$ and focus at F' , where $FF' = D_s dP/P$ with D_s the spectrometer dispersion. To know the excitation energy it is sufficient to measure FF' with a particle localisation system. This is only possible if the two image widths do not overlap, i.e.,

$$FF' \geq M_s T \quad \text{or} \quad \frac{dP}{P} \geq \frac{M_s T}{D_s} \quad 3.1.$$

Using relativistic kinematics, and specialising to the case of 1 GeV protons, this gives an energy resolution

$$\frac{\Delta E}{E} = \frac{3 M_s T}{2 D_s}$$

Now $M_s T$ can be made of the order of 1 mm, the limitation being the precision of the particle-localisation system and the slit scattering problems. The desired energy resolution of 10^{-4} can be attained if D_s is of the order of 15 cm/%, and this is a value typical of the experimental set-ups described in table 3. This energy resolution can only be used fully if the very important aberrations appearing in magnets are cancelled. Great improvements have been made on the basis of calculations, either using a matrix formulation starting from theoretical fields (50), or using point by point calculations of the trajectories starting from actual fields (51-53).

3.3. - Kinematics and Beam Energy Dispersion

In a nuclear reaction the momentum of the scattered proton does not depend just on the excitation energy, but also on the scattering angle θ , and the variation of P over the aperture $2d\theta$ could spoil the final ΔE . For a 1 GeV proton scattered through 20° with $d\theta = 1^\circ$, the energy spread is 3 MeV for a carbon target but 0.18 for lead. Fortunately this effect can be compensated (54-57) by a shift of the focal plane. Another important limitation could come from the energy dispersion of the incident beam, which may spread ΔE by MeV, depending upon the accelerator. Cohen (58) showed that this may be cancelled if the incident beam is magnetically analysed and satisfies some dispersion condition at the target. This is known as the energy loss mode. Other refinements are also important (48,49).

3.4. - Angular Resolution and Counting Rate

Given a sufficient energy resolution, it is then necessary to look at two other parameters, the angular resolution (AR) and the counting rate. Differential cross-sections vary so rapidly with θ in the intermediate energy region that AR has to be of the order of $\pm 0.1^\circ$. Uncertainties in both the incoming and outgoing beam contribute to AR.

A high precision experiment requires a reasonable counting rate. The differential cross-section for a given transition is $(d\sigma/d\Omega) = (N/nQd\Omega)$ where N is the number of counts recorded per second, n the number of target nuclei per square centimetre, Q the number of incident protons per second and $d\Omega$ the solid angle of the spectrometer. To get an idea of the relative strength of different experimental arrangements, we estimate the cross-section $(d\sigma/d\Omega)_T$ required to give a counting rate of 1 event/second for a target density of $n = 6 \times 10^{21}$ nuclei/cm² (e.g., 120 mg/cm² of ¹²C). The results are to be found in table 3.

3.5. - Block Diagram of an Experiment

In the experimental block diagram of Figure 2, there are two parts of equal importance, the analysing system before the target, and the spectrometer after. The incident beam enters an object block, defining a width compatible with the restriction 3.1, after which the analyser magnet introduces some dispersion. The tuning block furnishes the right focusing and dispersion properties at the target to compensate the kinematic effects and the beam energy dispersion. Q is determined by a beam measurer, and the target is viewed by a monitor. The scattering angle is varied by rotating the spectrometer arm. The position of the particles is measured by the localisation block, set in the focal plane of the magnet and from this the excitation energy can be derived, using as calibration the energies of well-known excited states.

3.6. - Various Experimental Set-Ups

The parameters characterising the major experimental set-ups can be found in table 3.

In the Brookhaven experiment (59), stopped by the shut-down of the Cosmotron in 1967, an energy resolution of 3-2 MeV was achieved for 1 GeV protons. The arrangement differs from that of Figure 2, in that the scattering angle was varied by adjusting the analysing magnets. Also the energy loss mode was not used, but a correction was made for

the time variation of the incident beam. Particle localisation was by means of digital wire spark chambers sited before and after the spectrometer magnets, which allowed the 1° full horizontal aperture to be divided into three or six adjacent angular bins. Absolute cross-sections were obtained by measuring the integrated Q using the $^{12}\text{C}(p,pn)^{11}\text{C}$ reaction. This is known to $\pm 5\%$ (60), and the final quoted error is $\pm 13\%$.

The Leningrad (Gatchina) (37) 1 GeV proton synchrocyclotron also uses the Brookhaven method of compensating for the time variation, and in fact the over-all energy resolution is comparable with that obtained at the Cosmotron. The horizontal aperture is sufficiently small (0.4°) that the kinematic effects are negligible. Particles are detected at only one point of the focal plane at any time, using photoscintillators in coincidence. A complete excitation spectrum therefore requires that the measurement be repeated 10 or 20 times, depending upon the required energy band-width, and this accounts for a factor of 15 in $(d\sigma/d\Omega)_T$ in table 3. The normalisation is done relative to hydrogen, which is measured to $\pm 10\%$ (14,61,62). The final error is expected to be of this magnitude.

Saturne, like the Cosmotron, is a variable energy machine (0.5-3 GeV), and using the SPES1 facility a ΔE between 100 and 250 keV is realised at 1 GeV (63), as can be seen from the spectra of Figures 3 and 4. Optical aberrations of the magnet have been made negligible in the 100 keV limit by simple shimming, and the 500 keV spread of the incident beam compensated by energy loss. To save running time, the values of seven magnet parameters are calculated by computer before the experiment (64) for each nucleus and angle, and the subsequent tuning is done automatically. Particles are localised on the focal plane by six drift counters (65), each 50 cm long, which enable a 50 MeV band of excitation energies to be studied. Although the horizontal aperture is 2.8° , the exit angle is measured and binned in 0.05° intervals. These are later summed into angular slices corresponding to the experimental angular resolution. The

over-all normalisation, including uncertainty in the carbon activation measurement, is known to $\pm 7\%$. In the preliminary results so far obtained, there is a possible displacement of the scattering angle by up to 0.3° .

The Los Alamos experiment planned to start in 1974 will use the 800 MeV H^- beam of LAMPF, and should have an energy resolution at least twice as good as Saclay. The lay-out of the high resolution spectrometer (HRS) (66) differs from Figure 2 since the analysis and spectrometer systems are in the vertical plane, while the scattering takes place in the horizontal, the VHV configuration. (Actually the analyser is horizontal, but a steering system rotates the dispersed beam from the vertical.) This allows the energy loss mode to be used with a good angular resolution. The counting rate has been estimated using a flux Q of 1-10 μA , an upper limit being probably set by the background problems.

4. - THEORETICAL MODELS

In contrast to the electron case, Born approximation is not even qualitatively correct for proton scattering. Multiple scatterings are introduced usually in one or two different ways.

4.1. - Glauber Theory

The Glauber approach has been discussed in innumerable articles (67-70), but none better than the original Boulder notes (4), where almost all of the later refinements are foreshadowed. There are two basic approximations. At high energies the proton traverses the nucleus in such a short time that the nucleons cannot rearrange themselves until the probe has left. It is then reasonable to calculate the transition amplitude $F(q, r_1, \dots, r_A)$ as if the A target nucleons were "frozen" at the co-ordinates $r_1, \dots, r_A$. The nuclear physics only enters when the physical amplitude for elastic scattering or nuclear excitation is calculated by averaging this over the nucleon distribution using the nuclear wave function

$$F_{fi}(q) = \langle f | F(q, \underline{s}_1, \dots, \underline{s}_A) | i \rangle \quad 4.1.$$

This statement may be put on a more rigorous footing, assuming that the proton transfers little energy to the nucleons, by means of closure (71). It is basic to many models, because it reduces the problem to the solution of a complex series of one-body problems for F , and the main differences among the models lies in how this one-body problem is solved. In the Glauber model use is made of the impact parameter description of the scattering amplitude

$$F(q) = \frac{ik}{2\pi} \int d^2 \underline{b} e^{i \underline{q} \cdot \underline{b}} \Gamma(\underline{b}) \quad 4.2.$$

Here the profile function $\Gamma(\underline{b}) = 1 - \exp[-i\chi(\underline{b})]$ with χ the phase function. For a spherically symmetric system (zero spin), there is no dependence on the angular variable, so that

$$F(q) = ik \int b db J_0(qb) \Gamma(b) \quad 4.3.$$

The dynamical approximation of the model is that the phase function for a system of scatterers is equal to the sum of the phases from the individual scatterers.

$$\chi_A(\underline{b}) = \sum_j \chi_j(\underline{b} - \underline{s}_j) \quad 4.4.$$

Here we have interpreted the impact parameter $\underline{b}$ very geometrically, and also assumed that the proton trajectory is an essential straight (72), so that the impact parameter with respect to an individual nucleon is $\underline{b} - \underline{s}_j$ where $\underline{s}_j$ are the nucleon co-ordinates perpendicular to the average beam direction. This additivity would follow if the eikonal approximation were valid for all the scatterings near the forward direction from a set of local potentials. Of itself this

is not sufficient to justify the use of the theory for 1 GeV protons, but almost certainly many of the corrections vanish for small q when the averaging of Equation 4.1 is carried out. From the above it follows that the profile function for the transition between nuclear states i and f is

$$\Gamma_{fi}(\underline{b}) = \langle f | 1 - \prod_{j=1}^A (1 - \Gamma_j(\underline{b} - \underline{s}_j)) | i \rangle \quad 4.5.$$

Since the $\Gamma_j(\underline{b})$ can be obtained from the nucleon amplitudes by inverting Equation 4.2, the transition amplitude may be calculated with no free parameters. If the A fold product is expanded in powers of the individual amplitudes, then we generate a multiple scattering series where no nucleon is multiply struck. For deuterium and perhaps up to ${}^4\text{He}$ the many-body expectation value can be evaluated in some detail : for heavier nuclei a simpler approach is desirable. If the target nucleons were completely independent, then the multi-dimensional integral would factorise, so that for elastic scattering

$$\Gamma_{oo}(\underline{b}) = 1 - (1 - \langle 0 | \Gamma(\underline{b} - \underline{s}) | 0 \rangle)^A \quad 4.6.$$

In the case of equal proton and neutron densities, the required one-body matrix element can be found from the bare electron scattering form factor ² $S(\underline{q}) = \langle 0 | e^{i\underline{q} \cdot \underline{r}} | 0 \rangle$, and the nucleon amplitude $f(\underline{q})$.

$$\overline{\Gamma}(\underline{b}) \equiv \langle 0 | \Gamma(\underline{b} - \underline{s}) | 0 \rangle = \frac{1}{2\pi i k} \int e^{-i\underline{q} \cdot \underline{b}} S(\underline{q}) f(\underline{q}) d^2 \underline{q} \quad 4.7.$$

² A useful survey of the results of elastic and inelastic electron scattering from nuclei can be found in the books of Überall (72).

In the optical limit (73) the granularity due to the presence of distinct nucleons in the nucleus is neglected. Since $\bar{F} = O(A^{-2/3})$, exponentiating the formula 4.6, it is a reasonable approximation to take the nuclear phase shift as

$$\chi_1(b) = i A \bar{F}(b) \quad 4.8.$$

This is in fact the eikonal solution to a potential, the one-body optical potential ³

$$2m V(\underline{r}) = - \frac{1}{2\pi^2} \int e^{-i \underline{q} \cdot \underline{r}} S(\underline{q}) F(\underline{q}) d^3 q \quad 4.9.$$

In the iteration of this potential by solving the Schrödinger equation exactly, or in the eikonal limit, there is only the intermediate nuclear ground state between scatterings. One cannot excite the nucleus by hitting one nucleon and de-excite it by hitting a second if the two nucleons are uncorrelated. Since for an isoscalar nucleus the expression for $\chi_1(b)$ involves no free parameters, any significant deviations between theory and experiment could be a measure of the next order terms which depend upon the nuclear correlations. The correction to order f^2 is

$$\chi_2 = \frac{-i}{2A} \chi_1^2 + \frac{A(A-1)}{2(2\pi k)^2} \int e^{i(\underline{q}_1 + \underline{q}_2) \cdot \underline{b}} f(\underline{q}_1) f(\underline{q}_2) C(\underline{q}_1, \underline{q}_2) d^2 q_1 d^2 q_2 \quad 4.10$$

where the two-body correlation is defined by

³ This expression is written non-relativistically, but it is easy to introduce relativistic kinematics. Then the mass factor on the left-hand side is replaced by the total energy.

$$C(q_1, q_2) \equiv \frac{1}{A(A-1)} \langle 0 | \sum_{i \neq j} e^{i(q_1 \cdot \underline{r}_i + q_2 \cdot \underline{r}_j)} | 0 \rangle - S(q_1) S(q_2) \quad 4.11.$$

There are many possible contributions to the correlation function, and to estimate them it is of course necessary to have some kind of nuclear model. For a light nucleus the centre-of-mass (c.m.) constraint is very important. Thus the nuclear averaging in Equation 4.1 should really be over the $A-1$ internal nuclear co-ordinates, whereas we had assumed A independent particles previously

$$F_{fi}(q) = \int d^3 \underline{r}_1 \dots d^3 \underline{r}_A \delta^{(3)}(\underline{r}_1 + \underline{r}_2 + \dots + \underline{r}_A) \rho_{fi}(\underline{r}_1, \dots, \underline{r}_A) \times F(q, \underline{r}_1, \dots, \underline{r}_A) \quad 4.12.$$

For a harmonic oscillator shell model configuration of lowest energy one can neglect the delta function and integrate over all these A variables providing the densities are correctly renormalised and the resultant amplitude is multiplied by $\exp(q^2/4A\alpha^2)$ where α is the harmonic oscillator parameter (74). This factor may be cast in terms of a contribution to C , but because the constraint leads to significant three and higher body correlations it is best left as a factor. In view of the large momentum transfers being presently investigated where this factor is sizeable, it would be useful to improve upon the simple result, but except for the lightest nuclei this has not been done. Because of the Pauli principle the nuclear wave functions should be taken as Slater determinants rather than a simple product, and when this is done the $(1-\bar{\Gamma})^A$ term of Equation 4.6 is replaced by an $A \times A$ determinant, or sum of such terms (68). It is easy then to obtain the contribution to χ_2 providing the shell model configurations are specified. Thus for $^{16}_0$ the extra terms represent the transitions, say, between $|p_{22}^{33}\rangle$ and $|p_{22}^{11}\rangle$ and back again, which have been included in Equation 4.6, but are forbidden by the Pauli principle and must be subtracted.

There are, of course, more dynamical correlations such as short range repulsion between target nucleons, but it is likely that long range deformation is more important. The most studied case is that of ^{12}C which has an oblate deformation. Estimates of the resulting two-particle correlation can be made in several ways. One can contract the z axis of the wave function with some phenomenological parameter β (75,76), or take Hartree-Fock on other nuclear wave functions which contain such an asymmetry naturally (77). An α cluster model is deformed (78-80). An elegant formalism has been derived treating the nucleus as a deformed liquid drop, determining the parameters so as to reproduce the excitation spectrum at least crudely (81,82). The different nuclear models give qualitatively similar results in that they reflect the effects of having a nuclear rotational band as intermediate states.

So far the f 's have been taken as scalars, but Glauber theory can handle the spin and isospin degrees of freedom. These do cause serious complications because the ordering of the interactions along the trajectory is then important. For a closed shell nucleus neither spin nor isospin flip of a target nucleon is allowed in lowest order, while the double scattering depends critically upon correlations. The simplest example of such effects is the charge exchange (c.e.) contribution to p - d elastic scattering, where the normal Glauber amplitude $f_p f_n$ is replaced by $f_p f_n - \frac{1}{2}(f^{c.e.})^2$. The $\frac{1}{2}$ comes from the ordering of the interactions, and the minus sign from the interchange of the target neutron and proton (83). Similar algebra can be carried out up to ^4He , after which a kind of α cluster model can be employed (84,85), where one only considers spin and isospin flip between states of identical spatial wave function. In the double charge exchange the intermediate nucleon is a neutron. It has been suggested that at high energies other inelastic states such as the nucleon isobars might also participate (86-88). In our energy region there are two features which diminish such effects. To excite a state of mass m^* there is a minimum (longitudinal) momentum transfer $q_z = (m^{*2} - m^2)/2k$. Since this must occur with opposite signs to two of the target nucleons, it tends

to lead to break-up of the nucleus. Secondly, the only resonance we need to consider has isospin $\frac{3}{2}$, and this can only participate via isospin flip in the nucleus, i.e., through correlations. For proton scattering, there has not been much work on implementing the full Glauber approach with spin and isospin complications, mainly due to uncertainty in the nucleon amplitudes. Projectile spin flip through a spin orbit force can take place in first order (4), but this will be discussed later.

For heavy nuclei the electromagnetic interaction is very important, and the simplest method (70) for including it is to add the eikonal Coulomb phase χ_c to the nuclear phase χ . This leads to an elastic amplitude

$$F(q) = f_{pt}(q) + iR \int J_0(qb) b db [e^{i\chi_{pt}} - e^{i(\chi + \chi_c)}] \quad 4.13.$$

where $f_{pt}(q)$ is the pure point Coulomb amplitude, and χ_{pt} the corresponding phase.

4.2. - Optical Potentials

The KMT form of the optical potential (5,89-91) differs from the one just derived in its treatment of the $1/A$ corrections. We can rearrange the no-correlation Glauber amplitude (Equations 4.8 and 4.3) in the form

$$F(q) = \frac{iR A}{A-1} \int J_0(qb) b db [1 - e^{i(A-1)\bar{F}(b)}] \quad 4.14.$$

such that the first term in Equation 4.10 for χ_2 vanishes, at the expense of introducing extra A dependent terms in the triple scattering χ_3 . The KMT prescription is therefore to replace the A in the one-body optical potential (Equation 4.8) by $A-1$, solve the wave equation and multiply the resultant amplitude by $A/(A-1)$.

To include some spin effects we first decompose the nucleon amplitudes à la Wolfenstein (92)

$$f(q) = A + B \underline{\sigma}_p \cdot \underline{\sigma} + C (\underline{\sigma}_p + \underline{\sigma}) \cdot \hat{n} + D (\underline{\sigma}_p \cdot \hat{K})(\underline{\sigma} \cdot \hat{K}) + E (\underline{\sigma}_p \cdot \hat{q})(\underline{\sigma} \cdot \hat{q}) \quad 4.15.$$

where $\underline{K}$ is the mean proton momentum and $\underline{n}$ the normal to the scattering plane. The σ_p acts on the projectile spin, and σ on the target nucleons. The amplitudes still have to be decomposed in isospin. Note also that we have to interpret these quantities in the nuclear c.m. frame (5,91), which for a heavy nucleus is very close to that of the laboratory. A check on the transformation is that for a very diffuse system (e.g., a hydrogen molecule), for which the scattering amplitude can be calculated in Born approximation, the total cross-section should be A times the nucleon value. For a spin zero nucleus only the amplitudes A and C enter the one-body optical potential

$$2mV = -\frac{(A-1)}{2\pi^2} \int e^{-i\underline{q} \cdot \underline{z}} [A(q) + \underline{\sigma}_p \cdot \hat{n} C(q)] S(q) d^3q \quad 4.16.$$

giving a central and spin-orbit term. It has been hoped that an exact resolution of a wave equation with this potential might get away from the restriction to small angles present in the eikonal approximation. It must be remarked though that for 1 GeV proton scattering from any except the lightest nuclei, it is only when one looks at fine detail or at large momentum transfer that there is much difference between an exact resolution and the eikonal approximation. As in the Glauber scheme, correlations imply second order terms in the potential (89-91), and it is standard practise to include the c.m. constraint in this way rather than as an approximate multiplicative factor. The Coulomb potential may be added directly to the optical potential before solving, although there are logical problems with the $1/A$ corrections here.

5. - ELASTIC SCATTERING

The simplest description of the small angle spin independent proton-proton (p) and proton-neutron (n) amplitudes is in terms of a total cross-section σ , relative real part ρ and slope β ,

$$F(q) = i \frac{k \sigma_{(1-i\rho)}}{4\pi} \exp(-\frac{1}{2} \beta^2 q^2) \quad 5.1.$$

Both σ_p and σ_n are well determined, as is ρ_p , but the values of ρ_n are very uncertain. β_p can be found from the p-p differential cross-section if we neglect spin, and this value can be used to describe the rather meagre n-p data as well. It is of course to be hoped that in the near future phase shift analysis will give us more information.

For the lightest nuclei the Glauber multiple scattering series may be evaluated term by term. Thus if we take as a model for ^4He four particles in a 1s state of a harmonic oscillator, then the explicit form for the series is (93)

$$F(q) = i k \exp(q^2/16\alpha^2) \left(\frac{1+2\alpha^2\beta^2}{2\alpha^2} \right) \sum_{m=1}^4 \binom{4}{m} \times \frac{(-1)^{m+1}}{m} \left[\frac{\sigma \alpha^2 (1-i\rho)}{2\pi (1+2\alpha^2\beta^2)} \right]^m \exp \left[- \left(\frac{1+2\alpha^2\beta^2}{4m\alpha^2} \right) q^2 \right] \quad 5.2.$$

The spring constant determined from low q^2 electron scattering data (94) after correcting for the finite proton size and the c.m. motion, is $\alpha^2 = 0.535 \text{ F}^{-2}$. Despite the simplifications this formula has most of the features of more complicated models. It is essentially an expansion in $(\sigma \alpha^2 / 2\pi)$ which is the ratio of the nucleon and nuclear sizes ($\sim 1/4$ for 1 GeV). Near the forward direction the single scattering term dominates, but it falls off faster with q^2 than the double. From some angle therefore the double takes over leadership until it is overtaken by the triple. The pattern to be observed in

the $p\text{-}^4\text{He}$ differential cross-section depends sensitively upon relative phase of single and double scattering, i.e., on the value of ρ . In our energy range $\bar{\rho}$ is negative and typically about 0.2-0.3 in magnitude, so that the interference is dominantly destructive, and a series of minima is predicted. This is illustrated in Figure 5 and compared with the Cosmotron (2) and Saclay (26) measurements. Despite the unknown absolute normalisation on the latter, it is hard to explain the discrepancies between the data. The minimum found at the Cosmotron is defined by three points corresponding to one measurement which was taken at the end of the run. On the other hand a very weak minimum is hard to explain theoretically. It would, for example, require an unreasonably large variation of ρ with angle, or alternatively a very strong spin dependence. It is claimed that an exact solution for an optical potential has a weaker minimum (95), but this paper is misleading in other respects, and deep minima have been found by other authors (91). It should be noted that at 587 MeV (24) the observed peak to valley ratio is at least 1.7 compared to the Saclay 1.15. Many papers have been written on the interpretation of the Cosmotron ^4He data (91,96-99). For $q^2 \sim 10 \text{ F}^{-2}$ the nuclear form factor has a zero (94), so that a Gaussian parametrisation is inadequate. One could assume that the deviation is due to short range dynamic correlations and introduce a Jastrow factor in the wave function, or suppose it is the single particle density at fault. Because of the c.m. constraint these possibilities are not completely independent. The calculations are now much more complicated, but in either case (99) if the parameters are chosen so as to fit the electron data, the height of the subsidiary maximum shown in Figure 5 is increased towards the experimental values. The main reason for this, however, is that the single scattering, which is destructive in this region, is reduced, and this can be stated independent of any nuclear model. A number of respectable dynamic correlations has been investigated using the optical potential approach (91), and the qualitative behaviour is rather similar to the Glauber calculations.

For medium-size nuclei the optical limit of Glauber theory provides a very satisfactory description. For the case of ^{58}Ni both theoretical and experimental minima (3,100,101) are quite pronounced. For heavy nuclei they are filled in considerably due to the distortion in the Coulomb field (102), and this is shown in the case of ^{208}Pb in Figure 6. The effect depends critically upon the sign of the real part, so that this experiment demonstrates the ρ is in fact negative. The positions of the minima are in good agreement, but it must be stressed that at present there is an uncertainty in the absolute zero of the scattering angle of $\pm 0.3^\circ$ connected in part with the settling down of spectrometer supports (3). It is tempting to ascribe the deviations in the height of the maxima to differences between neutron and proton distributions in the nucleus, but in the opposite way to that required to explain (103,104) the poor resolution experiment at 20 GeV. More definite conclusions cannot be drawn with the present N-N input.

An interesting experiment which investigates the density change between adjacent nuclei has been carried out in Leningrad (38). In Figure 7 is plotted the difference in the elastic differential cross-sections for ^{39}K and ^{40}Ca . It seems that the difference between the nuclear radii is not as large as that used in the Glauber prediction illustrated (38).

One of the features of the Cosmotron data (2) was the comparison of the scattering from ^{16}O and ^{12}C . The over-all agreement with theory was very good in the former case, but for the latter the theoretical subsidiary maximum was too high (68). Part of this is probably experimental since the new Leningrad and Saclay measurements are about 10% higher here. One could hope to explain the rest as being due to the deformation in ^{12}C . In Figure 8 are plotted the ensemble of ^{12}C results around 1 GeV. For the purposes of display they are multiplied by a factor $\exp(q^2)$, but it must be remembered that this converts angular uncertainty into an error in the ordinate. Note especially in this respect the possible systematic 0.3° error in the Saclay data. The optical limit of Glauber theory,

where only the c.m. constraint is kept, provides a reasonable description out to $q^2 \sim 5 \text{ F}^{-2}$. This is a little fortuitous since the Pauli correlations in second order increase the subsidiary maximum by about 14%. Deformation reduces this, but the amount depends on the nuclear model. Using Hartree-Fock wave functions (D. Vautherin, private communication) to calculate the two-particle correlation function, it is 10%, while in the collective model of Friar (82) it is about twice this. There is a greater sensitivity to correlations in the region of the next maximum, but at that stage one may have to worry about three-particle correlations. More importantly one will first have to explain such obvious discrepancies as the depth of the minima.

Only virtual deformations are allowed for ^{12}C , but for a nucleus with spin it can be real. Thus the deuteron D state makes this nucleus cigar shaped, with a principal axis depending on the spin orientation. There are now three independent scattering amplitudes (neglecting nucleon spin-flip) with differing effective radii, and since the position of an interference minimum depends sensitively on this quantity, we can expect the minima to be at different angles, as first remarked by Coleman and Rhoades (unpublished). Normally only the incoherent sum of the three cross-sections is determined, and this should show no minimum even in the absence of nucleon spin flip (105,106). This is very similar to the result for e-d scattering, where the minimum in the spherical form factor at $q^2 \sim 20\text{F}^{-2}$ is filled in by the quadrupole scattering (107). In this way we are able to explain the existence of a shoulder rather than a dip in the elastic p-d data (39). These also show a sharp backward peak due to a pick-up mechanism, but this almost certainly involves mesonic degrees of freedom in one form or another (108,109). Preliminary data on ^6Li also show only a shoulder (28), but here spin flip may have to be invoked because the quadrupole moment is quite small.

A total cross-section provides a measurement of the imaginary part of the forward elastic amplitude. If this is known as a function of energy the real part of the forward amplitude can be calculated from a dispersion relation (110), which can in turn provide a check on nuclear parameters providing the nucleon real parts are better determined. Neutron cross-sections at higher energies have been used to determine matter radii, which seem to be rather similar to the charge radii, but it must be remembered that only one number is thus extracted (111).

6. - POLARISATION EXPERIMENTS AND THE EFFECTS OF SPIN DEPENDENCE

For nuclei of non-zero spin it is possible to study quantities which depend upon the spin orientation, but except in special cases such experiments are difficult. It is much easier to look at the proton polarisation in nuclear scattering by using a polarised beam. It has been known for many years that in poor resolution scattering, say, from ^{12}C there is appreciable polarisation (30). One of the few experiments carried out in our energy range for elastic scattering (24) involved 544 MeV protons incident on ^4He . For the scattering from a spin zero system, the amplitude has two terms

$$\hat{f}(q) = f(q) + \sigma \cdot (\hat{k} \wedge \hat{q}) g(q) \quad 6.1.$$

and in impulse approximation f and g are related to two of the amplitudes of Equation 4.14.

$$\begin{aligned} f_I(q) &= A \times A(q) S(q) \\ g_I(q) &= A \times C(q) S(q) \end{aligned} \quad 6.2.$$

The single scattering amplitudes f_I and g_I may be used as driving terms in coupled (spin) channel Schrödinger equations to calculate such observables as the polarisation $P = \text{Re}(f^*g)$. For simplicity we shall here work in the eikonal limit, using the formalism described in the Glauber lecture notes (4,112). Define profile functions

$$\begin{aligned}\Gamma_c(b) &= \frac{1}{ik} \int J_0(qb) q dq f_I(q) \\ \Gamma_s(b) &= -\frac{1}{kb} \int J_1(qb) q dq g_I(q)\end{aligned}\quad 6.3.$$

Then the total spin dependent profile function for the nucleus is

$$\Gamma(b) = 1 - [1 - (\Gamma_c + \sigma \cdot (\hat{k} \wedge \hat{b}) \Gamma_s)]^A \quad 6.4.$$

Assuming the spin dependence to be weak, the primary effect of the spin terms will be to fill in the diffraction minima and induce polarisation. To lowest order in Γ_s , the spin non-flip amplitude is unchanged from the standard result of Equation 4.6, while the spin-flip becomes

$$g(q) = -kA \int J_1(qb) \Gamma_s(b) [1 - \Gamma_c(b)]^{A-1} b^2 db \quad 6.5.$$

It will be seen in a later section that this has exactly the form of a DWIA amplitude for an inelastic magnetic transition. We have relatively little information about $C(q)$, but if we take it as being proportional to $A(q)$ apart from the necessary kinematic factor, $C(q) = RqA(q)$, then it is easily shown by partial integration that the nuclear amplitudes should also be proportional, $g(q) = Rqf(q)$. Thus the polarisation would be just the same as for p-p, assuming that only A and C contribute. This, however, varies roughly like

$\sin(2\theta^*)$ (113). For P to show sharp structure as can be seen in Figure 9, the slope parameters of C and A must be quite different. Neglecting for the moment the real parts of the amplitudes, if C is much steeper than A , g will pass through zero for smaller angles than f , P will have two zeros close together, and the general shape of Figure 9 will be reproduced. Kujawski (114) obtained such a shape at 633 MeV, but his calculations showed a wide variation of quantitative behaviour due to the ambiguities in the $N-N$ phase shifts above 400 MeV. As a simple model, suppose that all the $p-p$ polarisation is due to a $C-A$ interference, and that the phases of C and A are the same, i.e., P is maximal. In this case P determines the magnitude and slope of C , and indeed at 544 MeV it is much steeper than A (113). In Figure 9 are shown the predictions for P on the basis of Equations 6.5 and 6.3, using the trivial density $\sum_i a_i \exp(-\alpha_i^2 \sum_j r_j^2)$, which can take the c.m. constraint into account arbitrarily well. The shape and rough magnitudes are quite encouraging. The spin flip amplitude of course also fills in the minimum in the cross-section somewhat, but because of our assumption of maximal P , this is only a lower bound.

The $p-p$ polarisation reaches a maximum at around 700 MeV (113), and already at 1 GeV the predicted repletion of the minimum is much less. Now at 1 GeV the diffraction minima seem experimentally to become deeper and deeper with increasing nuclear mass, and this is not well reproduced by the simple Glauber model. Exact resolution of the wave equation with a one-body optical potential does lead to such a feature. For large A the eikonal approximation is good, while for light nuclei it may have errors near the minima. On the other hand, a spin-dependent term would also give a similar effect, since for a large nucleus the slopes of the nucleon amplitudes play minor rôles. Note that P contains terms linear in the spin flip amplitudes, but the first contribution to σ is quadratic. There are many other quadratic terms that we have so far neglected. Apart from the continued expansion of Equation 6.4, C can induce double spin flip of the target nucleons. This is a correlation effect, but in the doubly closed shell of ^4He the spin correlations are very strong. The same

is true for any of the other spin dependent amplitudes of Equation 4.14. Perhaps a careful study of the polarisation and the A and R parameters for, say, ^4He would give a lot of useful N-N information which we can re-use, but this will have to be supplemented by more direct N-N information at 1 GeV.

Polarisation in elastic p-d scattering has been measured at 544 (24) and 425 MeV (43), and the general structure is somewhat similar to that of the ^4He data. The situation is now much more complicated because of the deuteron spin, but this allows experiments such as the measurement of the asymmetry when using a polarised deuteron target (one measurement at 584 MeV) (44). Only qualitative agreement with both types of data has been obtained using Glauber theory (115). Even neglecting the N-N spin dependence, there can still be spin flip in p-d scattering due to the quadrupole moment form factor, and as we previously remarked, this fills in the minimum in p-d scattering in much the same way as it does for e-d. However, if we scatter from a deuteron target aligned (with $m = \pm 1$ states being distinguished from the $m = 0$) in a suitable way, we could expect to find the dip again. This was the basis of an ingenious experiment suggested by Harrington (116). Take an unpolarised deuteron beam and scatter it from a hydrogen target. In the region of the putative minimum ($q^2 \sim 7F^{-2}$), the deuteron should emerge in a state of almost pure $m = 0$ in the direction of $\underline{q}$. This can be tested by scattering these deuterons from a second hydrogen target and measuring the dependence on φ , the angle between the momentum transfers in the two experiments. Thus for $\varphi = 90^\circ$, and $q^2 \sim 7F^{-2}$ there should be almost complete extinction. An experiment has been carried out with 3.6 GeV/c deuterons (45). The analysis is somewhat complicated by the nucleon spin flip amplitudes, and to extract the data in Reference (45), the authors used a $C(q)$ proportional to $qA(q)$, which we have seen has no influence near the minimum. The maximum value of the alignment obtained was 0.7 which leads to an estimate of a D state probability of $6.5 \pm 2.0\%$, but a relative real part $\rho = 0.65 \pm 0.15$. This large value for ρ means that the minimum was never as deep as one would have guessed. But we have found this

with all the light nuclei, and may be due to the oversimplistic spin dependence assumed. A secondary result of this experiment was the deuteron vector polarisation, but with large error bars.

7. - THE EXCITATION OF DISCRETE NUCLEAR LEVELS

Since medium energy proton scattering is not seriously limited by the bremsstrahlung problem, it is possible to measure the excitation of nuclear levels to larger momentum transfer with greater accuracy than that obtained in electron scattering at a comparable energy. Of course the disadvantage is that the interpretation of electron data, at least for light nuclei, is almost model independent, whereas for proton scattering we still do not know how reliable the models are.

Consider the ^{12}C excitation spectrum shown in Figure 4. The only level clearly distinguished in the Brookhaven experiment (2) was the first excited state at 4.4 MeV ($J^P = 2^+, T=0$), the next two being blurred together because of the limited energy resolution. The situation is somewhat similar with the Leningrad data (37). The Saclay group (3) have determined angular distributions for the first three states out to $q^2 \sim 14\text{F}^{-2}$, and it is only a question of time before more is extracted from the type of spectra shown in Figure 4.

It was claimed in Section 4 that Glauber theory should be as applicable to inelastic scattering as to elastic, but to evaluate the theory fully one needs some microscopic model for the transition. The simplest description of the 4.4 MeV state in the shell model would be as a $(p_{\frac{3}{2}})^{-1}(p_{\frac{1}{2}})$ particle hole. However, such a description underestimates the electromagnetic transition rates by a factor of four. A Glauber calculation of the excitation of this level with 1 GeV protons found a cross-section too low by a rather similar factor (68). At this level the two probes are telling us the same thing ; a simple particle-hole cannot account for this state which is strongly collective in character. To be able to extract more quantitative results, we need a more phenomenological way of using the theory, and this we can derive providing we neglect all correlations (117).

The profile function corresponding to the excitation of a state n is

$$\Gamma_n(\underline{b}) = - \langle n | \Pi_j (1 - \Gamma_j(\underline{b} - \underline{s}_j)) | 0 \rangle \quad 7.1.$$

As for elastic scattering, the initial wave function can be taken as a product of similar factors, but for the final state one of the components must correspond to the excited state so that

$$\Gamma_n(\underline{b}) = \int \psi_n^*(\underline{r}) \Gamma(\underline{b} - \underline{s}) \psi_0(\underline{r}) \mathcal{D}_{A-1}(\underline{b}) \quad 7.2.$$

where $\mathcal{D}_A(\underline{b}) = (1 - \langle \Gamma \rangle_0)^A$ is just a kind of damping factor. This leads to a scattering amplitude

$$F_n^{lm}(\underline{q}) = \frac{i \underline{k}}{2\pi} \int e^{i \underline{q} \cdot \underline{b}} \rho_n(\underline{r}) Y_l^{m*}(\hat{\underline{r}}) \mathcal{D}_{A-1}(\underline{b}) \Gamma(\underline{b} - \underline{s}) d^3 \underline{b} d^3 \underline{r} \quad 7.3.$$

where we have introduced a transition density $\rho_n(\underline{r})$ and a spherical harmonic Y_l^m to describe the excitation of a state of angular momentum l and polarisation m : $\rho_n Y \equiv \psi_n^* \psi_0$. If we can neglect the finite size of the proton, then $\Gamma(\underline{b})$ is proportional to a delta function, and the above reduces to

$$F_n^{lm}(\underline{q}) = F(0) \int e^{i \underline{q} \cdot \underline{b}} \rho_n(\underline{r}) Y_l^{m*}(\hat{\underline{r}}) \mathcal{D}_{A-1}(\underline{b}) d^3 \underline{r} \quad 7.4.$$

The same transition density is measured in electron scattering. Neglecting the multiple scatterings ($D=1$), and quantising along the q direction, only $m=0$ states are excited, and the result is proportional to the transition form factor

$$F_n(q) = \int e^{i\vec{q} \cdot \vec{r}} \rho_n(r) \gamma_e^m(\hat{\vec{r}}) d^3r \quad 7.5.$$

At this point we can neglect the restriction to single particle transitions inherent in the derivation, and use the formulae with phenomenological form factors (transition densities). Before we do this we must clarify some of the approximations. In general we cannot neglect the nucleon size, but we can interpret the ρ_n of Equation 7.4. as effective densities $\tilde{\rho}_n$ in which the "size" of the nucleon (as measured for example by the slope parameter of Equation 5.1) is folded in.

Explicitly,

$$f(0) \tilde{\rho}_n(r) \gamma_e^m(\hat{\vec{r}}) = \frac{i\hbar}{2\pi} \int_{z=z'} d^3s' \rho_n(\vec{r}') \gamma_e^m(\hat{\vec{r}}') \Gamma(\vec{s}-\vec{s}') \quad 7.6.$$

In contrast to electron scattering, the incident proton interacts almost as strongly with the neutrons as the protons. Thus for an isoscalar ground and a $T=0$ excited state, the $F(0)$ of Equation 7.4 should be interpreted as $(f_p + f_n)$ but for a final $T=1$ state as $(f_p - f_n)$, which is proportional to the charge exchange amplitude. With the parameters used in Section 5, this suggests that $T=1$ states should be suppressed relative to the $T=0$ by at least a factor of ten. If the target has non-zero spin there are additional form factors, but we shall not go into these complications here. Since we have neglected nucleon spin flip, our discussion so far is not valid for magnetic transitions, an example of which will be treated at the end of this section.

Equation 7.4 can be considered as a very simple form of the distorted wave impulse approximation (DWIA) which is much used in lower energy nuclear physics. Here we assume that the transition is direct

from the ground to excited state, but we include the distortion of the initial and final proton waves in the optical potential of the residual nucleus. Replacing the distorted waves by plane ones gives identically the single scattering result of Equation 7.5, while calculating the distortion from the eikonal approximation to the one body optical potential leads to the Glauber result of Equation 7.4. It has been shown numerically that even for 156 MeV protons there is in fact little merit in resolving the optical potential exactly rather than using the eikonal results (118).

To evaluate Equation 7.4 it is convenient to let k lie in the z direction and q the x . For unpolarised cross-sections we can just sum the squares of such amplitudes, and predictions for polarisations of excited states can be made in terms of linear combinations of them. With this choice only states with $(\ell-m)$ even are excited, and the integral over the impact parameter angle can be done leaving only a Bessel transform of $J_m(qb)$ to be done numerically. In view of the large momentum transfers investigated in the Saclay experiment, the c.m. constraint is probably quite important. With little justification this can be introduced as a multiplicative factor in the same way as for elastic scattering (Equation 5.2). We look first at the case of ^{12}C . The transition form factor for the first level has been measured (119,120) out to $q^2 \sim 10\text{F}^{-2}$, but the situation with the next two levels is much less favourable. For small q , the transition form factors must behave like q^ℓ except for the monopole excitation where q^2 is appropriate. The experimental data on the form factors, assuming that Born approximation is valid, could be fit in the form

$$F_\ell = B Z q^n (1 - C q^2) e^{-d q^2} \quad 7.7.$$

where n is the appropriate power as discussed above. The values of the parameters are displayed in table 4; only in the 2^+ case did the data justify a non-zero C . F_ℓ must be multiplied by the c.m. factor before comparing with experiment.

In Figure 10 are shown the predicted cross-sections for the excitation of the first three levels in ^{12}C compared with the experimental data of Reference (3). The agreement is reasonable in the 2^+ case, the theory being 20% too high on the first maximum and too deep at the minima. It can easily be imagined that the inclusion of spin dependent amplitudes would improve this significantly. Furthermore, at this level of accuracy it is not clear how valid it is to extract the form factors in Born approximation. The situation for the 0^+ level is even more encouraging, but the 3^- seems disastrous. Apart from the large discrepancy at the maximum, there is no sign of the predicted minimum, apart from a slight change in slope. The behaviour at the maximum could be due to a poor determination of the electromagnetic transition, caused by insufficient energy resolution. In order to test this possibility, the parameters C and d were varied to fit the proton scattering in this region ($C = 0.088\text{F}^3$, $d = 0.62\text{F}^2$), but the predicted minimum still remains in Figure 10. Since it cannot be filled in significantly by energy resolution, nor by incoherent excitation of other nuclear levels (for example the wide state at 10.3 MeV), it is possibly the spin dependent amplitudes that should be invoked, but a detailed analysis is still lacking.

CORRECTIONS TO THE DWIA Correlations can come in in two distinct ways ; they can modify the distorted waves or they can allow two-step processes, such as exciting the 0^+ level in ^{12}C via the 2^+ . The effects of the Pauli principle and the deformation discussed for elastic scattering do not change the distortion greatly. Two-step processes promise to be much more important. In the Glauber calculation of the 2^+ level based on the $(p_{\frac{3}{2}})^{-1}(p_{\frac{1}{2}})$ model they are taken into account (68), but because of the limited configuration the model tends to underestimate grossly transition matrix elements, so that it is not surprising that the effects are small. A special case of such terms is the excitation of the 2^+ with $M=0$, being converted to $M=2$ by a second interaction. Second order processes can be investigated more thoroughly in a macroscopic collective model (82,101), assuming for example that ^{12}C is a vibrating liquid drop. Although crude, such a model reproduces tolerably the low-lying levels. The 2^+ and

3^- are ascribed to single surfon excitations, while the 0^+ is a two-surfon state. The form factors of the first two can be fit reasonably, but the magnitude of the third is too low. By treating the optical potential as a function of creation and annihilation operators in surfon space it is possible to predict the excitation of these levels, as well as the feedback of the inelastic scattering into the elastic channel (the deformation correction of the type shown in Figure 8). Working to only first order in the transition, providing the model reproduces the correct form factors, the results should be identical to the DWIA. The fit (101) to the first excited state of $^{208}\text{Pb}(3^-)$ is shown in Figure 11, where a nuclear radius somewhat larger than the electron scattering value was used on the naïve assumption that the neutron excess just fills higher orbitals. The agreement is a little less impressive when it is realised that the diffraction minima coincide quite closely with those suggested by a deformed black disc model. In contrast to elastic scattering, such a model also predicts that, to the extent that we can neglect the surface thickness, there should be no Coulomb distortion filling in the minima. This explains why the minima are much sharper for the excited state than for the ground state. In the vibrational model one can calculate plausibly the two-step process $\text{g.s.} \rightarrow 2^+ \rightarrow 3^-$ in ^{12}C , which must be added coherently to the direct amplitude (82). This displaces the theoretical minimum in the 3^- case from $q^2 \sim 5.3\text{F}^{-2}$ to a little over 6, without removing it, and it is expected the contribution of the 0^+ intermediate state would be even weaker (J. Friar, private communication). The suggestion of Alexander and Rinat (preprint) that this latter might weaken the minimum considerably was based on a too large $0^+ \rightarrow 3^-$ amplitude of a doubtful phase.

At lower proton energies the $J^P = 1^+, T = 1$ state at 15.11 MeV is clearly seen in the scattering from ^{12}C , being sharply peaked at small momentum transfers (121). This is a magnetic transition and corresponds to the spin flip of one of the target nucleons (apart from small orbital motion contributions). In order to obtain an amplitude which does not vanish in the forward direction, the projectile must undergo a spin flip also. In impulse approximation it is the amplitudes B and D which enter

$$F_n = B^{c.e.} \sigma_i \tau_j \langle n | \sigma_i \tau_j e^{i\mathbf{q} \cdot \mathbf{r}} | 0 \rangle \quad 7.8.$$

plus an analogous D term. Here M is the polarisation of the excited state, and because this has $T=1$ the B and D correspond to charge exchange. Now at low energies the np backward amplitude is very sharply peaked (113) due to single pion exchange, and this has just the correct quantum numbers to contribute to this transition. After taking distortion into account, one can explain reasonably well low energy proton data. Now compared to the spin non-flip amplitude, the pion exchange decreases like $1/k$, but at small angles the level can still be seen in the Saclay data. One can therefore determine a value for $B^{c.e.}$, and by using also the analogous $T=0$ level at 12.7 MeV, one can determine the corresponding isoscalar amplitude. By choosing the nuclear levels appropriately and, where necessary, using a polarised beam, it should be possible to deduce information about the spin and isospin structure of the nucleon-nucleon amplitude which can be fed into other multiple scattering calculations.

We have seen that for the strong transitions in ^{12}C we can calculate magnitudes reasonably well using the DWIA, and this leads to knowledge about $B(\text{EL})$ values and transition radii. For some of the higher excitations, about which we should have more experimental information soon, the reaction mechanism may involve dominantly two-step processes. If this be true, the results will be sensitive to correlations or transition rates between excited states.

8. - SUM RULES AND CORRELATIONS

It was one of the early hopes of Palevsky (122) that sum rules might provide a useful way of investigating two-nucleon correlations inside the nucleus. To see how this could come about, suppose that the high energy nucleon amplitudes are spin-independent and sufficiently weak for the single scattering approximation to be valid. Then the elastic p-A differential cross-section is proportional to the square of the one-body density, while in the same limit the cross-section for the excitation of some final nuclear state $|n\rangle$ is

$$\begin{aligned} \left. \frac{d\sigma}{d\Omega} \right|_n &= |\langle n | \sum f_i e^{i\mathbf{q} \cdot \mathbf{r}_i} | 0 \rangle|^2 \\ &= \langle 0 | \sum f_j^* e^{-i\mathbf{q} \cdot \mathbf{r}_j} | n \rangle \langle n | \sum f_i e^{i\mathbf{q} \cdot \mathbf{r}_i} | 0 \rangle \end{aligned} \quad 8.1.$$

We now assume that the incident proton energy is so high that all the important nuclear states are accessible, for then the total inelastic cross-section is the sum over all n except the ground state. Using closure, and taking for simplicity an isoscalar nucleus and equal proton and neutron amplitudes,

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{INELASTIC}} = A |F|^2 \left[1 - A (S(q))^2 + (A-1) \langle 0 | e^{i\mathbf{q} \cdot (\mathbf{r}_1 - \mathbf{r}_2)} | 0 \rangle \right] \quad 8.2.$$

Since this quantity depends upon the ground state expectation value of a two-body operator, it is in principle sensitive to two-nucleon correlations. In terms of the Fourier transform of the correlation function defined by Equation 4.11,

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{INELASTIC}} = A |F|^2 \left[1 - (S(q))^2 + (A-1) C(\mathbf{q}, \mathbf{q}) \right] \quad 8.3.$$

The same formula is valid for electron scattering with A replaced by Z .

Similar sum rules can be obtained including the effects of multiple scattering (99), but then the correlations play a dual rôle, influencing the size of the multiple scattering terms as well as the interference which is already present in Equation 8.3. Thus in a model calculation on ^{16}O , it has been shown (102) that the Pauli correlation decreases the inelastic proton cross-section by 20-30% for small q , but increases it slightly for large q . In the case of ^4He , the effects of the Pauli principle as well as any long-range deformation may be neglected and so this nucleus might be the most suitable for investigating the short-range nucleon-nucleon correlations. There is the complication of a strong c.m. correlation, but this could be useful if there is a significant interference between this and the dynamical correlations (91).

On quite general grounds $C(q, -q)$ tends to zero for both large and small q , and in the simple calculation to Kofoed-Hansen and Wilkin (99), who fitted the ^4He form factor including Jastrow factors, it could have a maximum value of the order of 13%. This is then enhanced for proton scattering by the $(A-1)$ factor to some 40%. Unfortunately this may be mainly due to the c.m. effect, in the sense that if one fits the form factor without correlations, but on a very extended [possibly overextended (123)] shell model basis ($1s+2s+\text{etc.}$), the differences between the correlation functions lead to only 10% effects, which are even further reduced by the multiple scatterings.

Unless the correlation is significantly stronger than that considered in Reference (99), we have to investigate few percent effects, so that it is pertinent to ask if the theory is accurate to this level. What is clear is that at present we cannot evaluate the theory to this level. In the case of deuterium, because of the c.m. constraint, we know the correlation function completely, and in Figure 12 is shown the break-up cross-section $p+d \rightarrow p+p+n$ at 1 GeV compared with the predictions of a simple Glauber calculation (124). There is not necessarily any disagreement between the two sets of data since the Brookhaven points sum only over the quasi-elastic peak,

and they are really a sum for fixed scattering angle rather than fixed momentum transfer. The correlations are most important for $q^2 \sim 2F^{-2}$, which is not excessively large. If C were a free parameter in fitting the data of Figure 12, one would try to increase it in this region. In fact the discrepancy is due to the neglect of the spin dependence of the nucleon amplitudes, which allows inelastic scattering in the forward direction even in impulse approximation. The moral is again clear ; we cannot hope to extract nuclear structure information unless the nucleon amplitudes are much better understood.

Energy weighted sum rules have not yet been applied to proton scattering. Apart from any of the difficulties discussed above, the subtraction of the pion production background would here be even more critical.

9. - CONCLUSIONS

Present-day theoretical calculations reproduce many of the features observed in proton-nucleus scattering, but fail in some of the details. It is fair to ask what the situation will be in a few years time when the data will be more numerous and of much better quality. To evaluate the predictions of the theoretical models seriously, we need first and foremost better nucleon-nucleon scattering data. It is commonly believed that at high energies spin effects become very small, but a recent experiment (126) at 6 GeV has found differences of up to a factor of two between proton-proton cross-sections with different spin alignments. Some information can be derived from nuclear targets, but it is pointless doing series of very precise measurements on nuclei unless there is a parallel program to study all ten complex amplitudes in the scattering from nucleons. Clearly this is the one big disadvantage of using protons as probes rather than pions.

Given the basic nucleon amplitudes, there are still many assumptions in the models described in Section 4 which should be investigated in greater detail. For example, are off-shell effects

important, or is the optical potential local ? The effects of binding and even the proper choice of kinematic variables for the single scattering approximation can only be answered when the dynamics of the target are included. The application of Faddeev equations to 1 GeV p-d scattering would be an enormous effort, and the effects that we are interested in would depend in detail upon the N-N potential (assuming it exists). Furthermore there are the mesonic degrees of freedom. It must be stressed that if small angle p-A scattering suffers severe complications from these effects the hopes of learning more about nuclear structure in this way will be dimmed.

Because of space limitations, this review has been concerned solely with (p,p') reactions and total cross-sections which are related to the elastic amplitude via unitarity. Quasi-elastic $(p,2p)$ reactions have been reviewed earlier (127), but there is now evidence for processes $(p,p\alpha)$ which might tell us about α clusters in the nucleus. Similarly the knockout of pions might tell us about the rôle of the pion or nucleon resonances in the nucleus. Less controversially, it has been shown that deuteron pick-up with high energy proton beams is a practical method of investigating nuclear levels of the residual nucleus with high precision at such large momentum transfers that we might reasonably expect new features to emerge.

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Table 1 Summary of total (T) and reaction (R) cross-sections of different nuclei for neutrons and protons. Note that for protons on heavy nuclei it is not always possible to extract a total cross-section because of the Coulomb problems.

Energy (MeV)	Targets	Cross-section	Reference
<u>Neutrons</u>			
380-1700	Be,C,Al,Fe,Ni,Cu, Zn,Ag,Sn,Pb,Bi,U	R,T	(7)
380-1700	D	T	(8)
380- 630	D,Be,C,O,Al,Cu,Sn, W,Pb,U	T	(9)
765	D,Li,C,O,Al,Cu,Sb, Pb	R,T	(10)
1400	Be,C,Al,Cu,Sn,Pb, Bi,U	R,T	(11)
<u>Protons</u>			
180- 560	D,He,C,O	T	(12)
220- 570	Be,B,C,O,Al,Fe,Cu, Ge,Sn,Pb	R	(13)
510-8000	D,Al	R,T	(14)
650	Be,C,O,Al,Cu,Sn,Pb,U	R,T	(15)
860	Be,C,Al,Cu,Sn,Pb	R,T	(16)
890	Al	R	(17)
900	C,Al,Cu,Sb,Pb	R,T	(18)
910	D,C,O	R,T	(19)
1000	He,Li,C,O,Pb	R,T	(20)

Table 2 Summary of proton-nucleus differential cross-section and polarisation data. In Column 5 are given the ranges of momentum transfer where the cross-section has been measured, while in Column 6 is to be found the angular range of the polarisation measurement.

Target	Ref.	E MeV	ΔE MeV	$\frac{d\sigma}{d\Omega} - q \text{ F}^{-1}$	P. θ Deg.	Comment a, b
^3He	(21)	580		1.6 -4.5		Recoil and coincidence experiment. Elastic scattering.
	(22)	600		1.5 -4.2		Recoil experiment. Elastic scattering
^4He	(23)	540			4 -42	Recoil and coincidence experiment. Polarisation in elastic scattering.
	(24)	587	< 20	0.4 -4.4		Recoil and coincidence experiment. Elastic scattering.
	(25)	725	20	0.55-2.17	4.5-18	MSE. Elastic scattering and polarisation
	(2)	1000	3	0.7 -5.5		MSE. Elastic scattering.
	(26)	1050	0.3 -2	0.45-6		MSE. Elastic scattering. No absolute normalisation.
^6Li	(27)	600		1.5 -3		Recoil experiment. Elastic scattering.
	(28)	1040	0.2 -0.6	0.3 -3.9		MSE. Elastic and 3^+ , 2.19 MeV level.
^9Be	(29)	635	> 20		3 -30	Polarisation of NPES.
	(25)	725	20	0.5 -1.6	4 -13	NPES and polarisation.
	(30)	970	> 20		3 -8	Polarisation of NPES.
	(31)	970	> 20		4 -12.3	Polarisation of NPES.
	(32)	1730	> 20	0.32-1.5	1.5-5	NPES and polarisation.

Table 2 Continued

Target	Ref.	E MeV	ΔE	MeV	$\frac{d\sigma}{d\Omega} - q \text{ F}^{-1}$	P. θ Deg.	Comment
^{12}C	(29)	635	> 20			9	One polarisation point of NPES.
	(33)	660	> 20		0.2 -1	2.1-10.7	MSE. NPES polarisation, A and R parameters.
	(25)	725	20		0.5 -1.6	4 -13	MSE, NPES, polarisation.
	(34)	950	> 20			7.2	Polarisation of NPES.
	(30)	970	> 20			4 - 8	Polarisation of NPES.
	(35)	970	> 20		0.2 -0.75	2 - 4.5	NPES and polarisation.
	(31)	970	> 20			4 -12.3	Polarisation of NPES.
	(2,36)	1000	3		0.9 -2.6		MSE. Elastic, 2^+ , 4.43 MeV level, inelastic.
^{16}O	(37)	1000	3.5		1.5 -4.2		MSE. Elastic, 2^+ , 4.43 MeV level.
	(3)	1040	0.16-0.25		0.5 -5		MSE. Elastic, 2^+ , 4.43 MeV, 0^+ , 7.65 MeV 3^- , 9.67 MeV, and other levels.
	(2,36)	1000	3		0.6 -4		MSE. Elastic, inelastic.
		1040	0.14-0.20		0.45-4.5		MSE. Elastic, 3^- , 6.13 MeV, 2^+ , 6.92 MeV, 2^- , 8.87 MeV, and other levels.
^{27}Al	(29)	635	> 20			9	One polarisation point of NPES.
	(25)	725	20		0.5 -1.6	4 -13	MSE, NPES and polarisation.
	(34)	950	> 20			7.2	Polarisation of NPES.
	(31)	970	> 20			4 -12.3	Polarisation of NPES.

Table 2 Continued

Target	Ref.	E MeV	ΔE	MeV	$\frac{d\sigma}{d\Omega} - q \text{ F}^{-1}$	P. θ Deg.	Comment
^{39}K	(38)	1000		1.8	0.75-2		MSE. Elastic.
^{40}Ca	(25)	725		20	0.5 -1.2	4 -10	MSE, NPES and polarisation.
	(38)	1000		1.8	0.75-2.6		MSE. Elastic.
Fe	(25)	725		20	0.5 -1.2	4 -10	MSE, NPES and polarisation.
^{58}Ni	(3)	1040	0.13-0.16		0.35-2.7		MSE. Elastic, 2^+ , 1.45 MeV, 4^+ , 2.45 MeV and other levels.
Ta	(25)	725		20	0.5 -1.2	4 -10	MSE, NPES and polarisation.
Pb	(29)	635	> 20			9	One polarisation point of NPES.
^{208}Pb	(25)	970	> 20			4 -12.3	Polarisation of NPES.
	(3)	1040	0.10-0.14		0.35-2.7		MSE. Elastic, 3^- , 2.62 MeV level.
^{209}Bi	(29)	635	> 20			9	One polarisation point of NPES.

^a MSE = magnetic spectrometer experiment.

^b NPES = non-pure elastic scattering.

Table 3 Parameters of various experimental arrangements.

Experiment	Brookhaven	Gatchina	SPES1 Saclay	HRS ^a LAMPF
year	1966	1972	1972	(1974)
D_s cm/%	~4.5	18	13.7	18.2
Kinematical compensation	Partly	No	Yes	Yes
Energy loss mode	No	No	Yes	Yes
ΔE keV	3000 - 2000	3500 - 1800	250 - 100	120 - 30
$d\Omega$ Sr	7.5×10^{-5}	2.3×10^{-4}	3.4×10^{-3}	3.6×10^{-3}
AR deg.	$\pm 0.2 - \pm 0.1$	± 0.25	$\pm 0.5 - \pm 0.1$	± 0.05
Q protons/sec.	2×10^9	5×10^{11}	5×10^{10}	$6 \times 10^{13} - 6 \cdot 10^{12}$
$(d\sigma/d\Omega)T$	1mb/Sr	20 μ b/Sr	1 μ b/Sr	1 - 10nb/Sr

^a Calculated values.

Table 4 Form factors for the first three levels in ^{12}C fit to the experimental data of References (119,120) using Equation 7.7 and assuming Born approximation

E MeV	$J^P ; T$	B_{F^n}	C_{F^2}	d_{F^2}
4.44	$2^+ ; 0$	0.240	0.130	0.63
7.65	$0^+ ; 0$	0.167	-	0.99
9.67	$3^- ; 0$	0.134	-	0.77

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FIGURE CAPTIONS

- Figure 1 Focusing and dispersion properties of a magnetic spectrometer.
- Figure 2 Block diagram of a single arm scattering experiment ;
OB object block, AM analyser magnet, TB tuning block,
TA target, BME beam measurer, BMO beam monitor,
SM spectrometer magnet, PL particle localisation.
- Figure 3 Energy excitation spectrum (3) of 1.04 GeV protons
scattered at 10° by ^{208}Pb .
- Figure 4 Energy excitation spectrum from Saclay (3) of 1.04 GeV
protons scattered at 7° by ^{12}C with 130 keV per channel.
The elastic peak has been suppressed, and the first three
excited levels ($2^+, 0^+, 3^-$) overshoot the vertical scale.
Level structure is seen clearly up to 30 MeV. The posi-
tion of the broad background enhancement indicated from
7 to 50 MeV seems to follow the kinematics of quasi-free
 $p-\alpha$ scattering when the angle is varied !
- Figure 5 Elastic $p-^4\text{He}$ scattering at 1 GeV, the data coming from
Brookhaven (2) (shaded circles) and Saclay (26) (open
circles), the latter being arbitrarily normalised. The
theoretical curves are the results of single scattering (1),
single plus double (2), etc., from the simple Glauber model
of Equation 5.2, using $\bar{\sigma} = 4.4F^2$, $\bar{p}^2 = 0.203F^2$, $\bar{\rho} = -0.325$.
- Figure 6 Saclay data on elastic $p-^{208}\text{Pb}$ scattering at 1.04 GeV,
compared with the theoretical predictions of L. Lesniak
(private communication) using a Glauber model with a
Woods-Saxon distribution with $R = 6.5F$, $c = 0.545F$. The
importance of the Coulomb interaction in filling the minima
is readily seen.
- Figure 7 Difference in cross-section for the scattering of 1 GeV
protons from ^{39}K and ^{40}Ca , data being taken from the
Leningrad work (38). The accompanying Glauber calculation
uses a parabolic Fermi distribution (72) with $R = 3.741F$,
 $a = 0.585F$, $W = -0.204$ for ^{39}K and $R = 3.725F$, $a = 0.591$,
 $W = -0.169$ for ^{40}Ca .

- Figure 8 Elastic $p-^{12}\text{C}$ scattering at 1 GeV, with data from Brookhaven (2) (triangles), Leningrad (37) (crosses) and Saclay (3) (circles). The Glauber calculations show the effect of introducing first the Pauli correlation (curve 2), and then the deformation (curve 3), using Hartree-Fock densities.
- Figure 9 Polarisation P in elastic $p-^4\text{He}$ scattering at 544 MeV. The Virginia data (24) are compared with the predictions of Equations 6.5 and 6.3, assuming that the polarisation is maximal.
- Figure 10 The cross-section for the excitation of the first three levels in ^{12}C of angular moments 2, 0 and 3. The Saclay data are compared with the DWIA predictions of Equation 7.4, where the transition form factors are taken from electron scattering (119,120). For the 3^- case a fit using Equation 3.7 was tried in the region of the first maximum of the proton data. For convenience of presentation the $L=0$ results are multiplied by a factor 100, and the $L=3$ divided by 10.
- Figure 11 Excitation of the $3^-(2.6 \text{ MeV})$ level in ^{208}Pb with 1.04 GeV protons. The Saclay data are compared with the predictions based upon a vibrational model, with $R=6.85\text{F}$, $c=0.57\text{F}$ and deformation parameter $\beta_3=0.1$.
- Figure 12 Differential cross-section for deuteron break-up by 1 GeV incident protons, as a function of the momentum transfer to the faster proton. Courter (125) (closed circles) and preliminary bubble chamber (124) (open circles) data are compared with the predictions (124) of Glauber theory neglecting spin.

FIG 1

FIG 2

FIG 3

FIG 4

FIG 5

FIG 6

FIG 7

FIG 8

FIG 9

FIG 10

FIG 11

FIG 12